THE SHELL MODEL

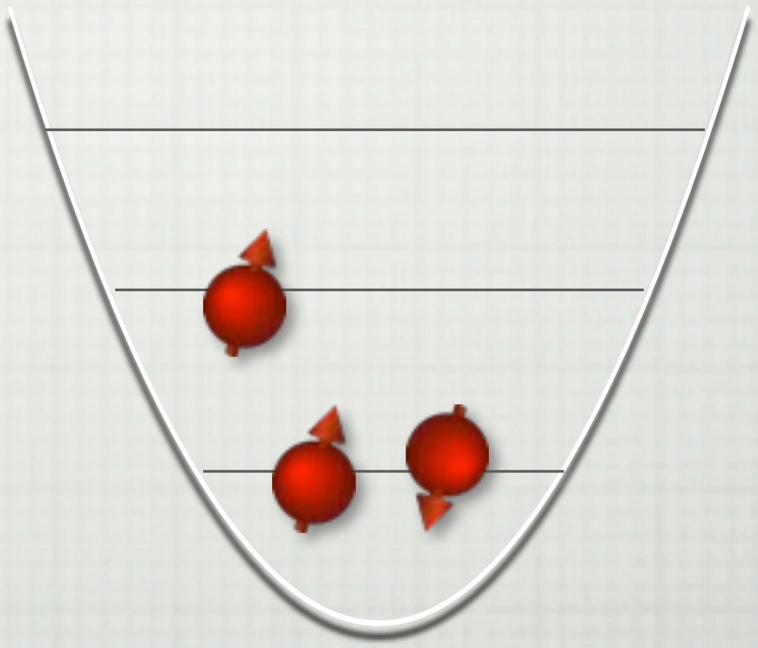
22.02

Introduction To Applied Nuclear Physics

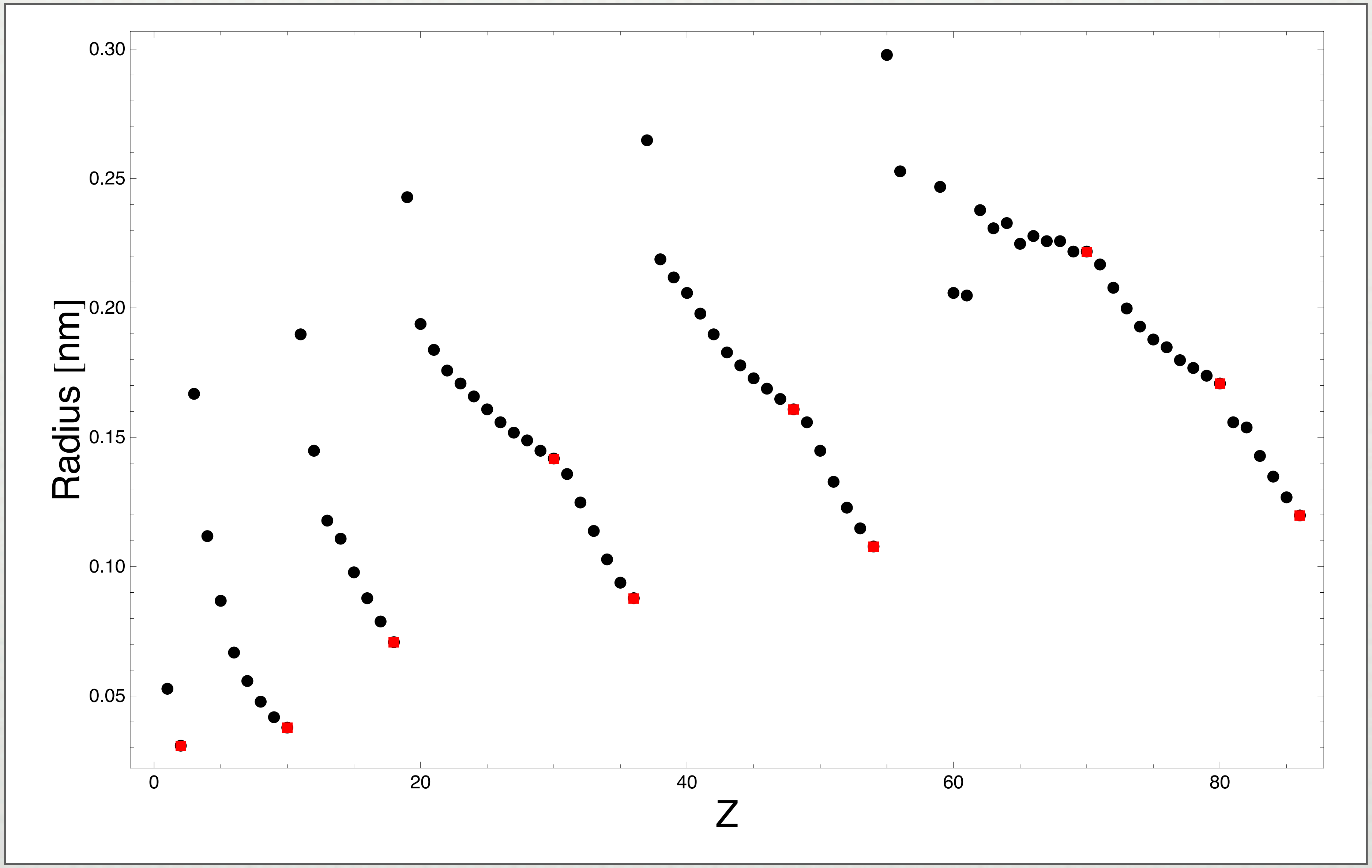
Spring 2012

Atomic Shell Model

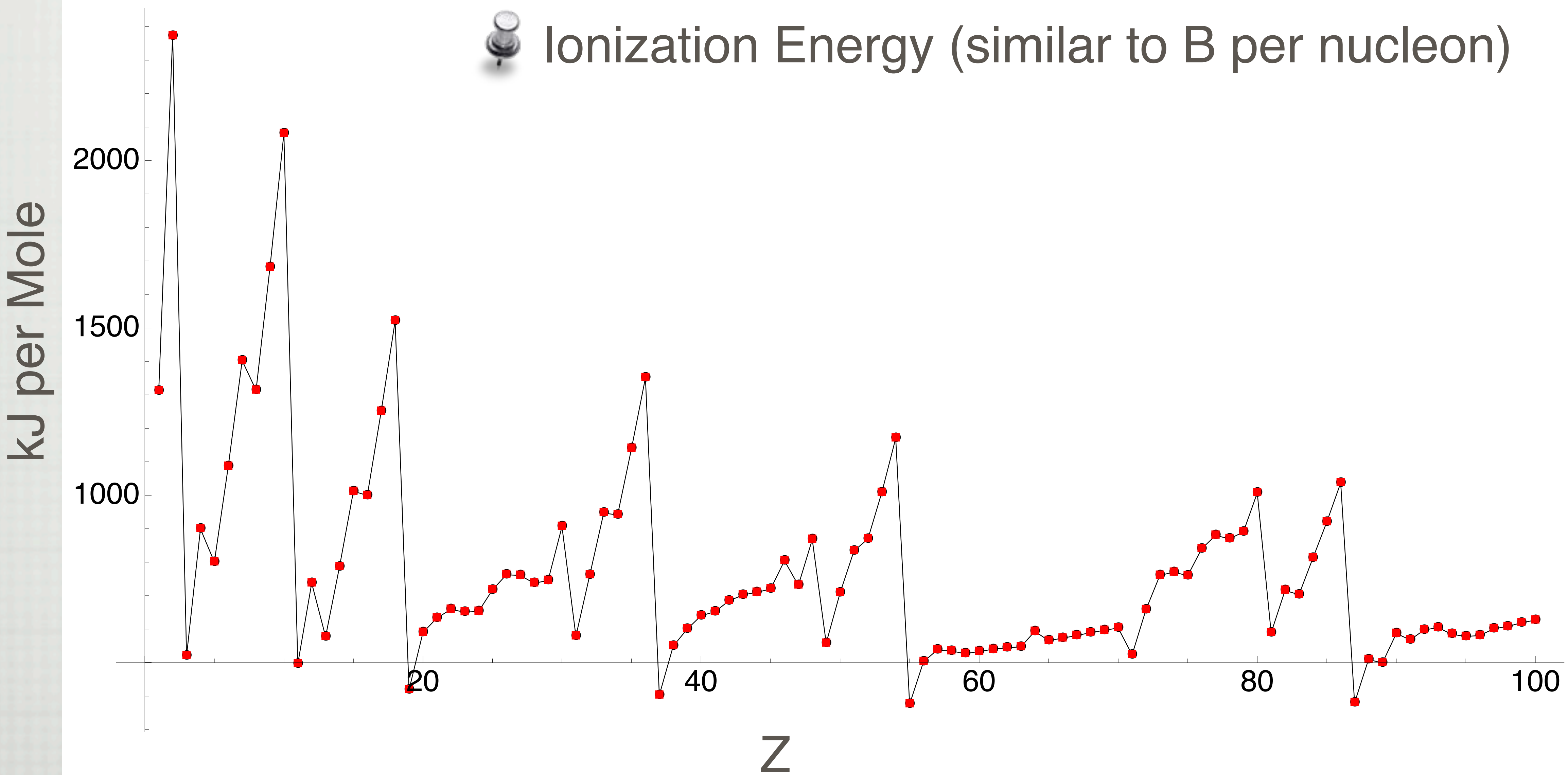
- Chemical properties show a periodicity
- Periodic table of the elements
- Add electrons into shell structure



Atomic Radius



Ionization Energy



ATOMIC STRUCTURE

- The atomic wavefunction is written as

$$|\psi\rangle = |n, l, m\rangle = R_{n,l}(r)Y_l^m(\vartheta,\varphi\,)$$

- where the labels indicate :
  - n : principal quantum number
  - l : orbital (or azimuthal) quantum number
  - m: magnetic quantum number
- The degeneracy is

$$\mathcal{D}(l) = 2(2l+1) \rightarrow \mathcal{D}(n) = 2n^2$$

AUFBAU PRINCIPLE

