Fall Term 2003
Plasma Transport Theory, 22.616
LITTLE Problem Set #3
Prof. Molvig

Passed Out: Sept. 25, 2003 DUE: Oct. 2, 2003

1. **Moment Equation Structure:** Develop the structure of moment equations (for a single species) that follow from the kinetic equation,

$$\frac{\partial f}{\partial t} + \mathbf{v}\cdot\nabla f + \frac{q}{m}\left(\mathbf{E} + \frac{1}{c}\mathbf{v}\times\mathbf{B}\right)\cdot\frac{\partial}{\partial \mathbf{v}}f = 0$$

assuming the distribution is close to Maxwellian,

$$f \simeq f^M = \frac{n}{(2\pi T/m)^{3/2}}\exp\left(-\frac{m(\mathbf{v}-\mathbf{V})^2}{2T}\right)$$

In particular show that the density moment gives,

$$\frac{\partial n}{\partial t} + \nabla\cdot n\mathbf{V} = 0$$

that the momentum moment gives,

$$mn\left(\frac{\partial}{\partial t}\mathbf{V} + \mathbf{V}\cdot\nabla\mathbf{V}\right) = -\nabla P + qn\left(\mathbf{E} + \frac{1}{c}\mathbf{V}\times\mathbf{B}\right) - \nabla\cdot\boldsymbol{\pi}$$

where the tensor, $\boldsymbol{\pi}$, is the stress moment resulting from deviations from Maxwellian,

$$\boldsymbol{\pi} \equiv \int d^3v m\mathbf{v}\mathbf{v}\left(f - f^M\right)$$

and the energy moment can be expressed as,

$$\frac{\partial}{\partial t}\left(\frac{3}{2}nT + \frac{1}{2}mnV^2\right) + \nabla\cdot\mathbf{Q} = qn\mathbf{V}\cdot\mathbf{E}$$

with, $\mathbf{Q}$, the total energy flux,

$$\mathbf{Q} = \frac{1}{2}mnV^2\mathbf{V} + \frac{5}{2}nT\mathbf{V} + \mathbf{q} + \boldsymbol{\pi}\cdot\mathbf{V}$$

This energy equation contains both internal and hydrodynamic flow energies. Eliminate the hydro flow energy by subtracting, $\mathbf{V}\cdot$Momentum Equation, from energy equation to yield,

$$\frac{\partial}{\partial t}\left(\frac{3}{2}nT\right) + \nabla\cdot\left(\frac{5}{2}nT\mathbf{V} + \mathbf{q}\right) - \mathbf{V}\cdot\nabla P = -\boldsymbol{\pi} : \boldsymbol{\nabla}\mathbf{V}$$

which can also be manipulated into a form for the temperature alone,

$$\frac{3}{2}n\left(\frac{\partial}{\partial t} + \mathbf{V}\cdot\nabla\right)T + P\nabla\cdot\mathbf{V} = -\nabla\cdot\mathbf{q} - \boldsymbol{\pi} : \boldsymbol{\nabla}\mathbf{V}$$

Finally show that with zero viscous stress, $\boldsymbol{\pi} = 0$, and zero heat flux, $\mathbf{q} = 0$, that this is equivalent to the adiabatic equation of state,

$$\frac{D}{Dt}\left(Pn^{-5/3}\right) = 0$$

implying that $Pn^{-5/3}$ is constant following the fluid.