Nuclear Shell Model

2, 8, 20, 28, 50, 82, and 126.

Simple Shell Model

$$V(r) = -\frac{V_o}{1+\exp[(r-R)/a]}$$ $V_o \sim 57$ Mev, R ~ 1.25A$^{1/3}$ F, a ~ 0.65 F.

harmonic oscillator potential, $V(r) = m\omega^2 r^2/2$ $\quad E_\nu = \hbar\omega(\nu + 3/2) = \hbar\omega(n_x + n_y + n_z + 3/2)$

Shell Model with Spin-Orbit Coupling

$$H = \frac{p^2}{2m} + V(r) + V_{so}(r)\underline{s}\cdot\underline{L} \quad \underline{j} = \underline{S} + \underline{L} \quad \underline{S}\cdot\underline{L} = (j^2 - S^2 - L^2)/2$$

$$|\ell - s| \le j \le \ell + s \quad -j \le m_j \le j$$

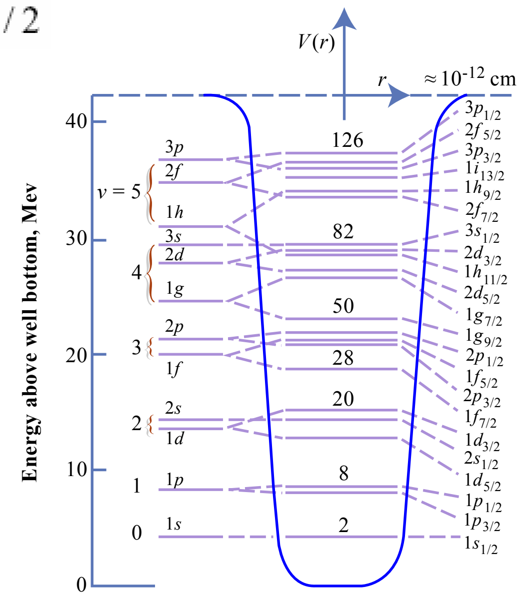
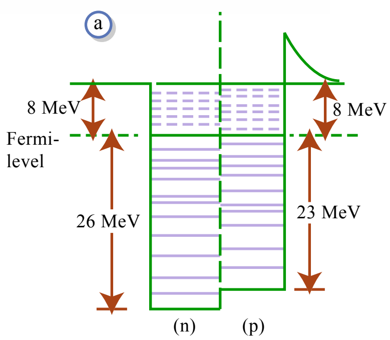


Figure by MIT OCW. Adapted from Meyerhof.

Potential Wells for Neutrons and Protons

$$\Delta V_s = \pm 27\frac{(N-Z)}{A} \text{ Mev}$$



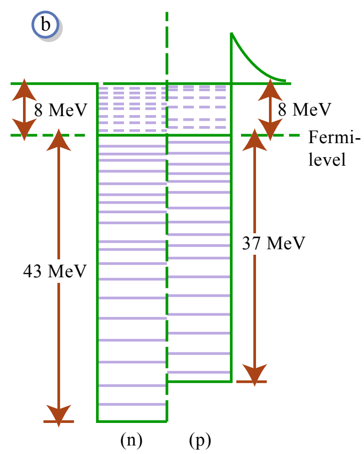


Figure by MIT OCW.
Adapted from Marmier and Sheldon.

Nuclear Binding Energy and Stability

$$B(A,Z) \equiv [ZM_H + NM_n - M(A,Z)]c^2$$

$$i + I \rightarrow f + F + Q \qquad Q = T_f + T_F - (T_i + T_I)$$

$$Q \equiv [(M_i + M_I) - (M_f + M_F)]c^2$$

$$Q = B(f) + B(F) - B(i) - B(I)$$

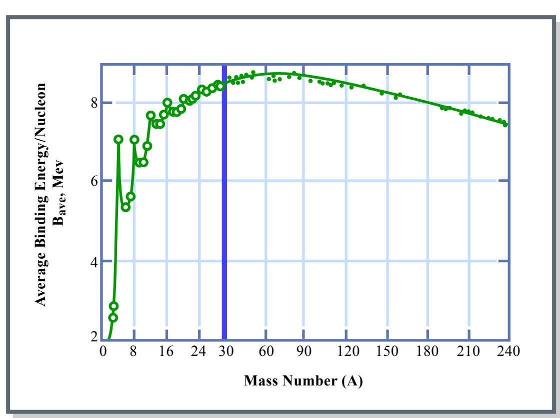


Figure by MIT OCW. Adapted from Meyerhof.

