Hints to Assignment #9 -- 8.022

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[1] Hollow wire.

We have worked on a similar problem in electrostatics by considering a hollow sphere. The name of the game here is **superposition**. In addition make sure you work in cylindrical coordinates and you do invoke the units vectors in this system.

- Assume that the current-free region results from the addition of a wire (radius a) carrying current **J** and another wire (of same radius a) carrying current **-J**.
- Pick a random point P inside the hollow region and express the field there as the superposition of two wires, one of radius R carrying current **J** and another one of radius a carrying current **-J**.
- There is one vector identify that is illuminating: phi^=k^ X rho^ where these are the three unit vectors of the cylindrical system.

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[2] Emf in a loop.

- Find the magnetic fields due to the two wires at any distance and invoke the superposition principle to find the total field across the loop.
- Perform a surface integral of B on the loop. Watch out, B changes with distance.
- Emf=-dPhi/dt and all the dPhi comes from the varrying I's.
- Convince yourself that all currents induced are in agreement with Lenz's law!

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[4] Time-varying electromotive force (Purcell 7.14).

Here are the steps that I see in this problem:

- Find (instantaneous) EMF from Faraday
- Find (instantaneous) current in circuit
- Find (instantaneous) magnetic force acting against the velocity
- Construct a differential eqn for the velocity using Newton's law- solve it with respect to u with the known initial conditions, u(t=0)=u_0, assume x=0 at t=0
- If you know u(t) you can find x(t)
- From the instantaneous value of current I(t) you can find the power spent on the resistor and after integration the total energy spent on it. You should find [m (u_o)^2]/2 the initial kinetic energy of the rod!

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[5] Charge moved by electromotive force (Purcell 7.18).

You have all the basic ingredients to show this interesting feature of flux "deposition" on a loop:

- Use Faraday to find the EMF -- watch out the flux coupling is N times the flux of one loop.
- Find current in the loop (I=EMF/R), insert it in Faraday
- Integrate both parts w/r/t time (from 0 when B_o to tau when B=0)

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[6] Angular momentum and electromagnetic fields (Purcell 7.22).

Very interesting problem as it shows that E/M field has angular momentum (and thus momentum) as well as how magnetic flux deposited on an atom may excite an electron in an atom (i.e., change its quantum numbers).

- Realize that azimuthal electric field sets up in order to make up for lost flux. Int(E dl)=-(1/c) d(Phi)/dt
- E field exerts a force on q and thus a torque around loop's axis: tau=R q E
- Newton's law for rigid body motion: tau=dL/dt where L is the angular momentum (work out directions of vectors prior to dealing with magnitudes) Integrate both sides (w/r/t time) to find out Delta L.
- recall that L=I omega where I is the moment of inertia Int(dm r^2) which for our charge q of mass m in circular motion will be mR^2. Substitute in previous result to find omega.

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[7] Mutual inductance of coaxial solenoids (Purcell 7.21).

The general strategy in finding mutual/self inductances is identifying the knows parts in the following two equations:

- Faraday: EMF = - (1/c) d(Phi)/dt and EMF = - M dI/dt By equating the RHS you have an expression for M. WATCH OUT in finding the flux linkage.
- Let's find the M12, i.e., the mutual inductance of the (inner) coil 1 due to the outer one (2). The equations from the above step read: (1/c) d(Phi12)/dt = M12 dI2/dt
- Find Phi12, i.e., the TOTAL flux on 1 due to field of 2. This will be Phi12=N1*(4pi/c)*(N2/b2)*I2*(pi*a1^2). Pick it from here.
- It AIN'T OVER yet. Convince yourself by REPEATING the above steps that M21=M12.

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[8] Coaxial conductors.

This is a very useful number for anyone wishing to improve his/her speaker's quality: self-inductunce of the cable. You have two options:

- Find the flux coupling on the suggested rectangle and calculate how it varies as a function of time when current varies. Once you find Phi(t) you can find EMF=-L di/dt and thus L. The integral for finding Phi is identical to problem [5]
- An alternative would be to find the energy of the magnetic field THROUGHOUT THE ENTIRE SPACE (there are three regions that this idealized coax creates) and then intergrate to find the energy. Once you have the energy you can double it, divide by I^2 and thus find L. Is it the same as in the previous approach?

In approach #1 where we calculate the flux coupling, we arbitrarily select a rectangular loop hanging somewhere from the top of the wire and living in the space in between the conductor. But we could have hung INFINITE such loops all around the conductor's outer shell over a 2*pi geometry. Are we calculating the correct flux coupling? Do we have to imagine all such loops similar to the one suggested and add them up (i.e., by performing an integration over azimuth angle). Have we missed something?