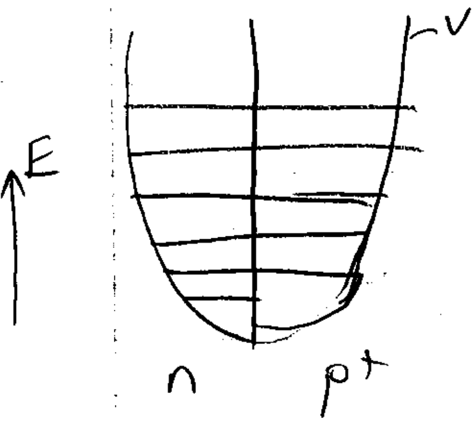
22.101 Quiz Review 11/12

Shell Model



spin-orbit coupling
→ splits energy levels

$$H = \frac{p^2}{2m} + V(r) + V_{so}(r)\, S \cdot L$$

To find $S \cdot L$,
define $J = L + S$
$\Rightarrow J^2 = (L+S)^2 = L^2 + S^2 + 2L \cdot S$
$\Rightarrow L \cdot S = \frac{1}{2}(J^2 - L^2 - S^2)$

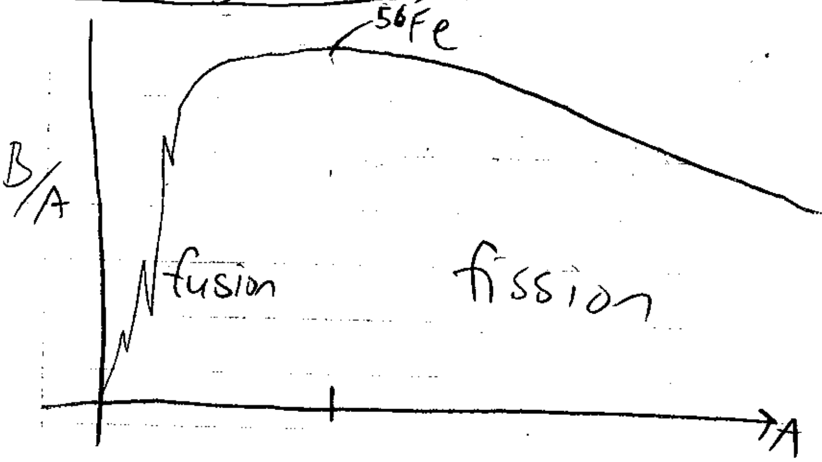
Transform $|l, m_l, s, m_s\rangle \longrightarrow |j, m_j\, l\, s\rangle$

$J^2 |j\, m_j\, l\, s\rangle = \hbar^2 j(j+1) |j\, m_j\, l s\rangle$, $|l - s| \le j \le l + s$
$J_z |j\, m_j\, l\, s\rangle = \hbar m_j |j\, m_j\, l s\rangle$, $-j \le m_j \le j$
L^2 ...
S^2 ...

- * values differ by 1 e.g. $-\frac{7}{2}, -\frac{5}{2}, -\frac{3}{2}, \ldots \frac{7}{2}$ or $-3, -2, -1, \ldots 3$
- * degeneracy of $2j+1$ for level j

$3p_{3/2} \Rightarrow$ 3rd p state, $l = 1$, $j = \frac{3}{2} \Rightarrow l, s$ parallel

Binding Energy



Empirical Mass Formula

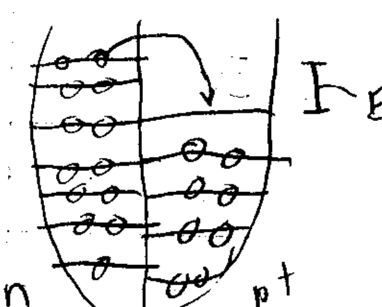
$$B(A,Z) = a_v A - a_s A^{2/3} - a_c \frac{Z(Z-1)}{A^{1/3}} - a_a \frac{(N-Z)^2}{A} + \delta$$

Volume term: Each nucleon contributes a constant energy due to its binding w/ surrounding nucleons

surface term: Nuclei on the surface contribute less because they are not completely surrounded, The number of nucleons on the surface is related to r^2 and $r \sim A^{1/3} \Rightarrow$ surface $\propto A^{2/3}$

Coulomb term: p^+ repel others removing binding. This energy is related to Coulomb potential $\frac{q^2}{r} \propto \frac{Z(Z-1)}{A^{1/3}}$

assymetry term: Owing to Pauli exclusion, neutrons and protons fill nuclear well separately. Extra energy must be included if p^+ well more full than n well



pairing term - if nucleons pair up, can reduce energy

Radioactive decay

$A \xrightarrow{\lambda} B \qquad \frac{dN_A}{dt} = -\lambda N_A \Rightarrow N_A = N_{A,0}\, e^{-\lambda t}$

