MICHAEL SHORT: So I'd like to do a quick two or three minute review of the stuff we did last time to get you back into where we were. We were talking about different types of technologies that use the stuff you'll be learning in 22.01. Everything ranging from nuclear reactors for producing power, and the Cherenkov radiation that tells you-- well, that the beta particles are moving faster than the speed of light in water.

It's a neat thing, too. I've actually been to this reactor at Idaho, to the spent fuel pool. Even the spent fuel, once it comes out of the reactor, is still giving off betas and still giving off Cherenkov radiation. And you can tell how long it's been out of the reactor by how dim the glow gets, which is pretty cool. So you can tell how old a fuel assembly is by the blue glow. So remember, I told you guys, if someone says, oh, you're nuclear, do you glow green? You can be like, no, it's blue. That's the right way of things.

We talked about fusion energy and got into some of the nuclear reactions involved in fusion energy. And I'll be teaching you more about these today. Why fission and fusion work, it all has to do with the stability and binding energy of the nuclei involved. And that'll be the main topic for today, is excess mass, binding energy, nuclear stability.

We looked at medical uses of radiation, from implanting radioactive seeds called brachytherapy seeds in certain places to destroy tumors, to imaging, to X-ray therapy, to proton therapy using accelerators, or cyclotrons, to accelerate protons and send them into people. If you remember, last time we ran the SRIM code of the stopping range of ions and matter, and actually showed that protons all stop at a certain distance in tissue, depending on their energy and what you're sending them into.

Let's see-- we talked about brachytherapy. We talked about radiotracers, and this is going to be one of the other main topics for today, is these decay diagrams, and figuring out not only what products are made, but what energy levels do the nuclei have, and how do you calculate the energy of the radioactive decay products and the recoil nuclei, which do take away some of the energy.

We talked about one way to get rich. If you guys can figure out one of the ways to solve this moly-99 shortage. Right now, it's mostly made in reactors. The future has got to be accelerators or some sort of switchable device where you don't need to construct a reactor to make these medical isotopes for imaging, and tracers, and such.

And finally, we got all the way up to space applications, shielding, crazy, different types of shielding, like electromagnetic shielding, to protect from high energy protons, all the way to radiothermal generators, which use alpha decay to produce a constant amount of energy on the order of one to 200 watts for like 100 years. And finally, to a different configuration of nuclear reactors, where you can design them to produce thrust, not necessarily electricity. And that's where we stopped on Friday.

So let's move on to one of the things I'd alluded to earlier, which is semiconductor processing. This is actually a diagram from the MIT reactor, because we have this beam port here. Has anyone got to see the silicon beam port at the MIT reactor? Oh, seven, eight-- OK, about half of you. For those who haven't, who here has not had a nuclear reactor tour? Oh man. OK.

Well, you'll get one when you get to control the thing in early October. So you actually get to go down to the control room and see the rest of what's going on. So make sure to ask them, show us the beam port for silicon ingots. And I think I already told you the story about the poor UROP who held the ingots up to their chest, getting about 10 months of dose, which is not dangerous, but it meant that for 10 months, they could have no radiation exposure, and they had to answer the phone. So that's how we ensure safety around here.

There's other ones that-- applications that you're probably carrying around in your pocket. You can use the fact that charged particles have very finite ranges in matter to separate little bits of that matter from other things. So this is actually how single-crystal sapphires can be separated in little slivers for protective phone covers, because sapphire is one of the most-- the hardest, or the most scratch-resistant materials there is. Single-crystal sapphire is exceptionally strong, and optically transparent, and expensive.

I know that because on one of our experiments, we use a single-crystal sapphire window to see into reactor conditions at like 150 atmospheres, and 350 degrees Celsius, and pretty corrosive chemistry. So you want to use as little as possible. So you can use a big, expensive accelerator to limit the amount of sapphire that you use. And this is actually done here in

Boston. There's a facility not far from here that uses an accelerator. And this is their super detailed diagram of what the radiation looks like-- yeah, whatever.

But what they do is they take-- they accelerate protons. They send them through bending magnets to steer that beam path. And then they send them into a large piece of single-crystal sapphire, which is exceptionally expensive to make. And they can actually lift off a thin sliver with micron precision. The reason for that is the same reason that we showed with that SRIM code. If you have this exact energy of protons going into well-known matter, you know what its range is going to be with an uncertainty or so of about a micron. So you can have things that come out thin, uniformly thick, and smooth, right away.

