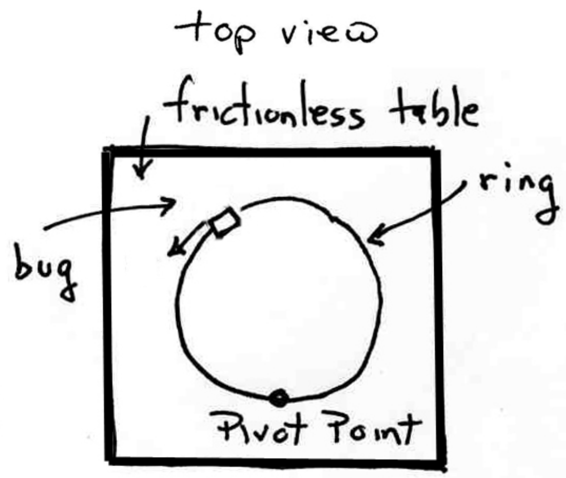
Problem Set 11

1. **Bug Walking on Pivoted Ring**

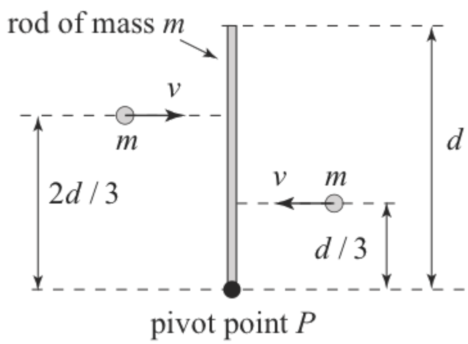
A ring of radius R and mass m_1 lies on its side on a frictionless table. It is pivoted to the table at its rim. A bug of mass m_2 walks on the ring with constant speed v relative to the ring, starting at the pivot, when the ring is initially at rest. Take $\hat{k}$ to point out of the page.



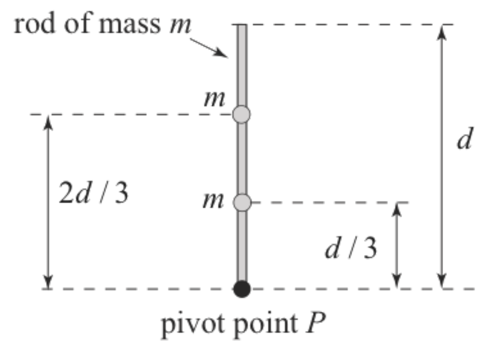
(a) What is the angular velocity of the ring when the bug is halfway around? Express you answer in terms of some or all of the following: m_1, m_2, v, R and $\hat{k}$.

(b) What is the angular velocity of the ring when the bug is back at the pivot? Express you answer in terms of some or all of the following: m_1, m_2, v, R and $\hat{k}$.

2. **A Rigid Rod**



view from above frictionless surface
before collision



view from above frictionless surface
after collision

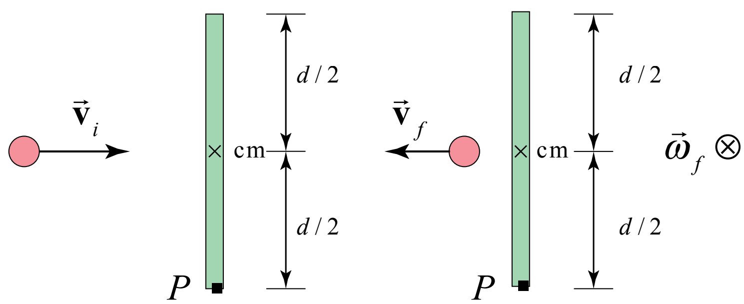
A rigid uniform rod of length d and mass m is lying at rest on a horizontal frictionless table and pivoted at the point P. A point-like object of mass m is moving to the right with speed v. It collides and sticks to the rod at a distance $2d/3$ from the pivot. A second point-like object of mass m is moving to the left (see figure) with speed v and collides with the rod at exactly the same instant as the first particle at a distance $d/3$ from the pivot. The moment of inertia of a rod for axis through the center of mass and perpendicular to the plane of the rod is $I_{cm} = \frac{1}{12}md^2$. After the collision, the rod and the two particles all rotate about the pivot point with angular speed ω_f.

(a) What is the component of the angular speed ω_f of the two particles and the rod immediately after the collision? Express your answer in terms of d, m, and v, as needed. Assume clockwise (into the page) to be positive.

(b) What is the ratio of the change in kinetic energy to the initial kinetic energy of the system, $\frac{K_f - K_i}{K_i}$? Express your answer in terms of d, m, and v, as needed.

