Sample questions for Quiz 3, 22.101 (Fall 2006)

Following questions were taken from quizzes given in previous years by S. Yip. They are meant to give you an idea of the kind of questions (what was expected from the class in previous times) that have been asked in the past. Percentage credit is indicated for each question based on 100% for a 90-min Quiz (closed book). Problem numbering means nothing here.

You will have to decide for yourself what connections, if any, there may be between these questions and the Quiz 3 that will be conducted on Dec. 13, 2006.

Problem 1 (20%)

Define concisely what is Compton scattering. Derive the relation between incident gamma energy $\hbar\omega$ and scattered gamma energy $\hbar\omega'$ for Compton scattering which also involves the scattering angle θ. What is the similarity (and difference) between this relation and the corresponding relation involving incident and scattered energies in neutron elastic scattering?

Problem 2 (30%)

Consider the measurement of monoenergetic gammas (energy $\hbar\omega$) in a scintillation detector whose size is small compared to the mean free path of the secondary gammas produced by interactions of the incident (primary) gammas in the detector.

(10%) (a) Sketch the pulse-height spectrum of low-energy gammas, say $\hbar\omega$ < 500 kev. Explain briefly the important characteristics of this spectrum in terms of the different interactions that can take place.

(10%) (b) Repeat (a) for higher-energy gammas, $\hbar\omega$ > 2 Mev.

(10%) (c) What other peaks can appear in the pulse-height spectrum if the detector were not small? Give a sketch and explain briefly.

Problem 3 (30%)

(15%) (a) You are told the reaction $^{13}C(d,p)^{14}C$ has a resonance at a deuteron energy E_d (LCS), and following this, ^{14}C undergoes $\beta-$ decay to ^{14}N. Draw the energy level diagram for this situation in which you show explicitly how the following energies can be calculated in terms of known masses and E_d: (1) kinetic energy available for reaction T_o, (2) Q value for the reaction, (3) deuteron separation energy, (4) proton separation energy, and (5) Q_β.

(15%) (b) On the basis of (a), predict whether or not the reaction $^{11}B(\alpha,n)^{14}N$ will have a resonance, and if so, at what energy of the α particle this will occur. (Since you are not given numerical values, you should leave your answer in terms of defined quantities such as masses and various energies.)

Problem 4 (20%)

Sketch the energy variation of an observed resonance in (a) neutron elastic scattering (resonance scattering in the presence of potential scattering), and (b) neutron inelastic scattering. Comment on the characteristic features in the cross sections, especially the low-energy behavior below the resonance. What is the connection between the energy at which the observed cross sections show a peak and the energy of the nuclear level associated with the resonance? (You may assume it is the same level in both cases.)

Problem 5 (20%)

Sketch the peaks that one would observe in the pulse-height spectra of a small detector in the presence of a 2-Mev gamma ray source, including any radiation from the background. For each peak identify the radiation interaction process that gives rise to it and indicate the energy at which this peak would appear.

Problem 6 (25%)

Consider the compound nucleus reaction of inelastic scattering of neutrons at energy T_1 (LCS) by a nucleus ${}_Z^AX$.

(a) Draw the energy level diagram showing the different energies that one can use to describe this reaction (including the Q value).