There's some other really wacky products-- like has anyone heard of these betavoltaic batteries? No? They rely on beta decay or the direct capture and electricity generation from a radiation source like radioactive tritium. So in this little chip is about 2 curies worth of tritium. You guys will learn in about a week, how to go from activity in curies to mass, or something like that.

And so this chip actually contains a lot of radioactive tritium that directly creates electricity. So you can hook into that chip and produce nanowatts for years. So it's one of these batteries that lasts-- well, as long as a couple half-lives of the isotope that's inside. Now there's a trade-off here. The shorter the half-life, the more active a given isotope will be for the same number of atoms, but the shorter it will last. So you can have higher power for lower time, or lower power, higher time. It's the classic energy trade-off-- works the same way with irradiation.

And so now I wanted to get into some of the more technical stuff, where we'll be talking about nuclear mass and stability. And this is where the nuclear stuff really begins in 22.01. First, I want to make sure that we all agree on notation. So I'll be writing isotopes in this sort of fashion, where we refer to A as the atomic mass, or just the total number of nucleons. This is not the exact mass of a nucleus. It just refers to the sum of the protons and neutrons in the nucleus itself.

And a lot of what we'll be talking about today is the difference between this nice integer mass number, and the actual mass of the nucleus, and that difference is given by the binding energy, or the excess mass, which are directly related. Z is just referred to as the atomic number, or the number of protons. It's what makes the element what it, which makes the name kind of redundant. But it's-- humans learned by association. It's easier to remember element names or symbols than which element is which just by the number of protons. So a lot of times we'll use the name, or at least the symbol just so we know what's going on. And anything up here is some sort of a charge.

I do want to warn you guys of the dreaded multiple symbol use or multiple use of symbols. I'll try to stick for a lower case q-- will be charged. And uppercase Q is going to refer to the Q value or the energy consumed or released by a nuclear reaction. So they're both Q's but we're going to keep one upper and lower case.

And like we mentioned before, let's say we were to write a typical nuclear reaction, like the capture of neutrons by boron to produce lithium-7, helium-4, better known as an alpha particle, and some amount of energy. There's two places where we actually use this reaction. One of them is as control rods. A lot of reactors use boron for carbide, or this compound B4C, which is conveniently solid, fairly dense, and contains a whole lot of boron in one place. Specifically, enriched in boron-10, because boron-10 has a high cross section, or probability, for neutron capture.

And the other one is in what's called boron neutron capture therapy. Have I discussed this with you guys already, BNCT? Good, because that's what we'll be talking about for a few slides. And to write this whole reaction is the same thing as writing this shorthand nuclear reaction. So this is often how you see them in the reading, and in papers, because it's shorter to do that. But it's the exact same thing.

So I have a couple of questions for you guys then. I have this extra Q here. Where does that Q actually go? So let's say boron and neutron absorb, it produces two nuclei with different binding energies. What happens to the excess energy created from the conversion of mass to energy? Yeah, Alex?

AUDIENCE: That could be heat.

MICHAEL SHORT: Yep. And heat, and more specifically?

AUDIENCE: Kinetic energy.

MICHAEL SHORT: Kinetic energy of the radiation released. And so that kinetic energy is actually used to our benefit in BNCT, or Boron Neutron Capture Therapy. The way this works-- once I hit play-- is you can either-- you can use any sort of source of neutrons, either a reactor or an accelerator, through a lower complex chain of events, like this. In this case, an accelerator-- so you don't need a whole reactor-- fires a beam of high-energy protons into a beryllium target.

Does that sound fairly familiar? Firing something at beryllium, releasing neutrons, like what Chadwick was doing? Except he was firing alpha particles into it. This releases neutrons. What they don't have labeled here is slowing down stuff, or probably hydrogenous material, so that the neutrons slow down to a lower energy. And their probability of capture increases or their cross section increases. And if you don't know what a cross section is, the definition is two slides away.

And the idea here is that these neutrons then enter the brain or wherever the tumor happens to be. And we rely on the fact that tumor cells consume resources much faster than regular cells, especially neurons, which after you're about five, don't tend to grow very much. So it's all downhill by the time you enter kindergarten. And we use that to our advantage so that the neutrons coming in will hit the cancer cells, which will preferentially uptake the borated compounds, leaving most of the normal cells intact.