Separation Energy

$$S_n = [M_n + M(A-1,Z) - M(A,Z)]c^2$$
$$= B(A,Z) - B(A\text{-}1, Z)$$

~ 1 Mev difference between the neutron absorbed being an even neutron or an odd neutron

N ~ Z for low A, but N > Z at high A.

(i) In the case of odd A, only one stable isobar exists, except A = 113, 123.

(ii) In the case of even A, only even-even nuclides exist, except A = 2, 6, 10,

14

Empirical Binding Energy Formula and Mass Parabolas

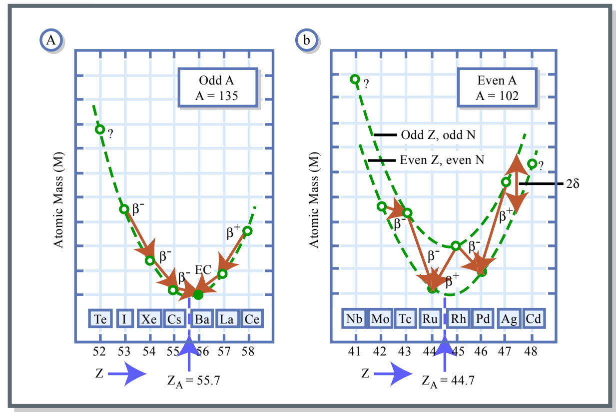
$$B(A,Z) = a_v A - a_s A^{2/3} - a_c \frac{Z(Z-1)}{A^{1/3}} - a_a \frac{(N-Z)^2}{A} + \delta$$

$\delta = a_p / \sqrt{A}$ even-even nuclei

$= 0$ even-odd, odd-even nuclei

$= -a_p / \sqrt{A}$ odd-odd nuclei

a_v	a_s	a_c	a_a	a_p	
16	18	0.72	23.5	11	Mev



$$M(A,Z)c^2 \cong A\left[M_n c^2 - a_v + a_a + a_s / A^{1/3}\right] + xZ + yZ^2 - \delta$$

Figure by MIT OCW. Adapted From Meyerhof.

$$x = -4a_a - (M_n - M_H)c^2 \cong -4a_a$$

$$y = \frac{4a_a}{A} + \frac{a_c}{A^{1/3}}$$

proton-rich. ${}_9F^{16} \rightarrow {}_8O^{16} + \beta^+ + \nu$ M(A, Z+1) > M(A,Z) + 2 m_e β^+ - decay

too many neutrons, ${}_7N^{16} \rightarrow {}_8O^{16} + \beta^- + \bar{\nu}$ M(A,Z) > M(A, Z+1) β^- - decay

A competing process with positron decay is electron capture (EC). M(A, Z+1) > M(A,Z)

twice the electron rest mass (1.02 Mev).

two exceptions, A = 113 123. Yet there are several exceptions, H^2, Li^6, B^{10} and N^{14}.

Radioactive-Series Decay

probability of a decay during a small time interval Δt. $P(\Delta t) = \lambda \Delta t$

the survival probability S(t). → $e^{-\lambda t}$ $S(t_{1/2}) = 1/2$ → $t_{1/2} = \ell n 2/\lambda = 0.693/\lambda$ $\tau = \frac{1}{\lambda}$

λ N(t), where N is the number of radioisotope atoms at time t, is called *activity*.

1 Ci = 3.7 x 10^{10} disintegrations/sec, 1 Bq = 2.7 x 10^{-11} Ci rate of radioactive decay.

Radioisotope Production by Bombardment

$$\frac{dN(t)}{dt} = Q_o - \lambda N(t) \qquad N(t) = \frac{Q_o}{\lambda}(1 - e^{-\lambda T})e^{-\lambda(t-T)} \qquad N(t) = \frac{Q_o}{\lambda}\left(1 - e^{-\lambda t}\right), \quad t < T$$