- The orbitals (or shells) are then given by the n-levels (?)

l	0	1	2	3	4	5	6
Spectroscopic notation	s	p	d	f	g	h	i
$\mathcal{D}(l)$	2	6	10	14	18	22	26
	historic structure				heavy nuclei		

n	$\mathcal{D}$(n)	e^- in shell
1	2	2
2	6	8
3	18	28

ATOMIC PERIODIC TABLE

H																	He
Li	Be											B	C	N	O	F	Ne
Na	Mg											Al	Si	P	S	Cl	Ar
K	Ca	Sc	Ti	V	Cr	Mn	Fe	Co	Ni	Cu	Zn	Ga	Ge	As	Se	Br	Kr
Rb	Sr	Y	Zr	Nb	Mo	Tc	Ru	Rh	Pd	Ag	Cd	In	Sn	Sb	Te	I	Xe
Cs	Ba	Lu	Hf	Ta	W	Re	Os	Ir	Pt	Au	Hg	Tl	Pb	Bi	Po	At	Rn
Fr	Ra																

AUFBAU PRINCIPLE

- The orbitals (or shells) are then given by close-by energy-levels

l	0	1	2	3	4	5	6
Spectroscopic notation	s	p	d	f	g	h	i
$\mathcal{D}(l)$	2	6	10	14	18	22	26
	historic structure				heavy nuclei		

