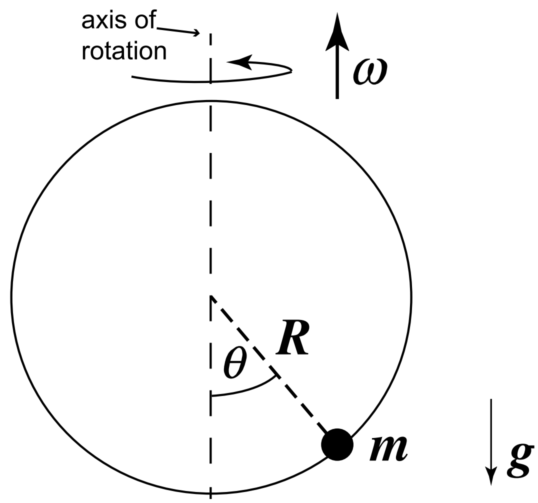
Problem Set 3

1. **Bead on a Rotating Hoop**

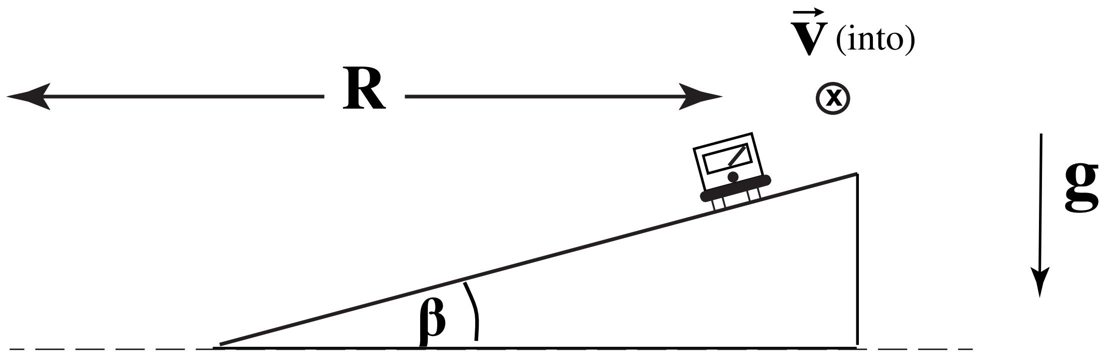
   A bead lies on a frictionless hoop of radius R that rotates around a vertical diameter with constant angular speed ω, as shown in the figure below.



   (a) What should ω be so that the bead maintains the same position on the hoop, at an angle θ with respect to the vertical? Express you answer in terms of some or all of the following: θ, R and g.

   (b) Analyzing the answer for Part A, you will find that there is a range of angular speeds, $0 < \omega < \omega_o$ for which the fixed angle $\theta = 0$ (meaning that the only balanced position is at the bottom of the hoop). Find the value of ω_o. Express you answer in terms of some or all of the following: R and g.

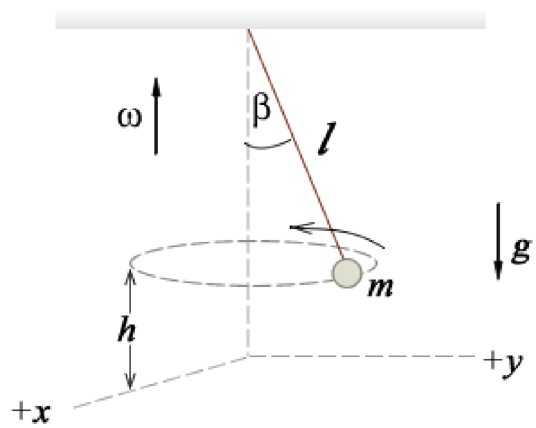
2. **Banked Turn**



A car of mass m is going around a circular turn of radius R, which is banked at an angle β with respect to the ground. Assume there is friction between the wheels and the road. Let μ_s be the coefficient of static friction and g the magnitude of the gravitational acceleration. You may neglect kinetic friction (that is, the car's tires do not slip). Derive an expression for the range of possible speeds $v_{\text{min}} \leq v \leq v_{\text{max}}$ necessary to keep the car moving in a circle without slipping up or down the embanked turn. Express your answer in terms of some or all of the following: μ_s, β, m, R and g.