Radioisotope Production in Series Decay

$$\overset{\lambda_1}{N_1 \rightarrow} \overset{\lambda_2}{N_2 \rightarrow} N_3 \text{ (stable)}$$

$$\frac{dN_1(t)}{dt} = -\lambda_1 N_1(t)$$

$$\frac{dN_2(t)}{dt} = \lambda_1 N_1(t) - \lambda_2 N_2(t)$$

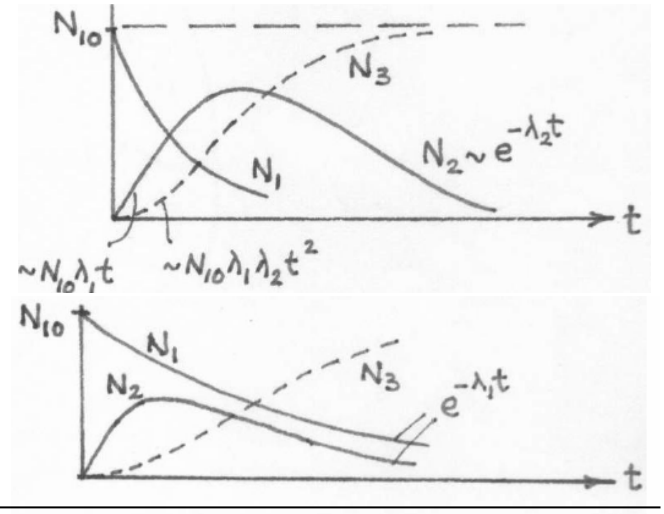
$$\frac{dN_3(t)}{dt} = \lambda_2 N_2(t)$$

$$N_1(t) = N_{10} e^{-\lambda_1 t}$$

$$N_2(t) = N_{10} \frac{\lambda_1}{\lambda_2 - \lambda_1}\left(e^{-\lambda_1 t} - e^{-\lambda_2 t}\right)$$

$$N_3(t) = N_{10} \frac{\lambda_1 \lambda_2}{\lambda_2 - \lambda_1}\left(\frac{1 - e^{-\lambda_1 t}}{\lambda_1} - \frac{1 - e^{-\lambda_2 t}}{\lambda_2}\right)$$

Series Decay with Short-Lived Parent



Series Decay with Short-Lived Parent

$N_2(t) \approx N_{10}\dfrac{\lambda_1}{\lambda_2}e^{-\lambda t}$ $\quad \lambda_2 N_2(t) \approx \lambda_1 N_1(t)$

secular equilibrium.

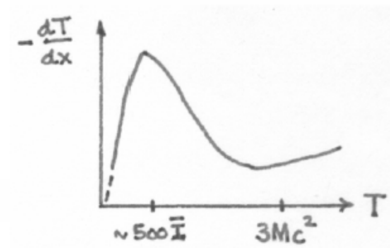
Charged-Particle Interactions: Stopping Power, Collisions and Ionization

Inelastic Collision with Atomic Electrons. ionization excitation of the atomic electrons

Inelastic Collision with a Nucleus. radiate (bremsstrahlung)

Elastic Collision with a Nucleus. Rutherford scattering.

Elastic Collision with Atomic Electrons



Stopping Power: Energy Loss of Charged Particles in Matter

$$-\frac{dT}{dx} = \frac{4\pi z^2 e^4 nZ}{m_e v^2}\ell n\left(\frac{2m_e v^2}{\bar{I}}\right)$$

nonrelativistic $\quad \ell n\left(\dfrac{2m_e v^2}{\bar{I}}\right) - \ell n\left(1-\dfrac{v^2}{c^2}\right) - \dfrac{v^2}{c^2}$

The result is valid

$$\frac{ze^2}{\hbar v} = \left(\frac{e^2}{\hbar c}\right)\frac{z}{v/c} = \frac{z}{137(v/c)} << 1$$

$$-\left(\frac{dT}{dx}\right)_{class} = \frac{4\pi z^2 e^4 nZ}{m_e v^2}\ell n\left[\frac{M\hbar v}{2ze^2(m_e+M)}\frac{2m_e v^2}{\bar{I}}\right] \text{ if } \frac{ze^2}{\hbar v} >> 1$$