n	$\mathcal{D}$(n)	e^- in shell
1	2	2
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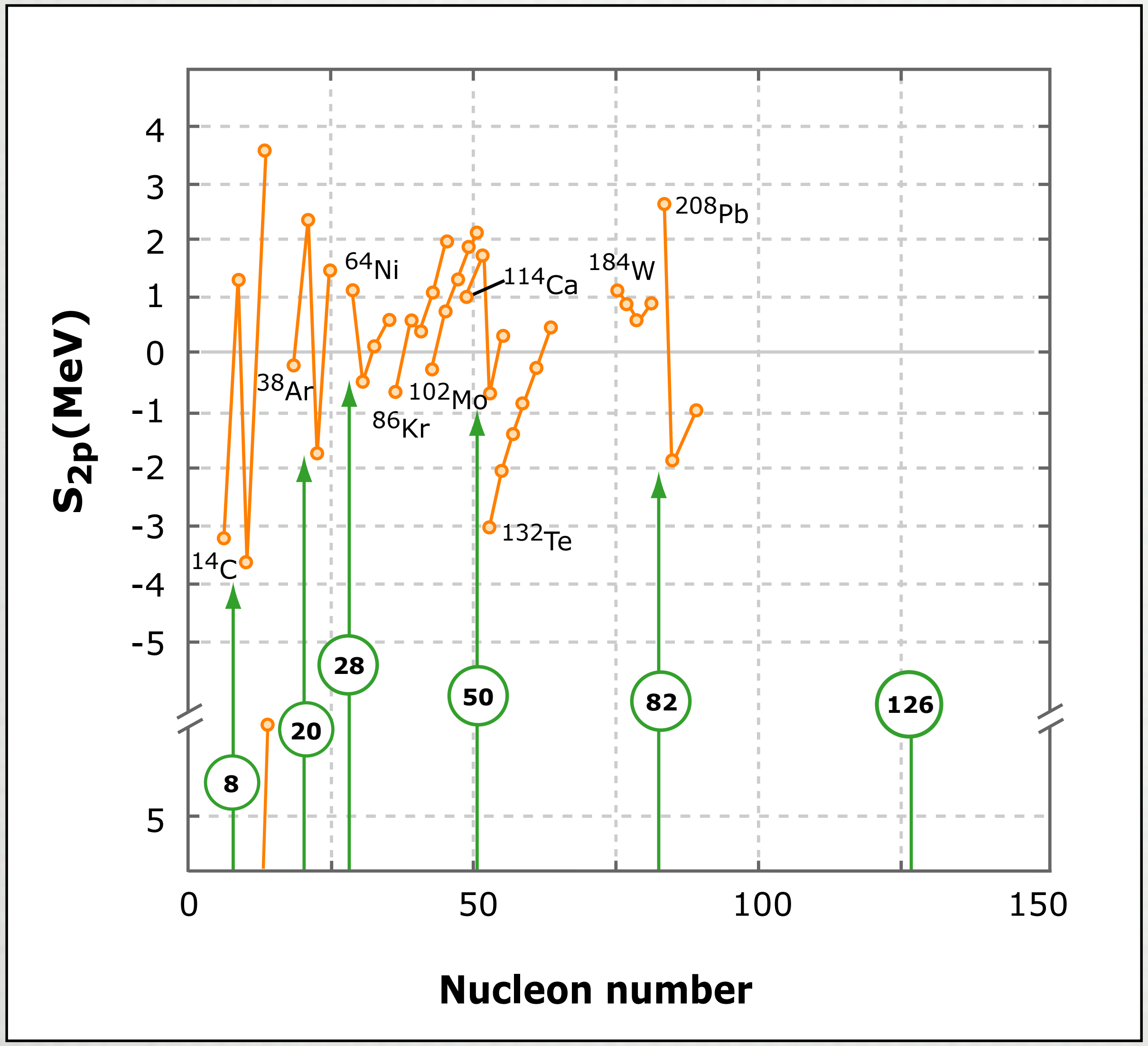
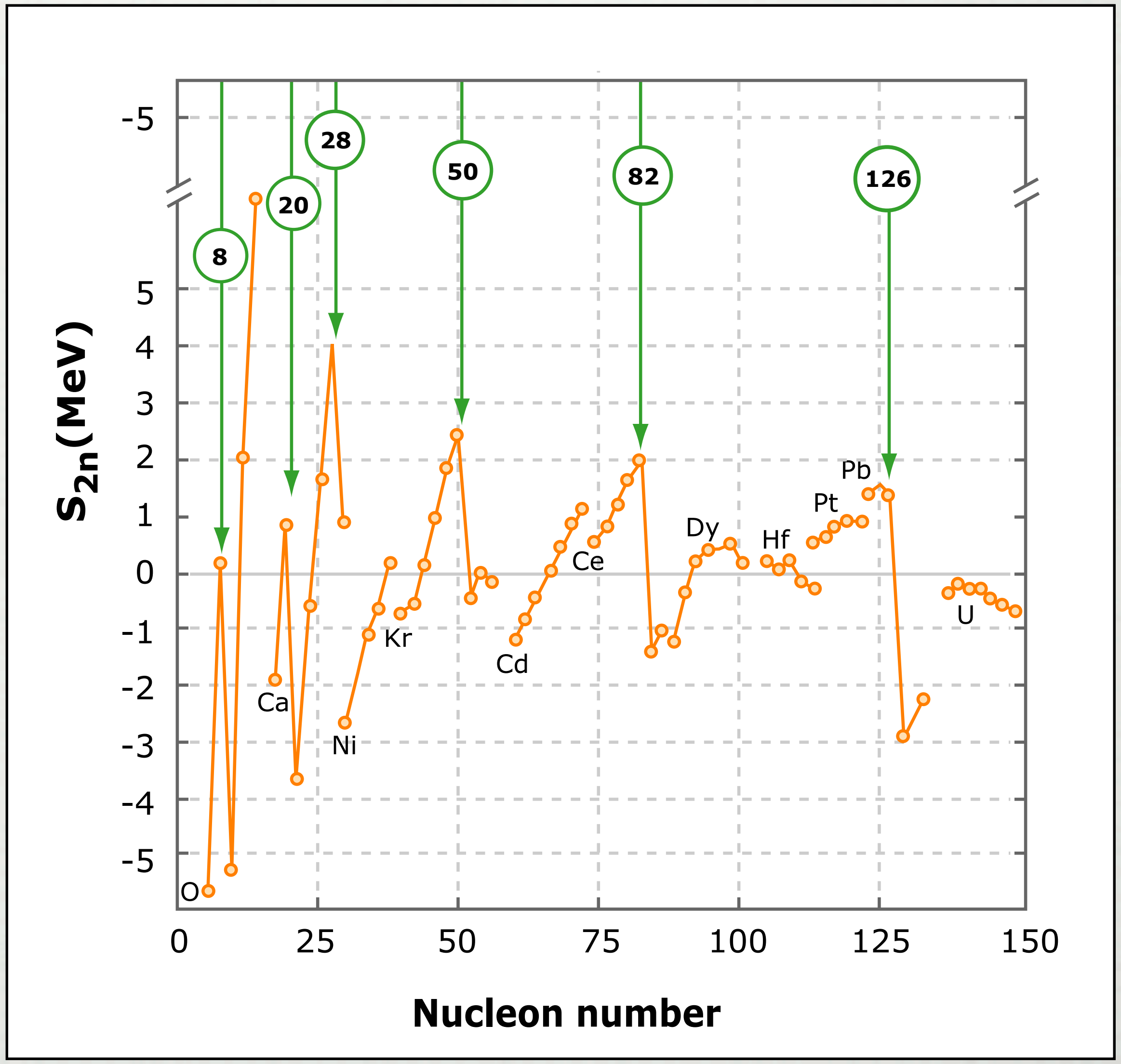
3s+3p form one level with # 10
4s is filled before 3d

Nuclear Shell Model

- Picture of adding particles to an external potential is no longer good: each nucleon contributes to the potential
- Still many evidences of a shell structure

