Let's consider examples of our principle that the external torque about a point, s, causes the angular momentum of a system to change about s.

We've examined central force problems in which we chose the point s to be the central point in which there was no torque.

Now, as examples, let's look at a case where we have a pivoted object.

So I could take an example of an object.

Let's see, this will be overhead view of a ring of radius r.

And I'm going to have a mass coming in.

I'll call this ring mass m1, this object m2.

This object is coming in with an initial velocity, and this is my pivot point.

Now, when the object m2 is hitting the ring, we have a force, F2 on 1.

This is the collision.

Here's our pivot point.

And when this hits the ring, we'll have some type of pivot force.

I'm just going to denote the pivot force.

We're not quite sure what it will point at, but I'll just say F pivot, for the moment, is holding this point in place.

At the same time, the object will have a force, F1,2, acting on the object.

Here is our pivot point, p.

And now, what point should we-- should we choose to see if there's no torque [? about? ?] Suppose we choose the pivot point.

Well-- p.

Well, clearly, the pivot force has no torque about the pivot, because the vector from the pivot point to where the pivot force is acting is 0.

Remember, pivot forces have no torque about the pivot point.

However, this collision force will produce a torque about the pivot.

And that's-- the angular momentum of the ring is not constant, because the ring will start to rotate.

Similarly, this object is reversing-- will reverse directions, or will do something due to the collision.

And again, this force will produce a torque that's equal and opposite to the torque on the ring.

This is the torque on the particle.

But if we make our system equal to the particle and the ring-- so we'll call this, our system is now both the particle and the ring-- then these torques are internal, and because they're equal and opposite forces, the rs vector is exactly the same rs vector.

The internal torques cancel in pairs.

The pivot force produces no torque.

So the torque on the system about the pivot is 0.

And that tells us that the angular momentum of the system about that pivot point, initially, will be equal to the angular momentum about that pivot point, finally, if we take two initial and final states.

In particular, suppose our initial state is-- here's the pivot.

Our object is coming in, that was m2 vi.

The moment arm is r.

And our vector from here to the object, rs initial, has a moment component that way.

If we put these vectors tail to tail and figure out that the angular momentum is pointing in this direction, l initial i, and the moment arm is r, then the initial angular momentum about this pivot point is just due to this moving object.

So that's m2 vi, and the moment arm is r, and we'll denote its direction that way.

Now, the final angular momentum-- let's imagine that it sticks.

So we have m2, pivot, and now our ring is going to be rotating with some omega final because this object hits it.

And notice that this object is a distance root 2 r from the pivot point.

So the final angular momentum can be two different pieces.

You can think of this as a system, where we have I of the system about p times omega final, because now it's just a rigid body.

And the angular momentum of the system is consisting of two pieces.

It's the angular momentum of the ring about the pivot, plus the angular momentum of the particle about the pivot, times omega final.

Now, this is the center of mass.

This is the pivot point, p.

The angular momentum of the ring is the angular momentum about the center of-- we'll use the parallel axis theorem-- angular momentum about the center of mass, plus the distance from the center of mass to the parallel axis, which is a distance, r.

So the first piece is Icm ring plus m r squared.

And the angular momentum about the particle, this is going in a circle of radius r2-- root 2r, so when we square that-- by the way this was, mass of the ring was 1.

Mass of the particle was m2 times r squared, which is this distance squared, which is 2 r squared times omega final.

The moment of inertia of the ring is m1 r squared about the center of mass.

We have the factor 2.

We have another factor of 2.

And so we get m1 plus m2 times a factor 2 r squared omega final, which we can call omega f, pointing out of the board.

And so now, we have an angular momentum condition, which is that m2 vi r equals 2 m1 plus m2 r squared omega final.

And that is the statement that the initial angular momentum of our system-- the ring is at rest here-- is equal to the final angular momentum of the system, where m2 is stuck to the ring, and they're all rotating about this pivot point with omega final.

That's the angular momentum of the system about p, omega final.

This is the initial angular momentum.

And so I can conclude that omega final is m2 vi over 2 m1 plus m2 times r.