Charged-Particle Interactions: Radiation Loss, Range $\quad \lambda_{min} = hc/T \quad \nu_{max} = T/h$

$$\sigma_{rad} \sim \frac{Z^2}{137}\left(\frac{e^2}{m_e c^2}\right)^2$$ cm^2/nucleus $\quad e^2/m_e c^2 = r_e = 2.818 \times 10^{-13}$ cm

$$\left[\frac{d\sigma}{d(h\nu)}\right]_{rad} = \sigma_o BZ^2\frac{T+m_e c^2}{T}\frac{1}{h\nu}$$

$$-\left(\frac{dT}{dx}\right)_{rad} = n(T+m_e c^2)\sigma_{rad}$$ ergs/cm

$$nZ = (\rho N_o/A)Z = \rho N_o(Z/A)$$

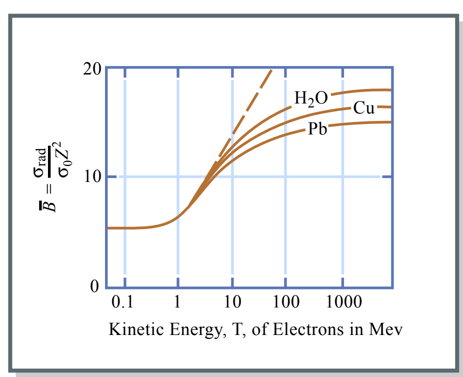


Figure by MIT OCW. Adapted from Evans.

Constant	Value	Unit	
		mks	cgs
Speed of light in vacuum c	2.997925(1)	x 10^{8} m s^{-1}	x 10^{10} cm s^{-1}
Elementary charge e	1.60210(2) 4.80298(7)	10^{-19}	C 0^{-20} emu 1 10^{-10} esu
Avogadro's number N	6.02252(9)	10^{26} $kmole^{-1}$	10^{23} $mole^{-1}$
Mass unit	1.66043(2)	10^{-27} kg	10^{-24} g
Electron rest mass m_0	9.10908(13) 5.48597(3)	10^{-31} kg 10^{-4} u	10^{-28} g 10^{-4} u
Proton rest mass M_P	1.67252(3) 1.00727663(8)	10^{-27} kg u	10^{-24} g u
Neutron rest mass M_n	1.67482(3) 1.0086654(4)	10^{-27} kg u	10^{-24} g u
Faraday constant Ne	9.64870(5) 2.89261(2)	10^{4} C $mole^{-1}$	10^{3} emu 10^{14} esu
Planck constant h $\hbar = h/2$	6.62559(16) 1.054494(25)	10^{-34} J s 10^{-34} J s	10^{-27} erg s 10^{-27} erg s
Charge-to-mass ratio for electron e/m_0	1.758796(6) 5.27274(2)	10^{11} C kg^{-1}	10^{7} emu 10^{17} esu
Rydberg constant 2 $^{2}m_0e^4/h^3c$	1.0973731(1)	10^{7} m^{-1}	10^{5} cm^{-1}
Bohr radius $\hbar^2/m_0e^2$	5.29167(2)	10^{-11} m	10^{-9} cm
Compton wavelength of electron h/m_0c $\hbar/m_0c$	2.42621(2) 3.86144(3)	10^{-12} m 10^{-13} m	10^{-10} cm 10^{-11} cm
Compton wavelength of proton h/M_pc $\hbar/M_pc$	1.321398(13) 2.10307(2)	10^{-15} m 10^{-16} m	10^{-13} cm 10^{-14} cm

Figure by MIT OCW. Adapted from Meyerhof, Appendix D.

Conversion Factor	Value
1 electron volt	1.60210(2) x 10^{-19} J 1.60210(2) x 10^{-12} erg 8065.73(8) cm^{-1} 2.41804(2) x 10^{-14} s^{-1}
$E_r\lambda_r$	12398.10(13) x 10^{-8} ev cm
1 u	931.478(5) Mev
Proton mass M_pc^2	938.256(5) Mev
Neutron mass M_nc^2	939.550(5) Mev
Electron mass m_0c^2	511006(2) ev
Rydberg $2\pi^2m_0e^4/h^2$	2.17971(5) x 10^{-11} erg 13.60535(13) ev
Gas constant	8.31434 x 10^{7} erg $mole^{-1}$ deg^{-1} 0.082053 liter atm $mole^{-1}$ deg^{-1} 82.055 cm^3 atm $mole^{-1}$ deg^{-1} 1.9872 cal_{th} $mole^{-1}$ deg^{-1}
Standard volume of ideal gas at NTP	22413.6 cm^3 $mole^{-1}$
$\frac{\text{Mass on physical scale } (O^{16} = 16)}{\text{Mass on unified scale } (C^{12} = 12)}$	1.000317917(17)
$\frac{\text{Mass on chemical scale } (O = 16)}{\text{Mass on unified scale } (C^{12} = 12)}$	1.000043(5)