Separation Energy



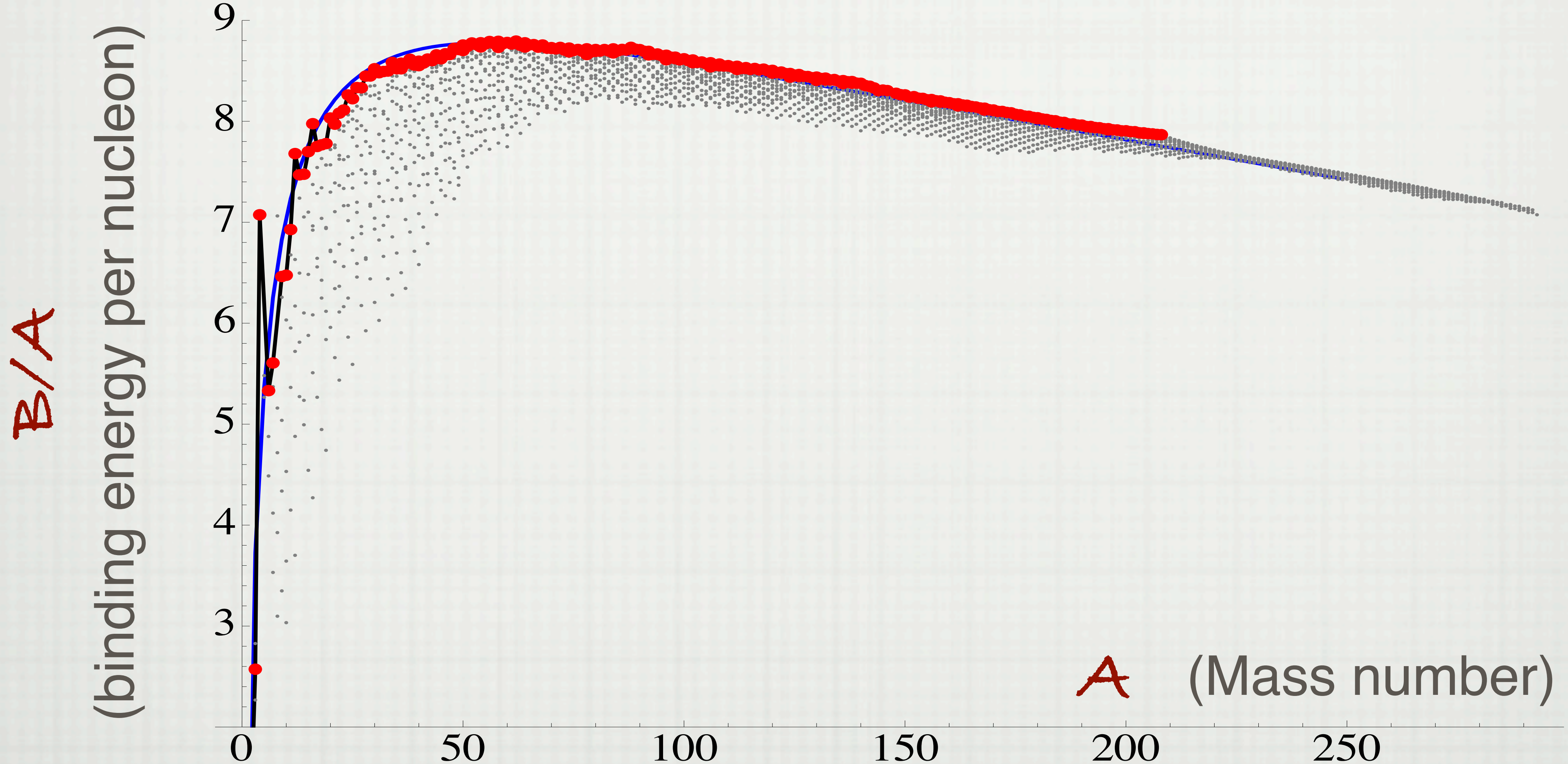


PROTON

NEUTRON

Image by MIT OpenCourseWare. After Krane.

B/A: JUMPS



- "Jumps" in Binding energy from experimental data

CHART of NUCLIDES (Z/A vs. A)

