Massachusetts Institute of Technology
Department of Physics
Physics 8.022 – Fall 2002
Quiz#2

- Total points in the quiz are 100.
- This is a closed book and closed notes exam. An equations table is given to you below.
- No programmable, plotting, integration/differentiation capable calculators are allowed.

Currents, Magnetism and Relativity Formulae

Ohm's Law: $\vec{J} = \sigma\vec{E}, V = IR$

Capacitance: $Q = CV$, $U = \frac{1}{2}CV^2$

Magnetic charges: $\vec{\nabla} \cdot \vec{B} = 0$

Biot-Savart's Law: $d\vec{B} = \frac{Id\vec{l}\times\hat{r}}{cr^2}$

Ampere's Law: $\oint_C \vec{B} \cdot d\vec{l} = \frac{4\pi}{c} I_{encl} = \frac{4\pi}{c} \int_S \vec{J} \cdot d\vec{a}$, $\vec{\nabla} \times \vec{B} = \frac{4\pi}{c}\vec{J}$

Faraday's Law: $\mathcal{E} = \oint_C \vec{E} \cdot d\vec{l} = -\frac{1}{c}\frac{d\Phi}{dt}, \vec{\nabla} \times \vec{E} = -\frac{1}{c}\frac{\partial\vec{B}}{\partial t}$

Mutual Inductance: $M_{12} = M_{21} = \frac{\Phi_{21}}{cI_1}, \mathcal{E}_{21} = -M_{21}\frac{dI_1}{dt}$

Self Inductance: $L = \frac{\Phi}{cI}, \mathcal{E} = -L\frac{dI}{dt}$

Magnetic Field Energy Density: $\frac{dU_B}{dv} = u_B = \frac{B^2}{8\pi}$

Relativistic Transformations:
All primed quantities measured in the frame F' which is moving in the positive x direction with velocity $u = \beta c$ as seen from F:

$$x' = \gamma(x - \beta ct) \quad p' = \gamma(p - \beta\frac{E}{c})$$
$$t' = \gamma(t - \beta\frac{x}{c}) \quad E' = \gamma(E - \beta cp)$$
$$E'_x = E_x \qquad E'_y = \gamma(E_y - \beta B_z) \qquad E'_z = \gamma(E_z + \beta B_y)$$
$$B'_x = B_x \qquad B'_y = \gamma(B_y + \beta E_z) \qquad B'_z = \gamma(B_z - \beta E_y)$$

Relativistic Mass, Energy: $m = \gamma m_0, E = mc^2$

Relativistic Doppler Effect: $f_o = [\frac{1-(u/c)}{1+(u/c)}]^{1/2} f_s$, u along the line joining o and s and u positive when s recedes from o.

1. **Short answers**

(a) SAT type question: charge is to electric field is to capacitor as current is to magnetic field is to ?

(b) A massless particle has energy E. Find its velocity. What is the enegy E' in the frame at which the particle is at rest?

(c) You are given a black box with two terminals labelled A and B. You wish to model the box as a Thevenin equavialent circuit with a battery V_{th} in series with a resistor R_{th}. You find V_{th} by measuring the potential difference between A and B. How do you find R_{th}?

(d) What did Einstein mean by postulating that the same laws of physics are valid for all frames moving with constant speed relative to each other: (a) laws of nature are the same because physical quantities are the same (b) laws of nature have the same form (invariant) but physical quantities transform or (c) none of the above.

(e) A speeding (at relativistic velocities) motorist drives toward an intersection where the traffic light signals permanently *green* (λ_{green}=530nm). Will he see the traffic light shifting (a) toward the blue (λ_{blue}=450nm) ("blueshift") (b) toward the *red* (λ_{red}=700nm) ("redshift") or (c) remaining unchanged?

(f) Two straight parallel wires carry currents $2I$ and I that run in opposite directions in them. The magnetic force acting on the first wire is such that it (a) will be repelled away from the second wire, (b) will be attracted toward the second wire, (c) will remain where it is, (d) none of the above.

2. **Inductor with constant current source**

Two resistors and an inductor are connected to a constant current source as shown in figure 1. The **constant** current source provides a current I no matter what voltage is required. The switch S is closed at $t = 0$; before then, no currents flow.