Figure by MIT OCW. Adapted from Evans

Lecture 9:

$$S_n = [M(A-1,Z) + M_n - M(A,Z)]c^2$$

$$-\frac{\hbar}{2m}\frac{d^2 u_\ell}{dr^2} + \left[\frac{\ell(\ell+1)\hbar^2}{2mr^2} + V(r)\right] u_\ell(r) = E u_\ell(r)$$

$$E_\nu = \hbar\omega(\nu + 3/2) = \hbar\omega(n_x + n_y + n_z + 3/2) \qquad H = \frac{p^2}{2m} + V(r) + V_{so}(r)\underline{s}\cdot\underline{L}$$

$$\left|\ell, m_\ell, s, m_s\right\rangle \equiv Y_\ell^{m_\ell}\chi_s^{m_s} \qquad S^2\chi_s^{m_s} = s(s+1)\hbar^2\chi_s^{m_s}, \qquad \text{s=1/2}$$

$$S_z\chi_s^{m_s} = m_s\hbar\chi_s^{m_s}, \qquad -s \le m_s \le s \qquad \underline{j} = \underline{S} + \underline{L} \qquad \underline{S}\cdot\underline{L} = (j^2 - S^2 - L^2)/2$$

$$j^2\left|jm_j\ell s\right\rangle = j(j+1)\hbar^2\left|jm_j\ell s\right\rangle, \qquad |\ell - s| \le j \le \ell + s$$

$$j_z\left|jm_j\ell s\right\rangle = m_j\hbar\left|jm_j\ell s\right\rangle, \qquad -j \le m_j \le j$$

$$L^2\left|jm_j\ell s\right\rangle = \ell(\ell+1)\hbar^2\left|jm_j\ell s\right\rangle, \qquad \ell = 0, 1, 2, \ldots$$

$$S^2\left|jm_j\ell s\right\rangle = s(s+1)\hbar^2\left|jm_j\ell s\right\rangle, \qquad s = ½$$

Nucleon occupation = 2j+1
J= $\ell \pm ½$

Lecture 10

$$B(A,Z) \equiv [ZM_H + NM_n - M(A,Z)]c^2 \qquad Q \equiv [(M_i + M_I) - (M_f + M_F)]c^2$$

$$T_i + M_i c^2 + T_I + M_I c^2 \rightarrow T_f + M_f c^2 + T_F + M_F c^2 \qquad Q = T_f + T_F - (T_i + T_I)$$

$$Q = B(f) + B(F) - B(i) - B(I)$$

$$S_a = [M_a(A',Z') + M(A-A',Z-Z') - M(A,Z)]c^2$$

$$S_n = [M_n + M(A-1,Z) - M(A,Z)]c^2 = \text{B(A,Z)} - \text{B (A-1, Z)}$$

Lecture 11

$\delta = a_p / \sqrt{A}$ even-even nuclei

$= 0$ even-odd, odd-even nuclei

$= -a_p / \sqrt{A}$ odd-odd nuclei

a_v	a_s	a_c	a_a	a_p	
16	18	0.72	23.5	11	Mev

$$B(A,Z) = a_v A - a_s A^{2/3} - a_c \frac{Z(Z-1)}{A^{1/3}} - a_a \frac{(N-Z)^2}{A} + \delta$$

Lecture 12

$P(dt) = \lambda\, dt$

$1 - P(dt)$

$[1 - P(dt)]^2$

$[1 - P(dt)]^n$

$$T = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda}.$$

Formulas for Energy Release (Q), in Terms of Mass Differences, Δ_P and Δ_D, of Parent and Daughter Atoms

Type of Decay	Formula
α	$Q_\alpha = \Delta_P - \Delta_D - \Delta_{Hc}$
β^-	$Q_{\beta^-} = \Delta_P - \Delta_D$
γ	$Q_{IT} = \Delta_P - \Delta_D$
EC	$Q_{EC} = \Delta_P - \Delta_D - E_B$
β^+	$Q_{\beta^+} = \Delta_P - \Delta_D - 2mc^2$