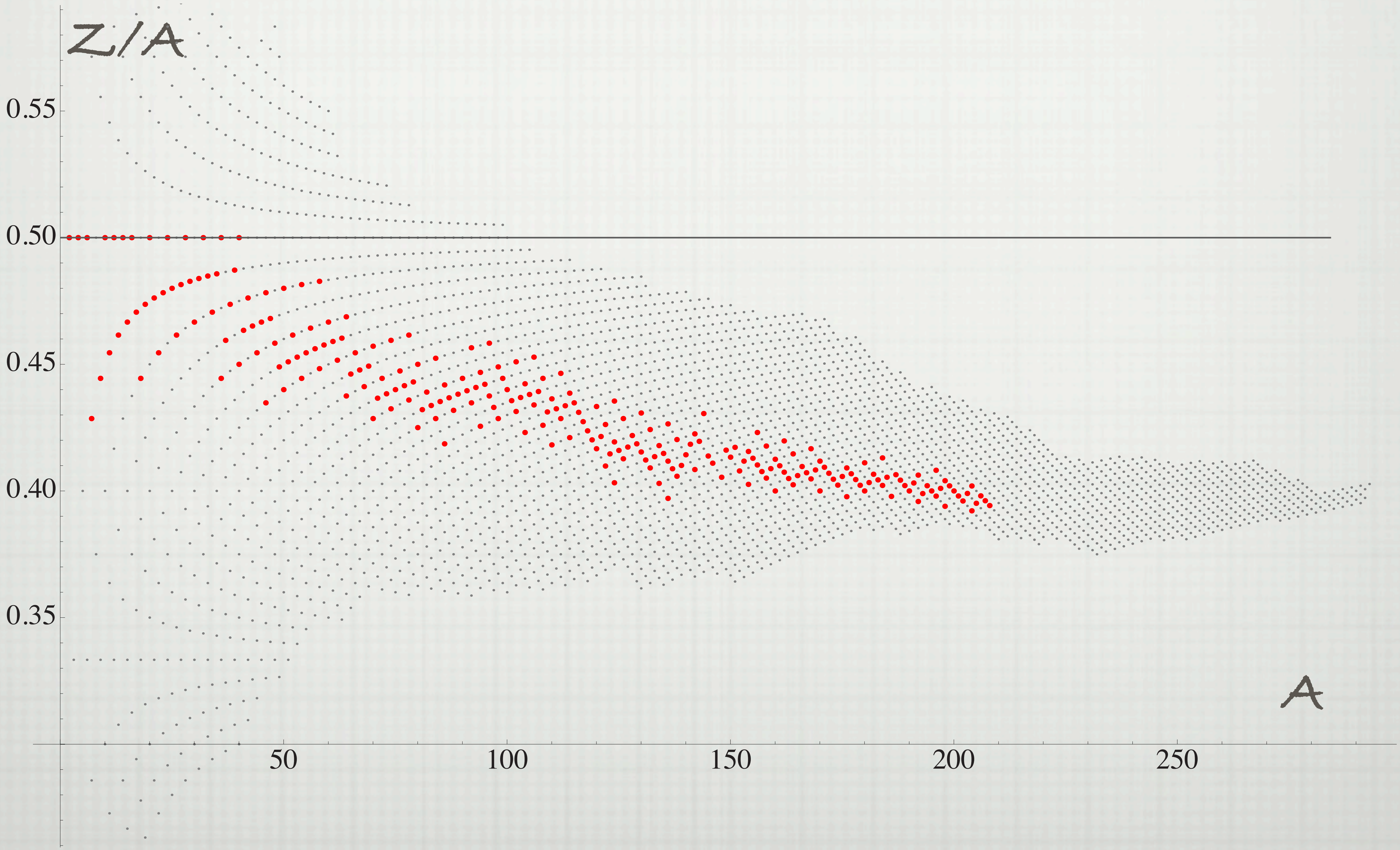


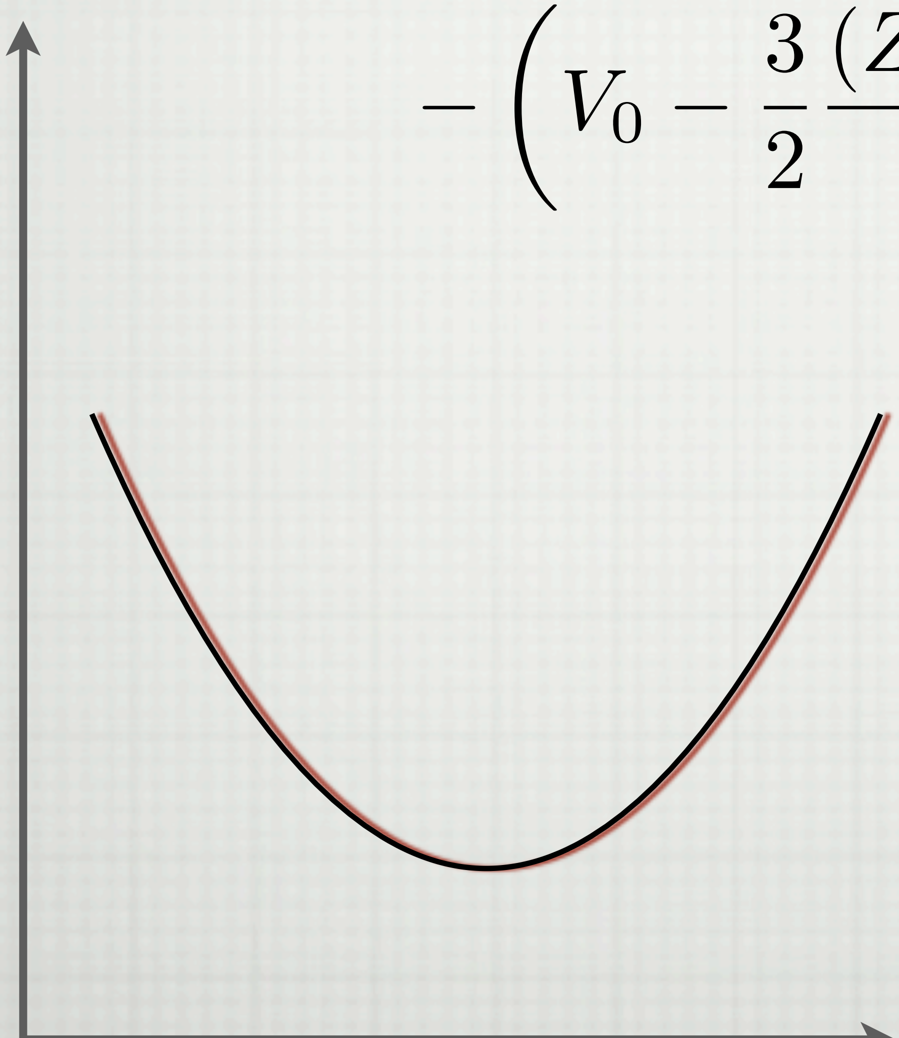
CHART OF NUCLIDES

- *"Periodic"*, more complex properties → nuclear structure

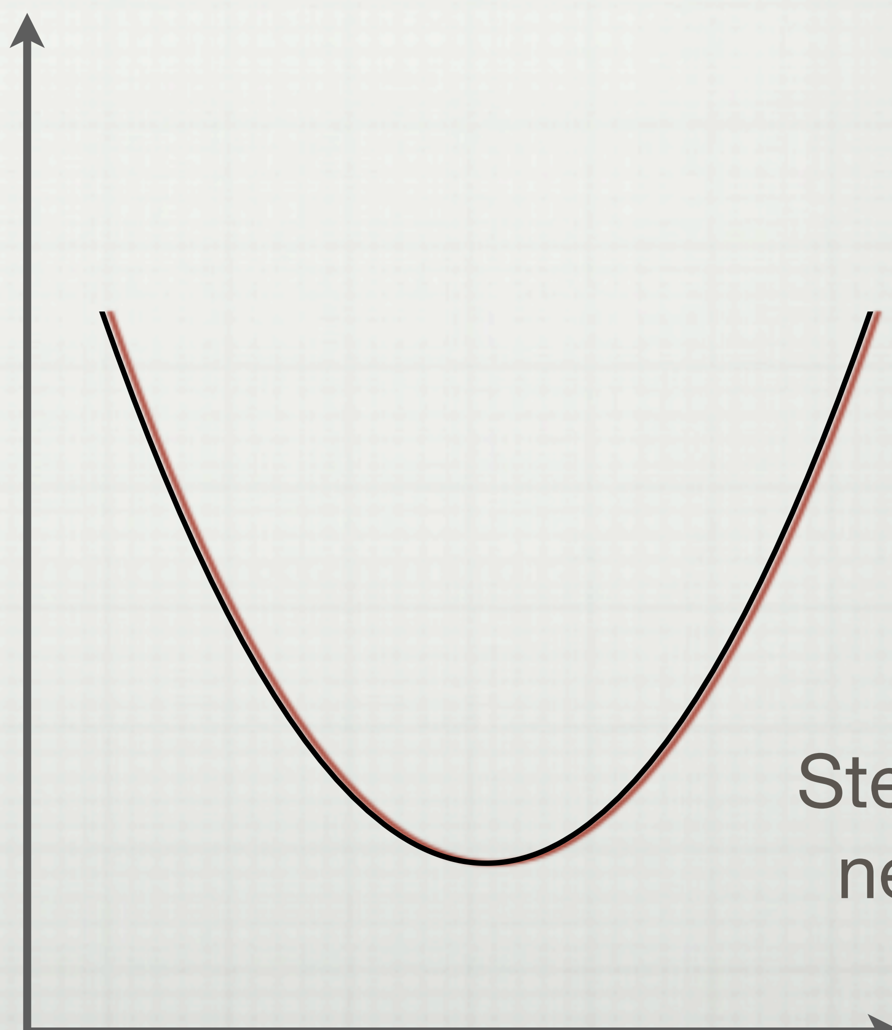
NUCLEAR POTENTIAL

$$V_{\mathrm{p}} = r^2 \left(\frac{V_0}{R_0^2} - \frac{(Z-1)e^2}{2R_0^3} \right) - \left(V_0 - \frac{3}{2} \frac{(Z-1)e^2}{R_0} \right)$$