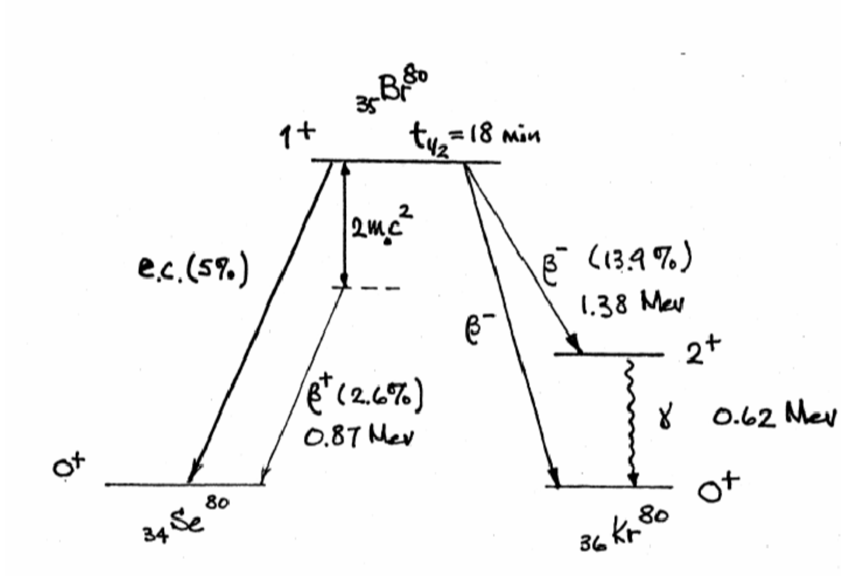
(b) Write down the corresponding Breit-Wigner cross section in terms of some of the energies shown in (a). Define all the parameters appearing in your expression.

Problem 7 (10%)

Consider the reaction $a + b \rightarrow c + d$, where Q is nonzero and particle b is stationary. What can you say about the magnitude and direction of the velocity of the center-of-mass before and after the reaction?

Problem 8 (20%)

The decay scheme of ${}^{80}Br$ is shown below. Classify the various decay modes and estimate all the decay constants that you can.



Problem 9 (15%)

At time t = 0 you are given an atom that can decay through either of two channels, a and b, with known decay constants λ_a and λ_b. Find the probability that it will decay by channel a during the time interval between t_1 and t_2, with t_1 and t_2 arbitrary. Interpret your result.

Problem 10 (20% total)

Consider a beam of collimated, monoenergetic neutrons (energy E) incident upon a thin target (density N atoms per cc) of area A and thickness Δx at a rate of I neutrons/sec. Assume the cross sectional area of the beam is greater than A. An energy sensitive detector subtended at an angle θ with respect to the incident beam direction is set up to measure the number of neutrons per second scattered into a small solid angle $d\Omega$ about the direction $\underline{\Omega}$ and into a small energy interval dE' about E'. Let this number be denoted by Π.

(a) (15%) Define the double (energy and angular) differential scattering cross section $d^2\sigma / d\Omega dE'$ in terms of the physical situation described above such that you relate this cross section to the scattering rate Π and any other quantity in the problem. (You may find it helpful to draw a diagram of the specified arrangement.)

(b) (5%) How is $d^2\sigma / d\Omega dE'$ related to the angular and energy differential cross sections, $d\sigma / d\Omega$ and $d\sigma / dE'$, respectively (no need to define the latter, assume they are known)?

Problem 11 (25%)

In neutron elastic scattering by hydrogen where the target nucleus is assumed to be at rest, the ratio of final to initial neutron energy is E'/E = (1/2)(1 + cos θ_c), where θ_c is the scattering angle in CMCS. Suppose you are told the angular distribution of the scattered neutrons is proportional to $\cos\theta_c$ for $0 \le \theta_c \le \pi/2$ and is zero for all other values of θ_c. Find the corresponding energy distribution $F(E \to E')$. Sketch your result and discuss how it is different from the case of isotropic angular distribution.

Problem 12 (20% total)

Give a brief and concise answer to each of the following.

(a) (7%) What is the physical picture of the model used to estimate the decay constant in alpha decay (give sketch). Why does the model give an upper limit for the decay constant?.

(b) (4%) What is electron capture and with what process does it compete?

(c) (4%) What is internal conversion and with what process does it compete?

(d) (5%) Give a sketch of the variation of the neutron cross section of C in the energy region below 0.1 Mev and explain the features.

Problem 13 (20%)

In neutron radiation damage studies an incident neutron with energy E collides with an atom in the crystal lattice, called the primary knock-on atom (PKA), causing it to recoil and displace a number of other atoms in the solid.