3. **Elastic Collision Between Ball and Pivoted Rod**

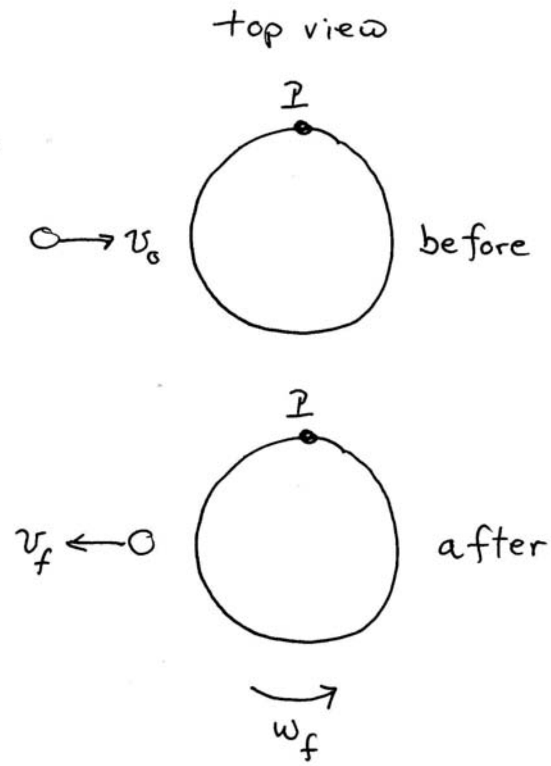
   A rigid rod of length d and mass m is lying on a horizontal frictionless table and pivoted at the point P on one end (shown in the figure). A point-like object of the same mass m is moving to the right (see figure) with speed v_i. It collides elastically with the rod at the midpoint of the rod and rebounds backwards with speed v_f. After the collision, the rod rotates clockwise about its pivot point P with angular speed ω_f. The moment of inertia of a rod about the center of mass is $I_{cm} = \frac{1}{12}md^2$.



   Find the angular speed ω_f. Express your answer in terms of d, m and v_i as needed.

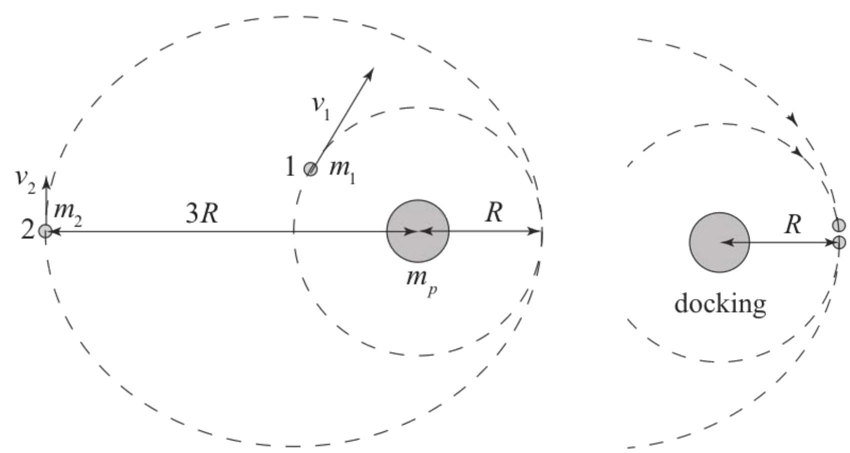
4. **Elastic Collision of Object and Pivoted Ring**

   A rigid hoop of radius R and mass m_R is lying on a horizontal frictionless table and pivoted at the point P (shown in the figure below). A point-like object of mass m is moving to the right with speed v_0. It collides elastically with the hoop at its midpoint. After the collision, the object is moving with an unknown speed v_f to the left and the hoop rotates counterclockwise about its pivot point with angular speed ω_f. The moment of inertia of a hoop for axis through the center of mass and perpendicular to the plane of the hoop is $I_{\text{cm}} = m_R R^2$.



   What is the speed v_f of the object immediately after the collision? Express your answer in terms of R, m, m_R, and v_0 as needed (do not use ω_f in your answer).

5. **A Spaceship and a Planet**



Spaceship 1 has mass m_1 and is moving with speed v_1 in a circular orbit of radius R around a planet of mass m_p. Spaceship 2 has mass m_2 and is moving in an elliptical orbit around the same planet. The mass of the planet is much, much greater than the mass of either spaceship. When spaceship 2 is at its furthest distance $3R$ from the planet, it is moving with speed v_2. When spaceship 2 is at its closest distance R from the planet, it is moving with speed v_p. The two spaceships are orbiting in the same plane as shown in the figures above. At a later time, both spaceships arrive nearly simultaneously at a point corresponding to the closest approach of spaceship 2. Spaceship 2 fires its rockets in order to reach the same speed v_1 as spaceship 1 in order to dock together. You may assume that the elapsed time interval for docking is very small compared to the orbital periods of the spaceships. Let G be Newton's universal constant of gravity.

What is the change in the speed, $\Delta v = v_1 - v_p$, of spaceship 2 in order for the two spaceships to dock together? (Does spaceship 2 speed up or slow down in order to dock?) Express your answer only in terms of G, R and m_p.