22.38 – PS #9 solutions

1) mean cycle unavailability : $\langle Q \rangle = \frac{1}{t_c}\left[(Q_0 t_s + (1-Q_0)\lambda/2\, t_s^2 + t_t + f_r t_r\right]$

Δt_r: $\langle Q \rangle_{.1} = \langle Q \rangle - \frac{.1 f_r t_r}{t_c}$

Δt_t: $\langle Q \rangle .1 = \langle Q \rangle - \frac{.1 t_c}{t_c}$

Δf_r: $\langle Q \rangle_{.1} = \langle Q \rangle - \frac{.1 t_r f_r}{t_c}$

ΔQ_0: $\langle Q \rangle .1 = \langle Q \rangle - \frac{.1 t_s Q_0}{t_c} - \frac{.1 Q_0 \lambda t_s^2}{2 t_c}$

given values (pg. 14) ; $t_r = 10^3$ hr , $t_t = 25$ hr, $f_r = .01$; $t_r = 60$ hr, $Q = .02$, $\lambda = 10^{-4}$, $t_s = 10^3$

$\Delta t_r = .06$ $\quad$ $\Delta t_t = 2.5$

$\Delta f_r = .06$ $\quad$ $\Delta Q = 2.2$ $\quad \Rightarrow$ t_t has the greatest impact

3) Loads & capabilities are both normally distributed,
$\therefore$ $D = C - L$ is also normally distributed with

mean = $15 - 8 = 7$

$\sigma_D = \sqrt{2^2 - 5^2} = 5.4 \quad \Rightarrow \quad S = \frac{0-7}{5.4} = -1.296$

$P(D \geq 0) = \phi(-S) = \phi(1.296) = .902$

4) assume β is small $\Rightarrow$ $F_{311_C} = q_I^3 + \beta q_I$

$F_{211_I} = q_I^2$

for $F_{311_C} = F_{211_I}$ $\Rightarrow$ $q_I^2 = q_I^3 + \beta q_I$ $\Rightarrow$ $\beta \approx q_I$

$\beta = \lambda_I = 10^{-3}$

Fraction of CCF contribution : $\frac{\beta}{q_I^2 + \beta}$

2)

States: (1) A & B both in tact

(2) A failed via 1

(3) A failed via 2

(4) B failed via 1

(5) B failed via 2

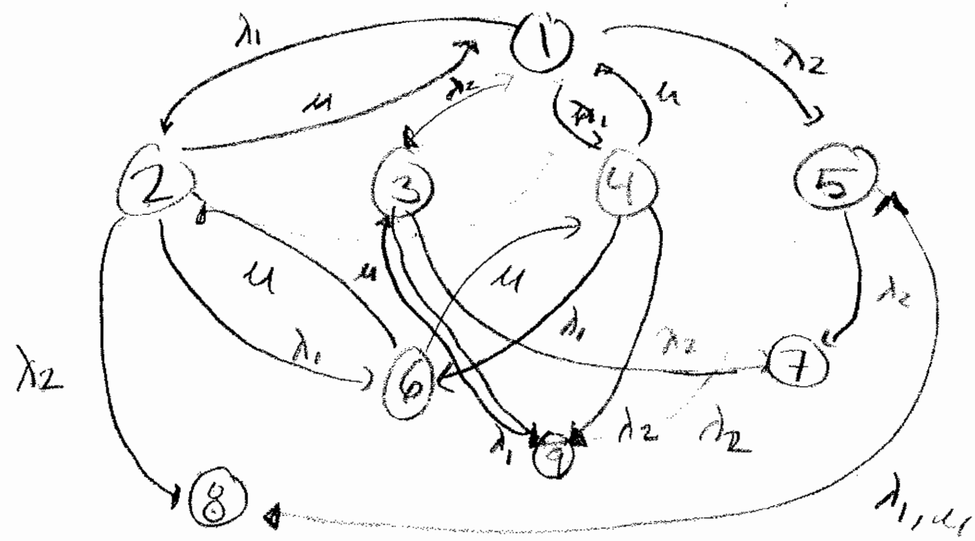
(6) A_1 B_1

(7) A_2 B_2

(8) A_1 B_2

(9) A_2 B_1

a)



b)

$$\begin{bmatrix}
-2(\lambda_2+\lambda_1) & \mu & 0 & \mu & 0 & 0 & 0 & 0 & 0 \\
\lambda_1 & -(\lambda_2+\mu+\lambda_1) & 0 & 0 & 0 & \mu & 0 & 0 & 0 \\
\lambda_2 & 0 & -(\lambda_2+\lambda_1) & 0 & 0 & 0 & 0 & 0 & \mu \\
\lambda_1 & 0 & 0 & -(\mu+\lambda_1+\lambda_2) & 0 & \mu & 0 & 0 & 0 \\
\lambda_2 & 0 & 0 & 0 & -(\lambda_2+\lambda_1) & 0 & 0 & \mu & 0 \\
0 & \lambda_1 & 0 & \lambda_1 & 0 & -2\mu & 0 & 0 & 0 \\
0 & 0 & \lambda_2 & 0 & \lambda_2 & 0 & \boxed{0} & 0 & 0 \\
0 & \lambda_2 & 0 & 0 & \lambda_1 & 0 & 0 & -\mu & 0 \\
0 & 0 & \lambda & \lambda_2 & 0 & 0 & 0 & 0 & -\mu
\end{bmatrix}$$

* state 7 is an absorbing state — no way out.

5) $f''(\theta) = \dfrac{P(E|\theta) f'(\theta)}{\int_{-\infty}^{\infty} P(E|\theta) f'(\theta) \, d\theta} = k L(\theta) f'(\theta)$

$f'(\lambda) = 1/[1\times 10^{-2} - 10^{-3}] = 111.111$

$L(\lambda) = \lambda^3 (e^{-300\lambda})(e^{-400\lambda})(e^{-800\lambda}) = \lambda^3 e^{-1500\lambda}$

$k = \left[\int_{-\infty}^{\infty} L(\lambda) f'(\lambda) \, d\lambda\right]^{-1} = \left[\int_{10^{-3}}^{10^{-2}} 111 \cdot \lambda^3 e^{-1500\lambda} \, d\lambda\right]^{-1} = (1.12\times 10^{-12})^{-1} \cdot \frac{1}{111}$

$$\boxed{f''(\theta) = \frac{\lambda^3 e^{-1500\lambda}}{1.12\times 10^{-12}}}$$

Lambda	f"(lambda)	P(lambda)
0.01	0.273127072	0.000273127
0.009	0.892347477	0.000892347
0.008	2.80878279	0.002808783
0.007	8.433037613	0.008433038
0.006	23.80046222	0.023800462
0.005	61.72816631	0.061728166
0.004	141.6429815	0.141642982
0.003	267.8061665	0.267806167
0.002	355.6219169	0.355621917
0.001	199.2233573	0.199223357
		1.062230346