(a) From qualitative arguments, find the currents at $t = 0$, i.e., $I_1(t = 0) = I_{o1}$ and $I_2(t = 0) = I_{o2}$ as well as at $t = \infty$, i.e., $I_1(t = \infty) = I_{\infty 1}$ and $I_2(t = \infty) = I_{\infty 2}$.

(b) Use Kirkhhoff's laws to write down two equations which relate $I_1(t)$ and $I_2(t)$ and their time derivatives.

(c) Assume $I_1(t) = \alpha + \beta e^{-\lambda_1 t}$ and $I_2(t) = \gamma + \delta e^{-\lambda_2 t}$ where $\alpha, \beta, \gamma, \delta, \lambda_1$ and λ_2 are constants. Use your result from 2a to first find α, β, γ and δ. Then use your result from 2b to find λ_1 and λ_2.

(d) Find the potential between A and B, $V_{AB}(t)$ (where $V_{AB} = V_B - V_A$).

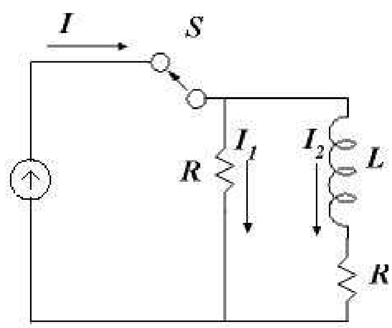

Figure 1

(e) After a long time, S is opened and the current is dissipated through the resistors. Find the total energy dissipated through the resistors in terms of I, R, L and constants.

3. **A different kind of coaxial line**

A coaxial line is made up of an inner (filled) conductor of radius R_a and an outer, infinitesimally thin conductor of radius R_b as shown in figure 2. The two conductors carry equal and opposite current I which, however, for the inner conductor is **not** uniformly distributed over its cross-sectional area but instead it is given by the current density $\vec{J} = J_0 exp(-\frac{\rho^2}{R_a^2})\hat{k}$.

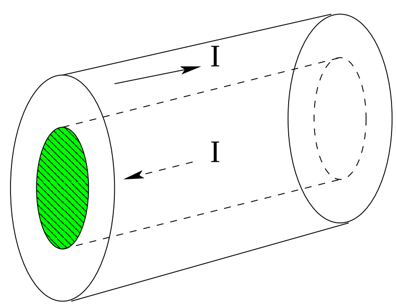

Figure 2

(a) Express I in terms of J_0, R_a and constants.

(b) Find the $\vec{B}$ field at any point in space.

(c) Give an expression for the magnetic field energy of space and briefly describe (1-2 sentenses, 1-2 formulae) how you can calculate the self-inductance of this coaxial line. **DO NOT** calculate either of them.

4. **Induced electric field**

An electric field has the form $\vec{E} = Ay\hat{\imath}$ where A is a constant.

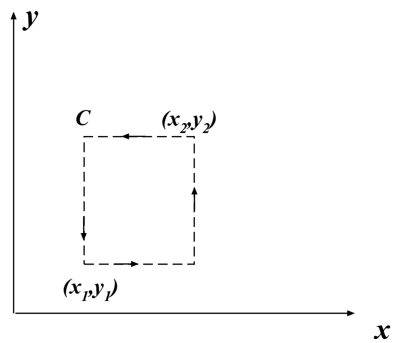

Figure 3

(a) Find $\oint_{\mathcal{C}} \vec{E} \cdot d\vec{l}$ for the curve $\mathcal{C}$ shown in figure 3.

(b) Assume there is a uniform in space magnetic field $\vec{B} = B(t)\hat{k}$. Find the flux through $\mathcal{S}$, the surface bounded by $\mathcal{C}$: $\Phi = \int_{\mathcal{S}} \vec{B} \cdot d\vec{a}$.

(c) Use Faraday's law to find $B(t)$.

(d) Evaluate $\vec{\nabla} \times \vec{E}$ and show that $\vec{\nabla} \times \vec{E} = -\frac{1}{c}\frac{d\vec{B}}{dt}$.

(e) Can the given electric field be due to the changing magnetic field alone?