Homework Assignment #4

22.105

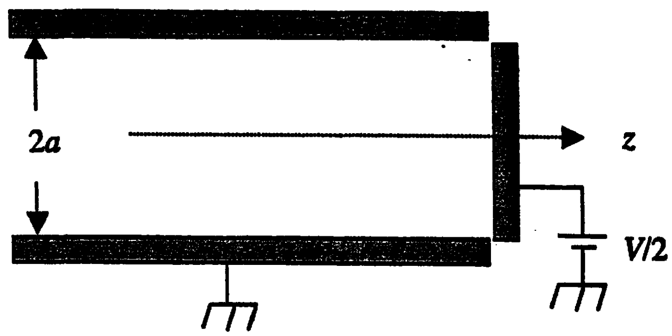
Electromagnetic Interactions

Fall 2005

Distributed: **Tuesday, October 17, 2005**

Due: **Thursday, October 26, 2005**

In this problem you will calculate the focusing properties of a cylindrical immersion lens by developing an analytic solution for the fields and a numerical solution for the particle motion. The geometry of interest is illustrated below.



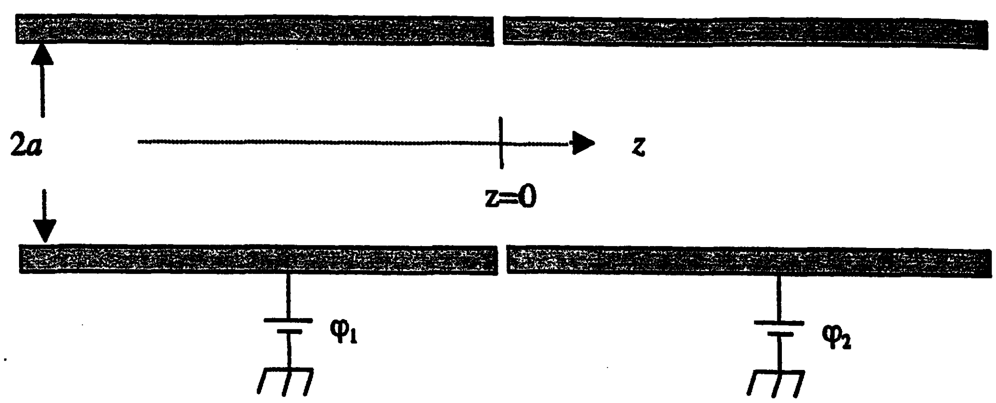
a. The first step is to solve for the electrostatic potential. The figure shows the cross section of a long $(L \gg a)$ cylinder of radius a excited by a source of potential at one end. Inside the cylinder the potential can be written in the form

$$\phi(r,z) = \sum_{n=1}^{\infty} A_n J_0(\beta_n r / a) \exp(\beta_n z / a) \qquad -\infty < z < 0$$

where the β_n are the zero's of J_0: $J_0(\beta_n) = 0$. Note that the excitation grid is located at $z = 0$. Find the expansion coefficients A_n. Helpful integral relations:

$$\int_0^1 x J_0(\beta_m x) J_0(\beta_n) dx = (1/2) J_1^2(\beta_m) \delta_{mn}$$
$$\int_0^z x J_0(x) dx = z J_1(z)$$

b. Now consider the complete immersion lens illustrated in cross section on the next page. Assume the potentials satisfy $\phi_2 > \phi_1 > 0$. Find the complete potential by applying the solution found in part (a) to this problem and recognizing that the potential at $z = 0$ is $\phi(0) = (\phi_1 + \phi_2)/2$. Keep in mind that the solutions must decay to zero at both $z = \pm\infty$.



c. An electron enters the lens from the left with velocity $\mathbf{u} = u_0 \mathbf{e}_z$ at a radius $r = r_0 = 0.2a$ where $u_0 = (2e\phi_1 / m)^{1/2}$. Introduce normalized variables as follows: $\tau = u_0 t / a, r(t) = a\rho(\tau), z(t) = ay(\tau), u_r(t) = u_0 w_r(\tau), u_z(t) = u_0 w_z(\tau)$, and $\phi(r,z) = \phi_1 \psi(\rho, y)$. Show that the equations of motion reduce to

$$\frac{dw_r}{d\tau} = \frac{1}{2}\frac{\partial \psi}{\partial \rho}$$
$$\frac{dw_z}{d\tau} = \frac{1}{2}\frac{\partial \psi}{\partial y}$$
$$\frac{d\rho}{d\tau} = w_r$$
$$\frac{dy}{d\tau} = w_z$$

d. Solve these equations numerically and find the focal point of the lens for the following three cases: $(\phi_2 - \phi_1)/\phi_1 = 0.5, 1.0$, and 4.0.