(a) (8%) Calculate the average recoil energy of the PKA (mass A). State all assumptions involved in your calculation.

(b) (12%) Let the number of atoms displaced by a PKA with energy T be denoted by ν (T) which behaves as follows

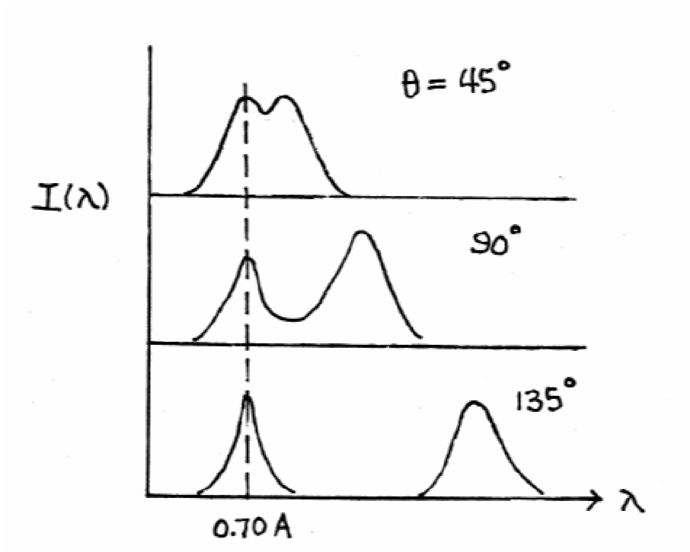
$$\nu(T) = 0 \qquad \text{for} \quad 0 < T < Ed$$

$$= T/2E_d \qquad E_d < T < L_c$$

where E_d is the energy to displace an atom, and L_c is the energy above which all the energy loss is by electronic excitation. Find the average number of atoms displaced as a function of E in the range, $0 < E < L_c$. Call this quantity N_E. (Note: Neither ν nor N_E are distributions.)

Problem 14 (15%)

When a beam of photons with wavelength 0.7 A is scattered by carbon at various scattering angles, the observed spectra behave in a way as sketched schematically. What is the process responsible for the peak at 0.7 A? What is the process responsible for the other peak in the spectrum? Comment on the variation with scattering angle of the two peaks.



Problem 15 (20%)

For the energy-level diagram shown below, identify all the transitions that can take place. For each transition determine the decay mode and the decay energy (Q-value).

$_{Z}X^{A}$

3^{+} ______________ 2.42 Mev

4^{-} ______________ 1.57 Mev

$_{Z-1}Y^{A}$

4^{+} ______________ 0 Mev

Problem 16 (15%)

Consider neutron inelastic scattering at incident energy T_1 (LCS) where the reaction can be written as

$$n + {}_{z}X^{A} \rightarrow ({}_{Z}X^{A+1})^{*} \rightarrow n' + ({}_{Z}X^{A})^{*}$$

Is the Q-value positive or negative? Draw the energy-level diagram depicting the kinetic and rest-mass energies involved in the reaction, and show how the kinetic energies are related to T_1 and any other energies in the problem. You may assume target nucleus is at rest.

Problem 17 (30% total, 6% each)
Give a short and concise answer to each of the following questions

(a) Suppose you are detecting a γ-ray at energy $\hbar\omega$ = 2 Mev using a large scintillation detector. Sketch the energy distribution that you would observe, indicating both the origin and position (the energy on a linear energy scale) of each peak.

(b) Sketch (as quantitatively as you can) the mass attenuation of lead for γ-rays with energy from 0.01 Mev to 100 Mev, showing the individual contributions from the various processes of interaction.

(c) Sketch the model used for calculating the decay constant for α-decay (no calculation), and explain the energies and distances in your sketch.

(d) Sketch the distributions of neutron energies for neutron elastic scattering at energy E by target nuclei with mass A in a medium at temperature T, for E/k_BT = 0.1, 1, and 4.

