

Problem Set No.2

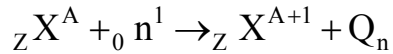
1. It is known that the proton and neutron in a deuteron can be broken apart by irradiating the deuteron with high-energy gamma rays. This is called the photo-nuclear reaction. It is found that the threshold energy of the photo-nuclear reaction is  $E_\gamma = 2.225\text{MeV}$ . Based on this energy and the known masses,  $M_H = 1.0078252\text{amu}$  and  $M_D = 2.0141022\text{amu}$ , calculate the neutron mass.
2. Show by the theory of electrostatic that the repulsive energy due to a uniform distribution of charge  $Ze$  in bulk of a sphere of radius  $R$  is  $E = \frac{3}{5} \frac{(Ze)^2}{R}$ . Show that the coefficient in this expression will become  $1/2$ , if the charge is instead uniformly distributed over the surface. What would be the coefficient, if the charge is uniformly distributed over a spherical shell of an inner radius  $R_1$  and an outer radius  $R$ ?
3. The semi-empirical mass formula with the Coulomb term derived from the assumption of a homogeneous volume distribution of charge yields values for the mass which, in the region of heavy nuclei, differ from measured values by at most 0.01 percent. Using  $U^{238}$  as an example ( $M = 238.12\text{ amu}$ ), show that this error limit absolutely rules out the other alternative of a uniform surface distribution of charge, even if the mass discrepancy were due exclusively to the Coulomb term. One could nevertheless conceive of other forms of charge distribution; for instance, a homogeneous distribution throughout the volume of a hollow spherical shell having an inner radius  $R_1$  and an outer radius  $R$  (equal to the nuclear radius). What is the largest value of the ratio  $R_1/R$  commensurate with the above mass?
4. Use the expression derived in class

$$Z_{\text{stable}} = -k_2/2k_3, \quad k_2 = -[4a_a + (M_N - M_H)], \quad k_3 = \frac{4a_a}{A} \left( 1 + \frac{A^{2/3}}{4a_a/a_c} \right)$$

to establish whether  ${}_{54}\text{Xe}^{142}$  is  $\beta^-$  unstable or  $\beta^+$  unstable.

5. Show that all stable nuclides with  $A \geq 140$  are unstable with respect to emission of an alpha-particle ( ${}^4_2\text{He}$ ). As an example, you can show that the binding energy of an alpha-particle in  ${}_{92}\text{U}^{235}$  is negative and equal to  $-4.64\text{ MeV}$ . Do the same thing for  ${}_{94}\text{Pu}^{239}$ .

6. When a thermal neutron (a neutron with practically zero kinetic energy in the energy scale we are considering in this problem) is captured in a nucleus with  $Z$  protons and  $N$  neutrons by a reaction:



the resultant  $Q$ -value is called the binding energy of the last neutron.

(A) Show that the  $Q$ -value can be calculated in terms of the binding energy by a formula:

$$Q_n = B(A+1, Z) - B(A, Z) .$$

In the case where  ${}_Z X^A$  is a fissionable nucleus, the  $Q_n$  can also be considered as the excitation energy available for inducing fission reaction of the compound nucleus  ${}_Z X^{A+1}$ .

You can use the semi-empirical formula

$$B(A, Z) = a_v A - a_c \frac{Z^2}{A^{1/3}} - a_s A^{2/3} - a_a \frac{(A - 2Z)^2}{A} + \delta$$

where

$\delta = a_p A^{-3/4}$  for an e - e nucleus,  $\delta = -a_p A^{-3/4}$  for an o - o nucleus,  $\delta = 0$  for an e - o or o - e nucleus

The constants in the semi-empirical formula have values (in unit of MeV):

$$a_v = 14.1, \quad a_c = 0.595, \quad a_s = 13.0, \quad a_a = 19.0, \quad a_p = 33.5 .$$

(B) Consider the case of thermal neutron induced fission reaction. Calculate the excitation energy available for the case of  ${}_{92} U^{235}$  explicitly using the given information above.

(C) Do the same thing for the case of  ${}_{92} U^{238}$  and show that the excitation energy available for this latter case is approximately 1 MeV less than the former case.

It is known that thermal neutrons can induce fission in  ${}_{92} U^{235}$ , but cannot do so in  ${}_{92} U^{238}$ . It is also known that in the latter case one needs to use high energy neutrons of energies greater than 1 MeV to induce the fission reaction. Your calculation is the explanation of this experimental fact.