22.101 Fall 2003 Professor Sow-Hsin Chen Due: Sept.29/03

Problem Set No.2

1. It is known that the proton and neutron in a deuteron can be broken apart by irradiating the deuteron with high-energy gamma rays. This is called the photo-nuclear reaction. It is found that the threshold energy of the photo-nuclear reaction is $E_\gamma = 2.225\text{MeV}$. Based on this energy and the known masses, $M_H = 1.0078252\text{amu}$ and $M_D = 2.0141022\text{amu}$, calculate the neutron mass.

2. Show by the theory of electrostatic that the repulsive energy due to a uniform distribution of charge Ze in bulk of a sphere of radius R is $E = \frac{3}{5}\frac{(Ze)^2}{R}$. Show that the coefficient in this expression will become 1/2, if the charge is instead uniformly distributed over the surface. What would be the coefficient, if the charge is uniformly distributed over a spherical shell of an inner radius R_1 and an outer radius R?

3. The semi-empirical mass formula with the Coulomb term derived from the assumption of a homogeneous volume distribution of charge yields values for the mass which, in the region of heavy nuclei, differ from measured values by at most 0.01 percent. Using U^{238} as an example $(M = 238.12\ \text{amu})$, show that this error limit absolutely rules out the other alternative of a uniform surface distribution of charge, even if the mass discrepancy were due exclusively to the Coulomb term. One could nevertheless conceive of other forms of charge distribution; for instance, a homogeneous distribution throughout the volume of a hollow spherical shell having an inner radius R_1 and an outer radius R (equal to the nuclear radius).What is the largest value of the ratio R_1/R commensurate with the above mass ?

4. Use the expression derived in class

$$Z_{\text{stable}} = -k_2/2k_3,\ \ k_2 = -\left[4a_a + (M_N - M_H)\right],\ \ k_3 = \frac{4a_a}{A}\left(1 + \frac{A^{2/3}}{4a_a / a_c}\right)$$

to establish whether ${}_{54}Xe^{142}$ is β^- unstable or β^+ unstable.

5. Show that all stable nuclides with $A \geq 140$ are unstable with respect to emission of an alpha-particle $({}_2He^4)$. As an example, you can show that the binding energy of an alpha-particle in ${}_{92}U^{235}$ is negative and equal to –4.64 MeV. Do the same thing for ${}_{94}Pu^{239}$.

6. When a thermal neutron (a neutron with practically zero kinetic energy in the energy scale we are considering in this problem) is captured in a nucleus with Z protons and N neutrons by a reaction:

$$_{Z}X^{A} +_{0} n^{1} \rightarrow_{Z} X^{A+1} + Q_{n}$$

the resultant Q-value is called the binding energy of the last neutron.

(A) Show that the Q-value can be calculated in terms of the binding energy by a formula:

$$Q_{n} = B(A+1, Z) - B(A, Z) \ .$$

In the case where $_{Z}X^{A}$ is a fissionable nucleus, the Q_n can also be considered as the excitation energy available for inducing fission reaction of the compound nucleus $_{Z}X^{A+1}$.
You can use the semi-empirical formula

$$B(A,Z) = a_{v}A - a_{c}\frac{Z^{2}}{A^{1/3}} - a_{s}A^{2/3} - a_{a}\frac{(A-2Z)^{2}}{A} + \delta$$

where

$\delta = a_{p}A^{-3/4}$ for an e - e nucleus, $\delta = -a_{p}A^{-3/4}$ for an o - o nucleus, $\delta = 0$ for an e - o or o - e nucleus

The constants in the semi-empirical formula have values (in unit of MeV):

$$a_{v} = 14.1, \ a_{c} = 0.595, \ a_{s} = 13.0, \ a_{a} = 19.0, \ a_{p} = 33.5 .$$

(B) Consider the case of thermal neutron induced fission reaction. Calculate the excitation energy available for the case of $_{92}U^{235}$ explicitly using the given information above.

(C) Do the same thing for the case of $_{92}U^{238}$ and show that the excitation energy available for this latter case is approximately 1 MeV less than the former case.

It is known that thermal neutrons can induce fission in $_{92}U^{235}$, but cannot do so in $_{92}U^{238}$. It is also known that in the latter case one needs to use high energy neutrons of energies greater than 1 MeV to induce the fission reaction. Your calculation is the explanation of this experimental fact.