And the difference in dose can be a factor of 5 or a factor of 10. So that the cancer gets fried while doing as little damage as possible to the remaining brain cells, of which we have fewer and fewer every day. So I say statistically speaking you guys are probably smarter than me if we go by number of neurons in your brain, because I think I'm the oldest person in the room.

And so now we can start to explain, how does BNCT work, and why did we make the choices that we did? For example, they use 30 MeV protons in order to induce these neutrons. So we have a nuclear reaction that looks something like this. We start off with beryllium-9 plus a proton-- let's just call it hydrogen to stick with our normal notation-- and becomes-- well, can someone help me balance this reaction? We know we get a neutron. What else is left? So Monica, what would you say?

AUDIENCE: Let's see--

MICHAEL SHORT: Even just say number of protons and neutrons, and we'll figure out the symbol later.

AUDIENCE: Number of protons should be--

MICHAEL SHORT: Sorry, oh, that's a 4.

AUDIENCE: The number of protons should be five.

MICHAEL SHORT: Yep. Five protons, which means it's boron. And number of neutrons in the nucleus? Someone else? Yeah?

AUDIENCE: Nine.

MICHAEL SHORT: Nine. So we have boron-9. Not a stable isotope of boron, but it doesn't really matter, because boron-9 almost immediately decays into an alpha, and an alpha, and a hydrogen. But this nuclear reaction right here is what we'll be studying for a little bit. And there'll also be some amount of energy. And this Q can actually be positive or negative. No one said there had to be energy released in a nuclear reaction, because in this case, we actually start off with 30 MeV protons and roughly zero MeV beryllium. If you want to get really exact, it's on the order of about 0.01 eV, which is why we neglect the kinetic energy of beryllium at room temperature.

There are other reactions that when you fire a proton into them will produce neutrons, such as the absorption of lithium. But can anyone think of why we'd want to use beryllium instead of lithium? Kristin, what do you think? What could be a bad thing about using lithium? You ever throw in water?

AUDIENCE: No.

MICHAEL SHORT: OK. Then I should show you what happens when you throw it in water. There's a few bad things about lithium. It does this when you throw it in water. It's one of the alkali metals. It's got an awfully low melting point. It reacts with oxygen to produce an oxide almost instantaneously. So if you ever take a lithium battery apart, which you shouldn't, but if you watch the video of somebody else doing it, you'll see that the lithium foil turns black almost instantly. It also has a pretty poor thermal conductivity and doesn't hold that structural integrity when it melts. So it's not that good of a target to use.

Beryllium's pretty cool in that it's the lightest structural material there is. Folks tend to make satellites out of it, because it costs a lot of money to launch things into space. And if you want something that has a high melting point, and is light, and is structural, beryllium's your way to go. It also happens to be a great neutron generator.

And then why 30 MeV? In this case, we're going to use a table called JANIS, which I've got open over here. And I just have to clone my screen so you can see it. This is a resource that I think you guys are going to be using quite a lot in this course. We have a link to it on the learning module site. And I'm going to show you how it works right now.

So I tend to use the web version because it works on any browser, any computer. And now you can start to pick which nuclear reaction you're looking at. And you can get tabulated cross sections. So I'm going to start by zooming all the way out. We can pick our incident particle. Since in this case, we're looking at the firing of protons into beryllium, I'll pick the incident proton data right here.

There's a lot of different databases with sometimes conflicting information. I tend to go with the most recent one you can find. And click on cross sections. And this is, again another table of nuclides, anything in green there's data for. Anything in gray, there isn't. So let's go all the way back to the light nuclei, zoom in, go back down to the light nuclei again until we find beryllium-9. Double-click on that, and let's look for the anything cross section.

And this is a pretty wide energy scale. So you can actually change your X minimum and maximum. So let's change it to a minimum and maximum-- I don't know-- a maximum of 50 MeV. We don't have to see all of that other stuff going on. 50 MeV and maybe a minimum of 10. If you notice-- actually, I'll go back to 1. And I want to point something out.

You can actually get a good yield of beryllium. Let's see-- you can actually get a good yield of neutrons by firing protons at beryllium in lower energies. But I notice there's this interesting feature right around there. The cross section's flatter. And so if you want to get an-- ensure that you get the right dose, you might want to deal with a flatter cross section or a flatter probability region, so that you have something more predictable instead of in a really high slope region. But some of these nuclear reactions actually take extra energy in order to move forward. And we'll show you another example pretty quickly. Let's go back to our slides here.