$$V_{\mathrm{n}} = r^2 \left(\frac{V_0}{R_0^2} \right) - (V_0)$$



Harmonic potential

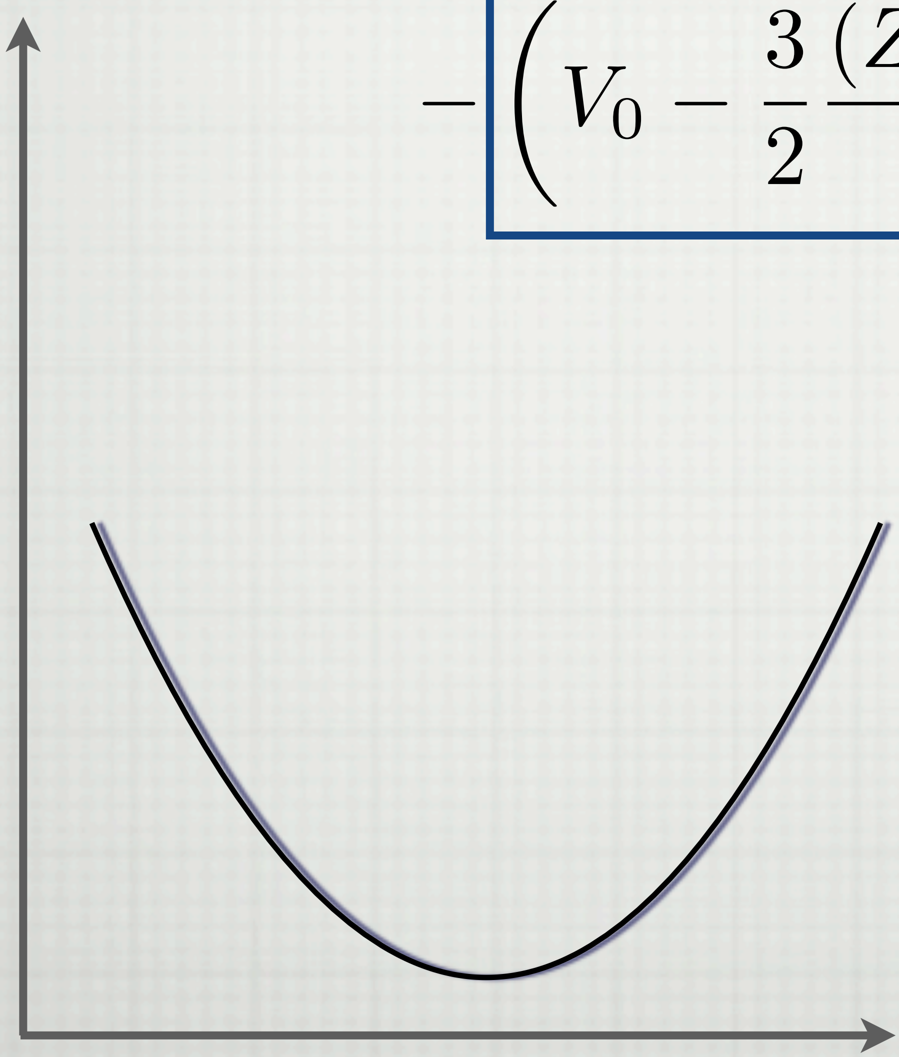


Steeper for neutrons

NUCLEAR POTENTIAL

$$V_{\rm p} = r^2 \left(\frac{V_0}{R_0^2} - \frac{(Z-1)e^2}{2R_0^3} \right) - \left(V_0 - \frac{3}{2} \frac{(Z-1)e^2}{R_0} \right)$$

$$V_{\rm n} = r^2 \left(\frac{V_0}{R_0^2} \right) - (V_0)$$

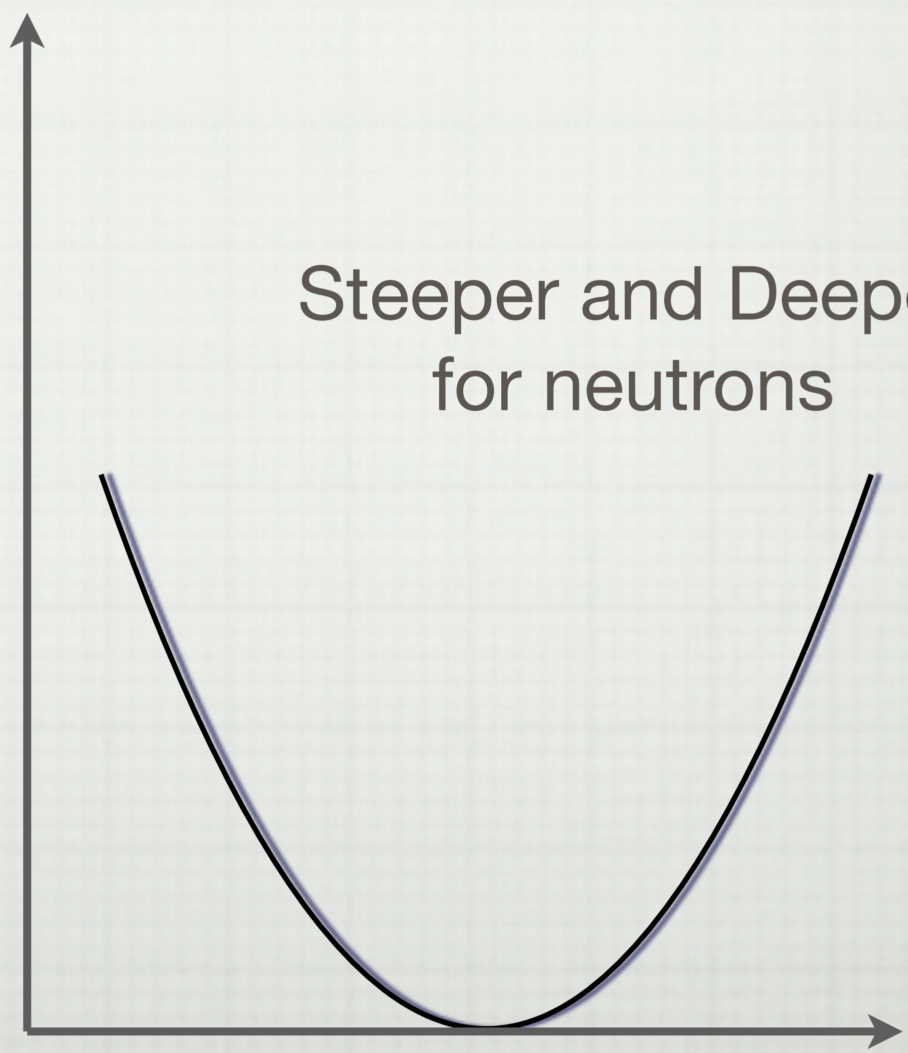


Harmonic potential

+

well depth

Steeper and Deeper for neutrons



Shell Mode

- Harmonic oscillator: solve (part of) the radial equation
  - including the angular momentum (centrifugal force term) we obtain the usual principal quantum number n = (N-l)/2+1

N	l	Spectroscopic Notation	$\mathcal{D}(N)$	$2\mathcal{D}(N)$	Cumulative #
0	0	1 s	1	2	2
1	1	1 p	3	6	8
2	0,2	2s,1d	6	12	20
3	1,3	2p,1f	10	20	40
4	0,2,4	3s,2d,1g	15	30	70

Spin-Orbit Coupling

- The spin-orbit interaction is given by $V_{SO} = \frac{1}{\hbar^2} V_{so}(r) \hat{\vec{l}} \cdot \hat{\vec{s}}$

- We can calculate the dot product

$$\left\langle \hat{\vec{l}} \cdot \hat{\vec{s}} \right\rangle = \frac{1}{2}(\hat{\vec{j}}^2 - \hat{\vec{l}}^2 - \hat{\vec{s}}^2) = \frac{\hbar^2}{2}[j(j+1) - l(l+1) - \frac{3}{4}]$$

- Because of the addition rules, $j = l \pm \frac{1}{2}$

$$\left\langle \hat{\vec{l}} \cdot \hat{\vec{s}} \right\rangle = \begin{cases} l\frac{\hbar^2}{2} & \text{for j=l+}\frac{1}{2} \\ -(l+1)\frac{\hbar^2}{2} & \text{for j=l-}\frac{1}{2} \end{cases}$$

Spin-Orbit Coupling

- when the spin is **aligned** with the angular momentum $j = l + \frac{1}{2}$
  the potential becomes more negative,
  i.e. the well is deeper and the state more tightly bound.

