MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics

# Problem Solving 6: Magnetic Force & Torque

## OBJECTIVES

1. To look at the behavior of a charged particle in a uniform magnetic field by studying the operation of a mass spectrometer

2. To calculate the torque on a rectangular loop of current-carrying wire sitting in an external magnetic field.

3. To define the magnetic dipole moment of a loop of current-carrying wire and write the torque on the loop in terms of that vector and the external magnetic field.

**REFERENCE:** Sections 8.3 – 8.4, 8.02 Course Notes.

## Mass Spectrometer

A mass spectrometer consists of an ionizer, which strips (ideally) a single electron from an atom whose mass you want to measure, an acceleration region, and a deflection region, as pictured in Fig. 1.

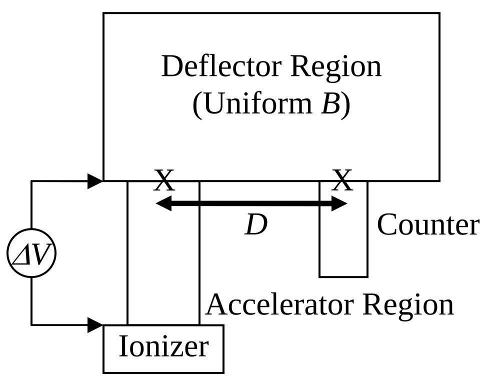



**Figure 1:** A mass spectrometer.

In Figure 1, the ions exit the ionizer at essentially zero velocity. They are accelerated by a potential difference through the accelerator region, where they enter the deflector region through a small orifice (X). The deflector region has a uniform B field which bends the ions around through another small orifice into the counter, where the ions are counted. By scanning the accelerating voltage $\Delta V$ a range of masses (or, more accurately, mass to charge ratios) can be measured, to determine the content of some unknown gas.

**Question 1 (*Answer this and subsequent questions on the tear-off sheet at the end*):** What should the polarity of the potential difference $\Delta V$ be (should it be positive at the top near the deflector or at the bottom near the ionizer)?

**Question 2:** In what direction should the magnetic field be pointed to guide the ions into the counter?

**Question 3:** Find an expression for the kinetic energy of the ions at the instant they enter the deflector region.

**Question 4:** What path do the ions follow in the deflector region? Derive an expression for the magnetic field that is needed to make sure that ions of mass $m$ end up in the counter (a distance $D$ away from where they enter the deflector region).

**Question 5:** About what potential $\Delta V$ is needed to get singly ionized carbon-12 ions into the counter if $B = 1$ T and $D = 20$ cm? You can assume that protons and neutrons have about the same mass given by $mc^2 \sim 1$ GeV.

## Magnetic Dipole Moment

In class we determined that a planar loop of area $A$ (with unit vector $\hat{\mathbf{n}}$ normal to the loop) and carrying current $I$, has a magnetic dipole moment $\vec{\boldsymbol{\mu}}$ given by:

$$\vec{\boldsymbol{\mu}} \equiv I\vec{\mathbf{A}} = IA\hat{\mathbf{n}}$$

The normal $\hat{\mathbf{n}}$ points in a direction defined by your thumb when you curl the fingers of your right hand in the direction of the current in the loop.

We calculated the torque on such a loop in a uniform magnetic field $\vec{\mathbf{B}}_{\text{ext}}$ to be