Then another question is, how does the boron only get into the cancer cells? Like we mentioned before, cancer cells are actively growing, which means they need a very large and active blood supply. And so it's one way for things to, let's say, not quite cross the blood-brain barrier. If the cancer cells are growing and your neurons aren't, then your cancer cells are going to use more energy, take in more sugar, which might be doped with boron, or some other compound doped with boron, and that's all you can get the boron into the cells that you want.

And then why was boron selected for the therapy? Let's think about that. What happens after the neutron is created? And let's write the next stage of the reaction. In boron neutron capture therapy, we rely on doping the patient with boron-10 to release an alpha particle, and lithium-7 and a gamma ray.

So now what we can start doing is look at the table of nuclides, which I'm going to teach you how to read now, to figure out-- let's say that this neutron had an energy of about zero eV and the boron nucleus had an energy of about zero eV. And in the end, all this stuff here has gained or lost some sort of energy cue, Q. And today we're going to teach you how to calculate this Q.

So I want to skip ahead to how to read the table of nuclides. So there's all-- this is like the poster you'll see in every nuclear building. It's kind of what makes us, us. What you'll notice is that there's a whole lot of nuclei at the lower left. They are the light ones. At the upper right, they are the heavier ones. And they're colored by half-life. In general, the blue ones will be stable, and the further away you get from blue, the less stable they get. So right away, without even delving deeper, what patterns do you guys notice here? Yeah, Alex?

AUDIENCE: As they get bigger, heavier, they're more unstable.

MICHAEL SHORT: Yeah. There's a whole section where there's no more blue. There are no stable elements. So stability drops off after a certain point. And what about in the region of stable isotopes? Does anybody notice any repeating patterns here? Take a look at every other row. There's a bunch of blues and then one, and then a bunch and then one, and then more and then none. That must be technetium, because that's the only element around there that doesn't have any. And then a bunch of blues.

So every other row-- and in this case, it's increasing number of protons-- has more or fewer stable isotopes. It turns out that the even numbered isotopes have a lot more stable ones, for reasons that we'll get into pretty soon. If you zoom in a little bit, you can see all the different isotopes so you can select which ones you want. And again, if you look really closely, that's-- let's say, neon right here has got a few stable ones. Sodium has one. Magnesium as three. Aluminum has one. And this pattern repeats all the way up to the point where you don't really get any more stable isotopes.

If you double-click on one of them, you get all the information that you'll need for the next three or so weeks of the course. So in this case, I picked on sulfur-32, one of the stable isotopes of sulfur. So if you notice it doesn't have any decay mechanisms here, but it does say its atomic abundance. So you can know how-- what percentage is normally found in nature. And then there's a few other quantities that is going to be the topic of what's going on here.

Let's start with the atomic mass. If you notice, the atomic mass is slightly less than 32, 32 being the mass number, or the total number of protons plus neutrons in the nucleus. The actual mass is a little lower by that amount right there, the binding energy. It might be a little funny because I've given you a mass in AMU, and a binding energy in kiloelectron volts. I want to remind you that these are the same thing.

The conversion factor you'll be using over and over again throughout this course, especially on the next p sets, is one atomic mass unit is 931.49 MeV c squared. Yeah-- I'm sorry. Yeah, never mind, put that there. So then, again, one, don't round-- because we've had times when folks said, ah, this is about 931. And when you're off by half an MeV, you could be at a totally different decay level or get a positive Q when it should be negative, or vice versa.

And let's take a quick look here to say, if this atomic mass is 31.9720707 AMU-- this is why I brought a calculator. Normally I do mental, math but since I told you guys don't round, I can't do eight significant digits in my head. So I'm going to get that in there-- 0707. If any of you guys want to follow along, I encourage you to. And say minus 32, which is the mass number.

So in this case we're taking the actual atomic mass minus the actual-- the mass number. In this case, it's 32. In this case it's, 31.9720707. And we end up with minus 0.0-- I'm going to put all the digits here-- 293 AMU. If we convert this to MeV-- times 931.49, we get minus 26.0159 MeV. See this number anywhere on the KAERI table? Right there-- that's the excess mass. And in this case, we usually give this the symbol delta for the excess mass. And these are how these quantities are directly related.

The excess mass-- well, actually, what does the excess mass really mean? It's the difference between the actual mass and a fairly poor approximation of the mass. So the excess mass doesn't really have that much of a physical connotation. But it is nice, because if you know very well the tabulated atomic number-- I'm sorry, the-- yeah, the mass number and the excess mass, you can figure out-- let's see-- yeah, you can figure out what the real atomic mass is.