$A \underset{\lambda_2}{\overset{\lambda_1}{\rightrightarrows}} B \qquad \frac{dN_A}{dt} = -\lambda_1 N_A - \lambda_2 N_A \Rightarrow \frac{dN_A}{dt} = -N_A(\lambda_1+\lambda_2)$

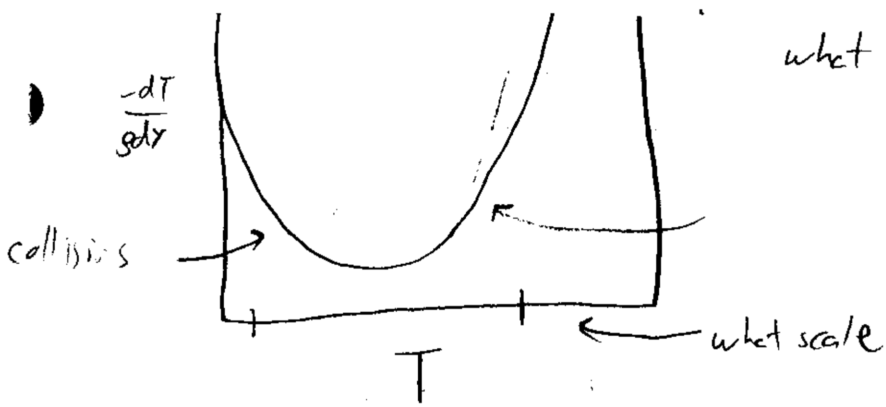
$\Rightarrow N_A = e^{-(\lambda_1+\lambda_2)t} \cdot N_{A,0}$

Stopping Power

$-\frac{dT}{dx} = \frac{4\pi z^2 e^4 n Z}{m_e v^2} \ln\left(\frac{2m_e v^2}{I}\right)$ (non-relativistic collisions)

$-\frac{dT}{dx} = \frac{2\pi e^4 n Z}{m_e v^2}\left[\ln\left(\frac{m_e v^2 T}{I^2(1-\beta^2)}\right) - \beta^2\right]$ (relativistic e^- collisions)

what is the effect of Z of absorber?

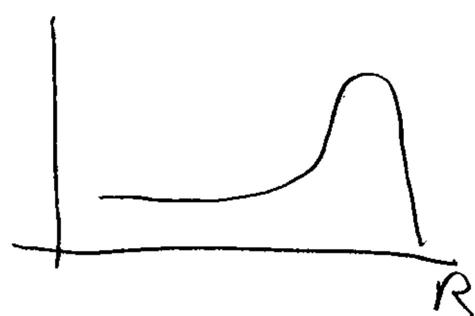


$\left(\frac{-dT}{dx}\right)_{rad} = n(T + m_e c^2)\,\sigma_{rad}$ (radiative loss of e^-'s)

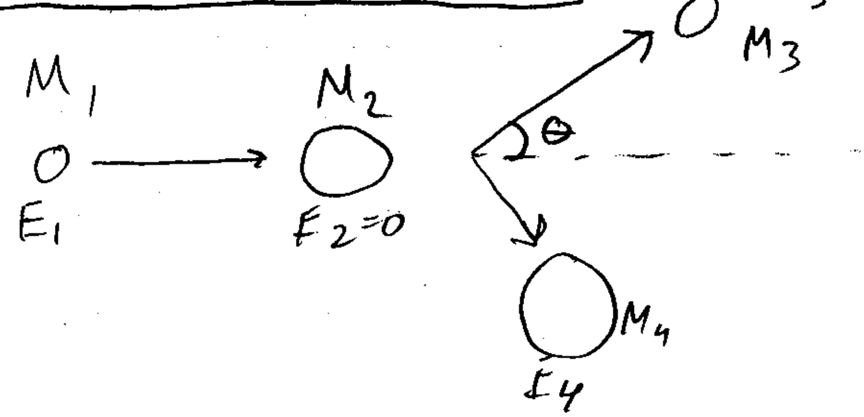
Range

$R = \int_0^{T_0} \left(-\frac{dT}{dx}\right)^{-1} dT$

$i = \frac{1}{W}\left(-\frac{dT}{dx}\right)$



Neutron Interactions



Conserve: energy, momentum

$-(E_1 + E_2) + (E_3 + E_4) = Q$

$Q > 0 \Rightarrow$ exothermic

$Q < 0 \Rightarrow$ endothermic (inelastic scattering)

$Q = 0 \Rightarrow$ elastic

elastic $\Rightarrow M_1 = M_3$

$M_2 = M_4$

endothermic $\Rightarrow$ threshold energy $E_{th} < E_1$, for reaction

$E_3, E_4 \geq 0$

Quiz: 6 Q's

- mostly short answers (calculation + a few sentences)