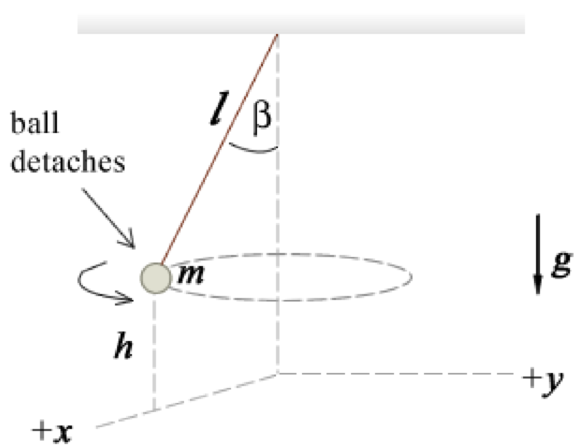
3. **Tetherball Breaking Off**

A small ball of mass m is suspended by a string of length l. The string makes an angle β with the vertical. The ball revolves in a circle with an unknown constant angular speed ω. The orbital plane of the ball is at a height h above the ground. Let g be the gravitational constant. You may ignore air resistance and the size of the ball.



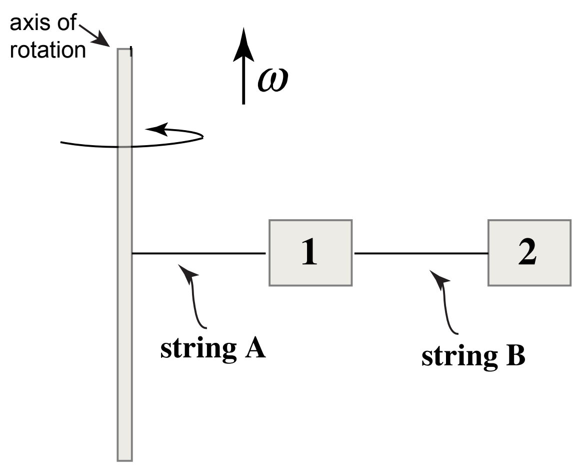
(a) Find an expression for the angular speed ω. Express you answer in terms of some or all of the following: l, β, and g.

(b) Later, the ball detaches from the string just as it passes the x-axis. It flies through the air and hits the ground at an unknown horizontal distance d from the point at which it detached from the string.



What horizontal distance d does the ball traverse before it hits the ground? Express you answer in terms of some or all of the following: l, β and h.

4. **Two Boxes Around a Shaft**

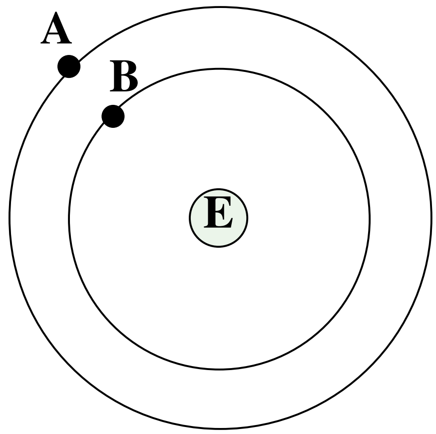


Box 1 and box 2 are whirling around a shaft with a constant angular velocity of magnitude ω. Box 1 is at a distance d from the central axis, and box 2 is at a distance $2d$ from the axis. You may ignore the mass of the strings and neglect the effect of gravity. Express your answer in terms of d, ω, m_1 and m_2, the masses of box 1 and 2.

(a) Calculate T_B, the tension in string B (the string connecting box 1 and box 2):

(b) Calculate T_A, the tension in string A (the string connecting box 1 and the shaft):

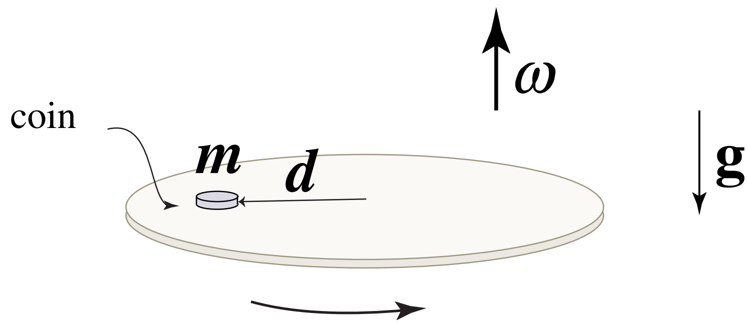
5. **Satellite**



(a) Two satellites are orbiting earth at different altitudes. Which satellite orbits at a higher speed v around earth? Assume that the orbits are circular and both satellites have the same mass.

(b) Which satellite orbits with a longer period, T, around earth? Assume that the orbits are circular and both satellites have the same mass.

6. **A coin on a rotating disk**



A coin of mass m is on a rigid disk at a distance d from the center of the disk. There is friction between the coin and the disk. The coefficient of static friction is μ_s. At time $t = 0$, the disk begins to rotate with a constant angular acceleration of magnitude α. The magnitude of the acceleration due to gravity is g.

Express you answers in terms of some or all of the given variables m, d, μ_s, α, t and g as needed.

(a) While the coin remains at rest relative to the disk, what is f_s, the magnitude of the force of static friction exerted by the disk on the coin as a function of time t?

(b) At what angular speed ω will the coin start to slip with respect to the disk?