(e) Sketch the total cross section of water for neutron energies from 0.001 ev to 10 ev. Explain the meaning of free-atom and bound-atom cross sections and indicate where they would appear in your sketch.

Problem 18 (25%)

(a) (15%) Angular differential cross sections (in CMCS) for C^{12} (left panel) and U^{238} (right panel) for two incident neutron energies, 0.5 Mev and 14 Mev, are shown below. Interpret (explain) all the features, including the difference between carbon and uranium.

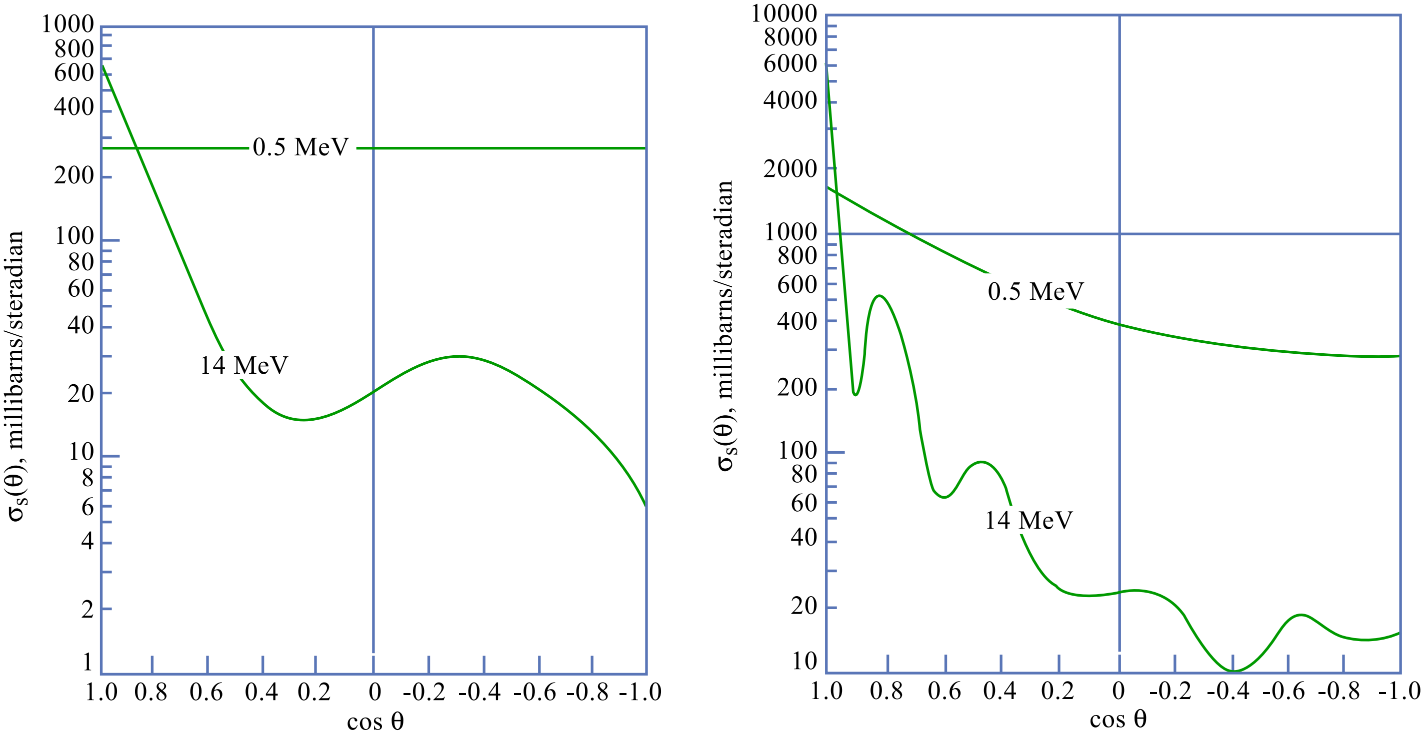


Figure by MIT OCW.