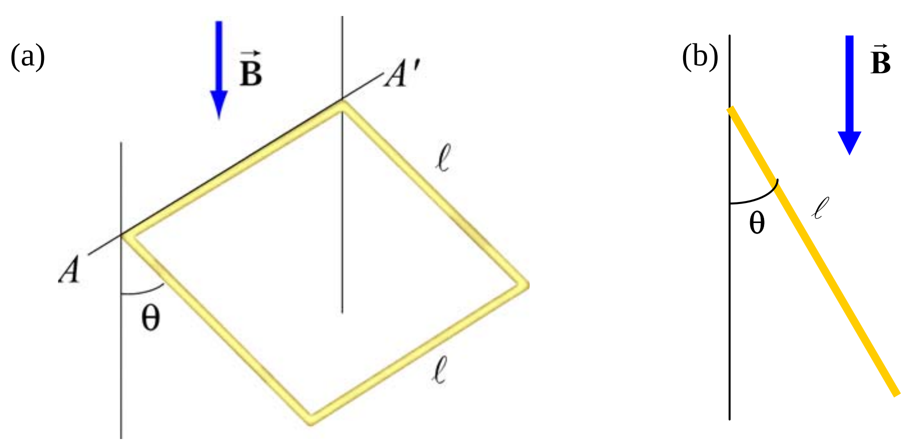
$$\vec{\boldsymbol{\tau}}_{\text{magnetic}} = \vec{\boldsymbol{\mu}} \times \vec{\mathbf{B}}_{\text{ext}}\,.$$

and the force on such a loop in a non-uniform magnetic field $\vec{\mathbf{B}}_{\text{ext}}$ to be:

$$\vec{\mathbf{F}}_{magnetic} = \left(\vec{\boldsymbol{\mu}} \cdot \vec{\nabla}\right)\vec{\mathbf{B}}_{\text{ext}}$$

(note that this evaluates to 0 if $\vec{\mathbf{B}}_{\text{ext}}$ is uniform – dipoles do not feel forces in uniform external fields).

**Problem:** A square loop of wire, of length $\ell$ on each side, pivots about an axis AA' that corresponds to a horizontal side of the square, as shown in Figure 4. A magnetic field of magnitude $B$ is directed vertically downward, and uniformly fills the region in the vicinity of the loop. A current $I$ flows around the loop.



**Question 6:** Calculate the magnitude of the torque on this loop of wire in terms of the quantities given, using our expressions above.

**Question 7:** In what direction does the current need to flow in order "levitate" the coil again the force of gravity (clockwise or counterclockwise viewed from above)?

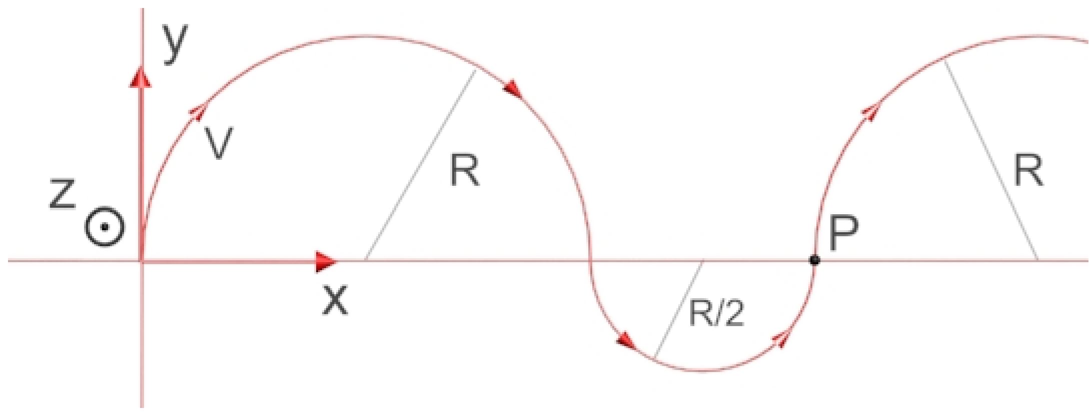
**Question 8:** Suppose that the loop ($\ell$ = 1 m) is essentially massless, but that a small child ($m$ = 20 kg) wants to hang from the bottom rung of the loop and be supported at θ = 45º. If we can push $I$ = 100 A through the loop, about how large a B field will we need to support the child?