Figure by MIT OCW. Adapted from Meyerhof, Appendix D.

$$\bar{t} = \frac{dt}{|dN|} = \frac{1}{\lambda N}$$

$$\left[1 - \frac{dt}{\bar{t}}\right]^{t/dt} \frac{dt}{\bar{t}} \xrightarrow[dt \to 0]{} e^{-t/\bar{t}} \frac{dt}{\bar{t}}$$

$$\frac{dN_\alpha}{dt} = \lambda_\alpha N$$
$$= \lambda_\alpha N_0 e^{-(\lambda_\alpha + \lambda_\beta)t}$$

$$-dN = dN_\alpha + dN_\beta$$
$$= \lambda_\alpha N\, dt + \lambda_\beta N\, dt$$

$$N = N_0 e^{-(\lambda_\alpha + \lambda_\beta)t} = N_0 e^{-\lambda_{tot} t}$$

$$\frac{dN_1}{dt} = -\lambda_1 N_1$$
$$\frac{dN_2}{dt} = \lambda_1 N_1 - \lambda_2 N_2$$
$$\frac{dN_3}{dt} = \lambda_2 N_2$$

$$N_1 = N_{10} e^{-\lambda_1 t}$$

$$N_2 = \frac{N_{10}\lambda_1}{\lambda_2 - \lambda_1}\left(e^{-\lambda_1 t} - e^{-\lambda_2 t}\right)$$

$$N_3 = \frac{N_{10}\lambda_1\lambda_2}{\lambda_2 - \lambda_1}\left(\frac{1 - e^{-\lambda_1 t}}{\lambda_1} - \frac{1 - e^{-\lambda_2 t}}{\lambda_2}\right)$$

$$\frac{dN}{dt} = Q - \lambda N$$

$$\frac{dN}{Q - \lambda N} = dt$$

$$\frac{d(Q - \lambda N)}{Q - \lambda N} = -\lambda\, dt$$

$$Q - \lambda N = (Q - \lambda N)_{t=0} e^{-\lambda t}$$

$$N = \frac{Q}{\lambda}\left(1 - e^{-\lambda t}\right)$$

Lecture 13

$$\frac{p_e^2}{2m_e} = \frac{2(ze^2)^2}{m_e b^2 v^2} \qquad -\frac{dT}{dx} = \frac{4\pi z^2 e^4 nZ}{m_e v^2} \ell n\left(\frac{2m_e v^2}{\bar{I}}\right)$$

$$-\frac{dT}{dx} = \frac{2\pi e^4 nZ}{m_e v^2}\left[\ell n\left(\frac{m_e v^2 T}{\bar{I}^2 (1-\beta^2)}\right) - \beta^2\right]$$

Lecture 14

$$\sigma_{ion} = \frac{2\alpha Z}{\beta^4} \ell n\left(\frac{\sqrt{2}T}{\bar{I}}\right)$$ ionization

$$\sigma_{nuc} = \frac{\alpha Z^2}{4\beta^4}$$ backscattering by nuclei

$$\sigma_{el} = \frac{2\alpha Z}{\beta^4}$$ elastic scattering by atomic electrons

$$\sigma_{rad}' = \frac{8\alpha}{3\pi}\frac{1}{137}\frac{Z^2}{\beta^2}$$ *bremsstrahlung*

$$nZ = (\rho N_o / A)Z = \rho N_o (Z/A)$$

$$nZ = (\rho N_o / A)Z = \rho N_o (Z / A)$$

$$\frac{(dT / dx)_{rad}}{(dT / dx)_{ion}} \approx Z\left(\frac{m_e}{M}\right)^2 \left(\frac{T}{1400 m_e c^2}\right)$$

$$i = \frac{1}{W}\left(-\frac{dT}{dx}\right)$$

$$R \propto \int_0^{T_o} T dT = T_o^2$$

$$R \propto \int_0^{T_o} dT = T_o$$

$$\frac{R}{R_1} = \frac{\rho_1 \sqrt{A}}{\rho \sqrt{A_1}}$$

$$R = 3.2x10^{-4} \frac{\sqrt{A}}{\rho} \text{ x } R_{air}$$