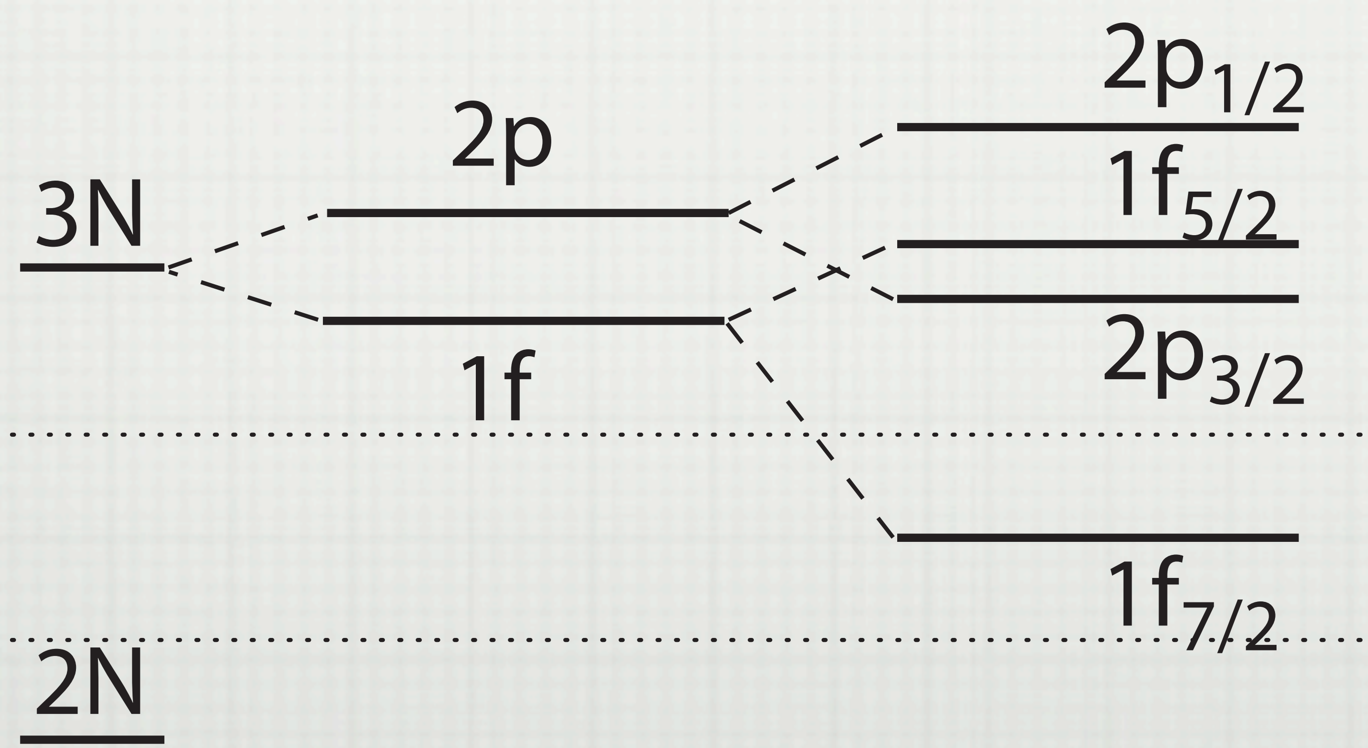
- when spin and angular momentum are **anti-aligned** $j = l - \frac{1}{2}$
  the system's energy is higher.

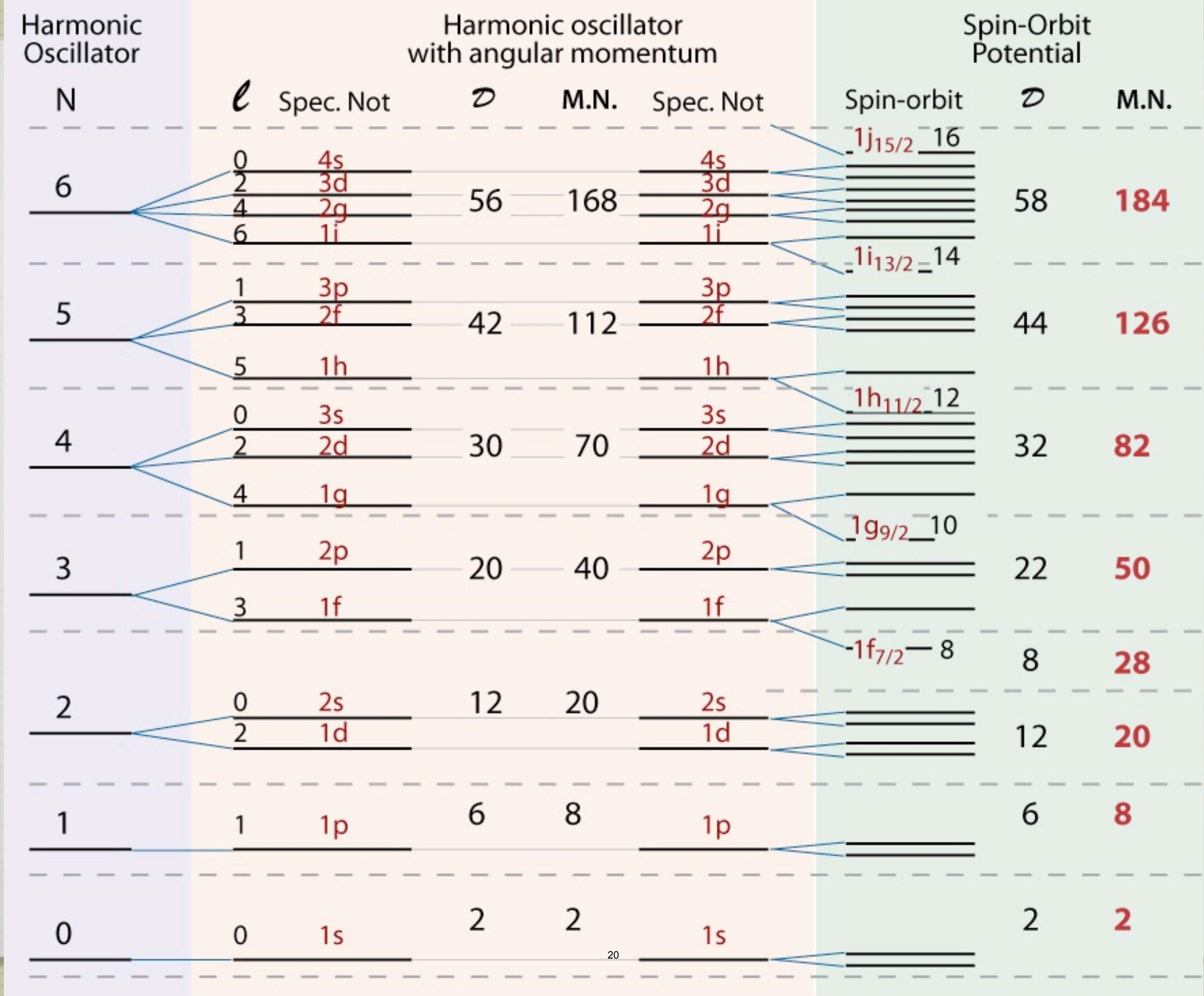
- The difference in energy is $$\Delta E = \frac{V_{so}}{2}(2l+1)$$

