Massachusetts Institute of Technology
Department of Physics
8.022 Fall 2004
Assignment 8: RLC circuits; AC circuits
Due date: Friday, Nov 19th

1. Purcell 8.4(10pts).
2. Purcell 8.7(15pts).
3. Purcell 8.8(10pts).
4. Purcell 8.12(10pts).
5. Purcell 8.13(15pts).
6. Charge in series RLC.

   In this problem, we will look at the behavior of $q(t)$, the charge on the capacitor in a series RLC circuit driven by a periodic EMF $\mathcal{E} = \mathcal{E}_0 \cos(\omega t)$.

   (a) Derive a differential equation showing the time evolution of the charge on the capacitor. Don't solve it!

   (b) Use the known solution for I(t) and the definition $I = \frac{dq}{dt}$ to find an expression for $q(t)$. Give the complex charge, $\tilde{q}(t)$, as well as the physical charge on the capacitor. What is the amplitude q_0 of q(t)?

   (c) Show that the maximum charge amplitude is at $\omega = \sqrt{\omega_0^2 - R^2/2L^2}$

7. Impedance of a RLC circuit.

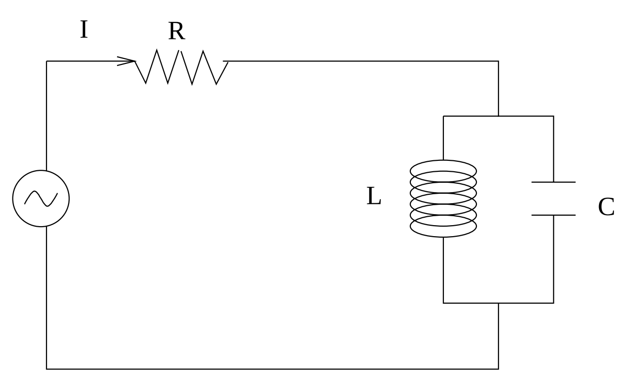

Figure 1: An AC driven RLC circuit.

   (a) What is the complex impedance of the combination of the circuit elements, R, L, and C shown in Fig 1? The AC voltage is given as $V_0 cos(\omega t)$ Please rationalize

the expressiion into separate real and imaginary parts. (Note: impedance in parallel behave in just the same way as resistor in parallel.)
(b) What is the current I, (the actual and not the complex current) flowing through the circuit? Give an expression for the phase angle.
(c)Explain the low and high frequency behavior of the phase shift of the current in terms of the currents through each of the circuit elements.

8. Levitating ring demonstration.

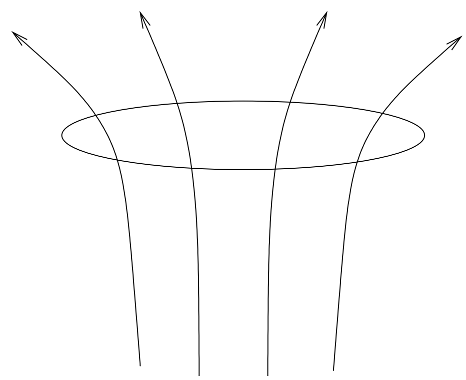

Figure 2: Sideview of the loop in the B field.

A conducting circular loop of radius a is levitated by a magnetic field (created by a coil which is not shown). the loop is centered on the axis of symmetry of the coil, so you may consider the component of the field pointing radially outwards from the center of the loop, B_r, as being constant around the loop. Two cases are sonsidered here. First you will consider what happens when the field strength is increased at a constant rate. Second, you will determine what happens in the AC case.

(a) If the field strength is increasing at a constant reate, a current will start to flow in the loop. Draw a diagram of the "top view" to show the direction of the current.

(b) The increasing magnetic field creates an increasing magnetic flux $d\phi/dt$ through the loop. The inductance of the loop is L, and the resistance of the loop R. Write down the correct differential equation describing the current flow in the loop. Your equation should incolve $\frac{d\phi}{dt}$,$\frac{dI}{dt}$,I, L,R and c. (Besides the external magnetic flux, the current will also produce a magnetic flux through the loop.)

(c) Write down the value for the current around the loop as $t \rightarrow \infty$.

(d) Now consider that the magnetic field oscillates in time. This means that the flux through the loop oscillates as $\phi = \phi_0 cos(\omega t)$. What is the current in the loop? What is the phase of the current compared to the phase of the flux? (Hint: Use complex number and note that $\phi = Re[\phi_0 e^{i\omega t}] = Re[\tilde{\phi}]$.Also use the fact that $\frac{d\tilde{\phi}}{dt} = i\omega\tilde{\phi}$.

(e) What is the force on the loop? Which way does it point? Use the expression for the current that you derived in part (d). Remember that the magnetic field is oscillating so that B_z oscillates as $B_z = B_{z0}cos(\omega t)$ and B_r oscillates as $B_r = B_{r0}cos(\omega t)$. For what value of hte resistance is the force maximized?