(b) (10%) Write down the relation between the neutron scattering cross section $\sigma_{meas}(v)$ measured in the laboratory (which depends on the temperature T of the target) and the theoretical cross section for s-wave scattering at low energy calculated using the method of phase shift. Define all the quantities. Sketch the behavior of $\sigma_{meas}(v)$ in the energy range from thermal up to the energy where one sees free-atom scattering. Briefly discuss the significance of your sketch.

Problem 19 (25%)

(a) (8%) You are given the Klein-Nishina cross section for Compton scattering.

$$d\sigma / d\Omega = \frac{r_e^2}{2}\left(\frac{\omega'}{\omega}\right)\left[\frac{\omega}{\omega'} + \frac{\omega'}{\omega} - 2\sin^2\theta\cos^2\varphi\right]$$

Find the cross section for Thompson scattering for unpolarized radiation. Sketch qualitatively your result and comment on any interesting feature.

(b) (8%) Indicate (do not carry out any complicated manipulations) how you would find the energy distribution of the Thompson scattered electrons using the result from (a).

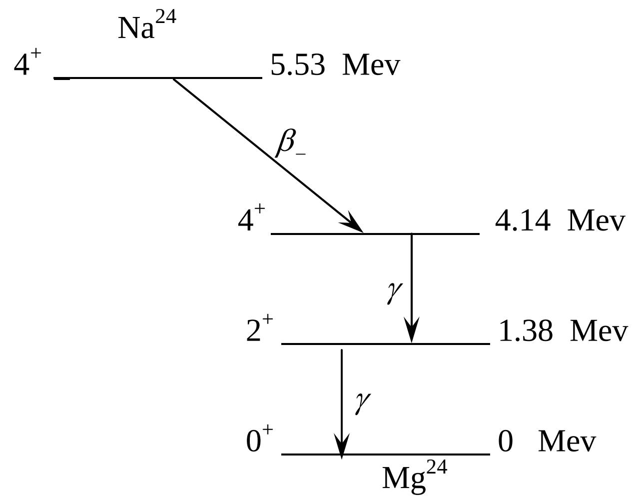
(c) (9%) Sketch the energy distributions for Compton scattering at two energies, say 0.5 Mev and 2.5 Mev, and comment on all the characteristic features. Do you expect the Compton and Thompson electron distributions to be similar or different (give some explanation)?

Problem 20 (25%)

Radioactive Na^{24} undergoes β_- decay to Mg^{24} (see energy level diagram below). In a measurement one finds a distribution of electrons having an end-point energy of 1.39

Mev, but in addition two groups of positrons with distinct end-point energies are also observed. There were no ${}_{10}Ne^{24}$ found.

(a) (8%) What could be the process giving rise to the positrons?
(b) (8%) What are the expected end-point energies of the two positron groups?
(c) (9%) What are the decay modes for the indicated transitions?



Problem 21 (25% total, 5% each) Give short and concise answer to each question.

(a) Sketch first the neutron scattering probability $F(E \rightarrow E')$ as a function of E'/E for the conditions of elastic scattering, target nucleus (mass M) at rest, and scattering is isotropic in CMCS. Then in the same sketch show what happens when the target is no longer at rest (consider several ratios of E/k_BT, where T is the target temperature).

(b) Sketch the mass attenuation coefficient of Pb for gamma radiation in the energy range 0.01 Mev to 100 Mev, showing the individual contributions from the various interactions considered in class.

(c) Explain the Gamow factor and its significance in describing nuclear decay.

(d) State the condition on the kinetic energy of the incoming particle (T_a) which is necessary and sufficient for the indicated reaction to occur (take $T_X = 0$),

$$a + X \rightarrow b + Y \quad .$$

(e) Draw the energy level diagram for the indicated reaction showing all the energies involved

$$n + {}_Z X^A \rightarrow ({}_Z X^{A+1})^* \rightarrow n' + ({}_Z X^A)^* \ .$$