  Thus it increases with *l* .

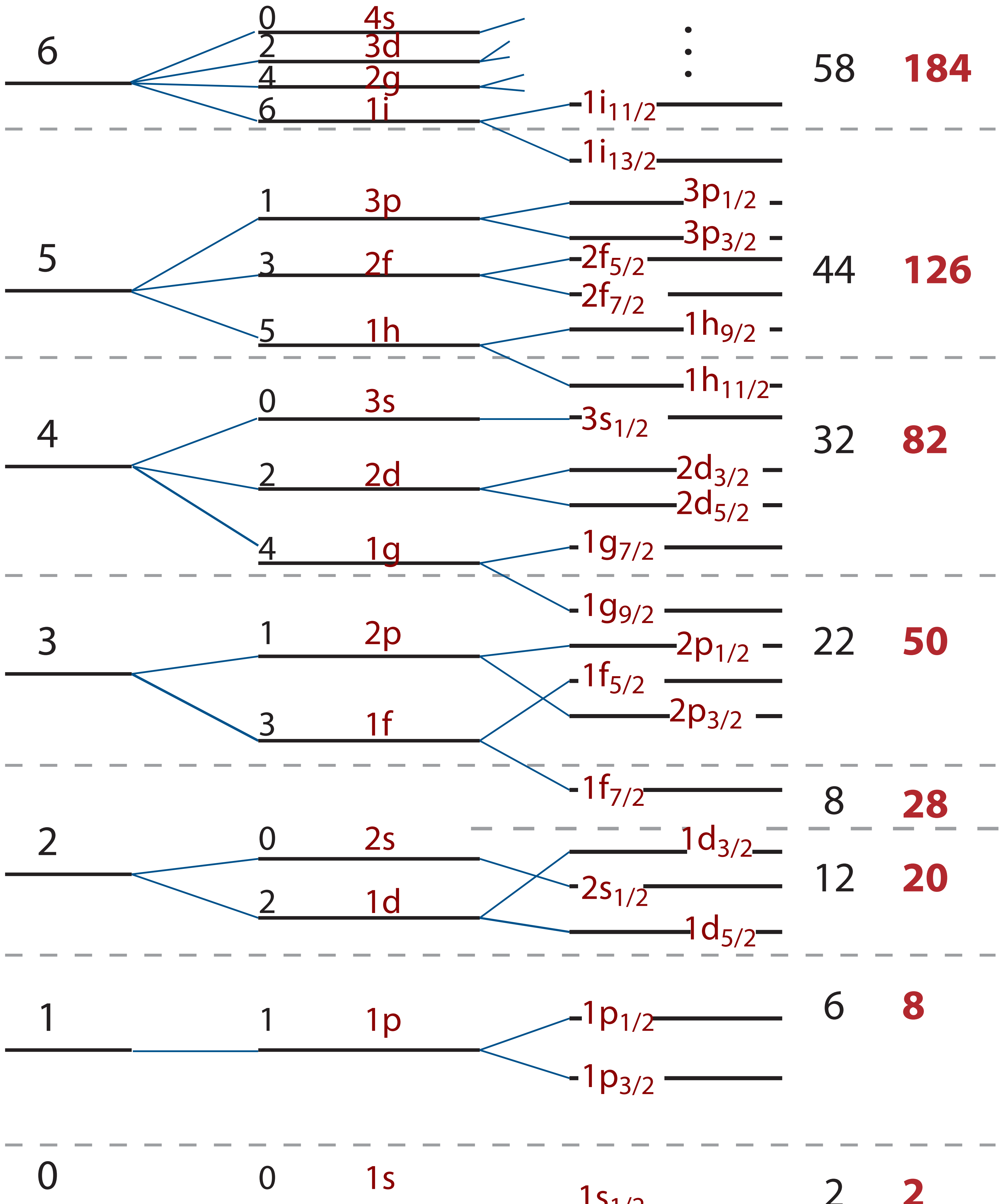
Example

- 3N level, with l=3 (1f level) j=7/2 or j=5/2
  - Level is pushed so down that it forms its own shell





Harmonic Oscillator
Harmonic oscillator with angular momentum
Spin-Orbit Potential
N
ℓ
Spec. Not
𝒟
M.N.
Spec. Not
Spin-orbit
𝒟
M.N.
6
0 4s
2 3d
4 2g
6 1i
56
168
4s
3d
2g
1i
1j15/2 16
1i13/2 14
58
184
5
1 3p
3 2f
5 1h
42
112
3p
2f
1h
1h11/2 12
44
126
4
0 3s
2 2d
4 1g
30
70
3s
2d
1g
1g9/2 10
32
82
3
1 2p
3 1f
20
40
2p
1f
1f7/2 8
22
50
8
28
2
0 2s
2 1d
12
20
2s
1d
12
20
1
1 1p
6
8
1p
6
8
0
0 1s
2
2
1s
2
2


6
0 4s
2 3d
4 2g
6 1i
1i11/2
1i13/2
58 184
5
1 3p
3 2f
5 1h
3p1/2
3p3/2
2f5/2
2f7/2
1h9/2
1h11/2
44 126
4
0 3s
2 2d
4 1g
3s1/2
2d3/2
2d5/2
1g7/2
1g9/2
32 82
3
1 2p
3 1f
2p1/2
1f5/2
2p3/2
1f7/2
22 50
8 28
2
0 2s
2 1d
1d3/2
2s1/2
1d5/2
12 20
1
1 1p
1p1/2
1p3/2
6 8
0
0 1s
1s1/2
2 2