**Question 9:** Now suppose the child starts fidgeting, causing the angle to slightly change. If the deviation is initially small, will the forces tend to cause the motion to run to larger excursions (i.e. to fall to θ = 0º or to snap up to θ = 90º) or will they tend to restore to 45º? If the former, about how long will it take to fall/rise? If the latter, what is the frequency of small oscillations of the angle about θ = 45º? HINT: You can expand the trigonometric functions about 45º using $\sin(x+y)=\sin x\cos y+\sin y\cos x$ and $\cos(x+y)=\cos x\cos y-\sin x\sin y$ and then use small angle identities sin(x) ~ x and cos(x) ~ 1. At 45º the two components of the torque are equal and opposite, so rewrite the total torque in terms of gravity g and length $\ell$. And, finally, the moment of inertia of a point mass a distance $\ell$ from a pivot is $m\ell^2$.

**Question 10:** If instead of balancing the child at 45º they wanted to ride at 60º, is it better to keep the field vertical or to switch to horizontal (i.e. which requires smaller B)?

## Sample Exam Question (If time, try to do this by yourself, closed notes)

A charge of mass $m$ and charge $q > 0$ is at the origin at $t = 0$ and moving upward with velocity $\vec{\mathbf{V}} = V\,\hat{\mathbf{j}}$. Its subsequent trajectory is shown in the sketch. The magnitude of the velocity $V = \left|\vec{\mathbf{V}}\right|$ is always the same, although the direction of $\vec{\mathbf{V}}$ changes in time.




(a) For $y > 0$, this positive charge is moving in a constant magnetic field which is either into the page or out of the page. Is that magnetic field for $y > 0$ into or out of the page?

(b) Derive an expression for the magnitude of the magnetic field for $y > 0$ in terms of the given quantities, that is in term of $q$, $m$, $R$, and $V$.

(c) For $y < 0$, the charge is moving in a *different* constant magnetic field. Is that field for $y < 0$ into or out of the page? What is the magnitude of that magnetic field in terms of in term of $q$, $m$, $R$, and $V$?

(d) How long does it take the charge to move from the origin to point $P$ (see sketch) along the x-axis? Give your answer in terms of the given quantities.

**MASSACHUSETTS INSTITUTE OF TECHNOLOGY**
**Department of Physics**

**Tear off this page and turn it in at the end of class !!!!**

**Note: Writing in the name of a student who is not present is a COD offense.**

# Problem Solving 7: Magnetic Force & Torque

Group ______________________________________ **(e.g. L02 6A Please Fill Out)**

Names ______________________________________

______________________________________

______________________________________

**Question 1:** What should the polarity of the potential difference $\Delta V$ be?

**Question 2:** In what direction should the magnetic field be pointed?

**Question 3:** Find the kinetic energy of the ions when they enter the deflector region.

**Question 4**: What path do the ions follow in the deflector region? Derive an expression for the magnetic field that is needed to make sure that ions of mass $m$ end up in the counter (a distance $D$ away from where they enter the deflector region).

**Question 5:** About what potential $\Delta V$ is needed to get singly ionized carbon-12 ions into the counter if $B = 1$ T and $D = 20$ cm? You can assume that protons and neutrons have about the same mass given by $mc^2 \sim 1$ GeV.

**Question 6:** Calculate the magnitude of the torque on this loop of wire in terms of the quantities given, using our expressions above.

**Question 7:** In what direction does the current need to flow in order "levitate" the coil again the force of gravity (clockwise or counterclockwise viewed from above)?

**Question 8:** Suppose that the loop ($\ell$ = 1 m) is essentially massless, but that a small child ($m$ = 20 kg) wants to hang from the bottom rung of the loop and be supported at θ = 45º. If we can push $I$ = 100 A through the loop, about how large a B field will we need to support the child?

**Question 9:** Now suppose the child starts fidgeting, causing the angle to slightly change. If the deviation is initially small, will the forces tend to cause the motion to run to larger excursions (i.e. to fall to θ = 0º or to snap up to θ = 90º) or will they tend to restore to 45º? If the former, about how long will it take to fall/rise? If the latter, what is the frequency of small oscillations of the angle about θ = 45º?

**Question 10:** If instead of balancing the child at 45º they wanted to ride at 60º, is it better to keep the field vertical or to switch it to horizontal (i.e. which requires smaller B)?