And I want to switch now to the actual table of nuclides and show you one example. If you want to very quickly jump between isotopes, you can type them in right up here. And does anyone know what the gold standard for atomic mass is? And I'll give you a hint, it's not gold. Yep?

AUDIENCE: Carbon-12.

MICHAEL SHORT: Carbon-12. What do you think the excess mass of carbon-12 is going to be without doing any calculations?

AUDIENCE: Zero.

MICHAEL SHORT: Exactly. Zero. So if we go to carbon-12, because that is set as the standard, the way atomic masses were done was carbon-12 weighs exactly 12 AMU. The excess mass here is zero. And that's why the atomic mass is 12.0 to as many decimals as we care to note. So is everyone clear on what excess mass is? Yep?

AUDIENCE: What's the point of c squared for that conversion?

MICHAEL SHORT: So mass does not actually equal-- oh right, and it's actually on the-- where did my chalk go? It's actually down below. The point is that energy is related to mass by c squared. So they're not the same units, but they're directly convertible.

AUDIENCE: OK.

MICHAEL SHORT: Yep. And so this way, you have an E over a c squared. You get an m, and there we go. I had the units upside down. However, carbon-12 does not have a zero binding energy. Yeah, Luke?

AUDIENCE: How come when you did that calculation, you didn't use the c squared? So like, it seems like then that would be 26.0159--

MICHAEL SHORT: MeV per c squared. Yeah.

AUDIENCE: But they don't say that up there-- or it didn't say that on the table.

MICHAEL SHORT: Yeah, that's true.

AUDIENCE: [INAUDIBLE]

MICHAEL SHORT: So it is funny, right? The binding energy is give it in keV, and that's correct. An energy is an energy. An excess mass, it really should say keV per c squared, because if we're talking in units of mass, it's got to be in m. Or in this way, you could say m is an energy per c squared. So this, to me, is a semantic inconsistency in the table. But you guys will know that a mass is always going to be an AMU, or kilograms, or MeV per c squared. And energies will be in MeV, keV, some sort of eV, usually, in this course.

The binding energy, though, that's correct. That's in keV, because that's an actual energy. Now then the question is, what does the binding energy actually represent? Does anyone remember from Friday or Thursday? I can refresh your memory, because that's what I'm here to do. The binding energy is as if-- let's say we're assembling carbon-12 from its constituent nucleons. There's going to be 12 of them. Let's say we had six protons and six neutrons.

We can calculate the total mass energy of this ensemble of nucleons when they're infinitely far apart from each other. And forgive the little-- it's not to scale. But they are infinitely far apart from each other. And we can say that-- let's say there is Z number of protons. So we'll say the binding energy is the number of protons times the mass of a proton plus the number of neutrons-- A minus Z-- times the mass of a neutron minus the energy of the assembled carbon-12 nucleus.

So there's actually a measurable difference in mass between six protons and six neutrons, and the actual mass of a nucleus with atomic number A and-- I'm sorry, with atomic number Z and mass number A c squared. So is everyone clear on how we arrived at this formula? It's effectively the energy released when you take the individual nucleons, assemble the nucleus. You don't have as much mass as when you started. Or in some cases, you might have a little more mass than when you started if things are particularly unstable.

And you can use the excess mass and binding energies in relative amounts to see, is a nucleus going to be stable? For example, let's look at iron-55 I'm going to jump here, make it a little bigger so the important stuff is easier to see. And if you notice, the binding energy of iron-55-- there's quite a bit of it. It's very well-bound. In fact, this is one of the most well-bound nuclei in the whole chart of nuclides.

Let's look at something that we know to be particularly unstable. Someone have any idea? Let's just add like 20 neutrons to iron let's see if it even exists. No-- doesn't happen. Let's try adding 10 neutrons to iron-- or go even crazier. What about 70? Too small-- all right, let's meet somewhere in the middle-- 68. Still a pretty high binding energy, but you can look at it as a difference in binding energy per nucleon.

So in this case, the binding energy per nucleon-- if you take the binding energy and divide by the total number of nucleons, will give you a relative measure of how tightly bound that nucleus is. Now these are not absolute things. You can't just say, certain binding energy leads to certain stability, but they do give you pretty good trends to follow. And we're actually going to be coming up with-- probably on Thursday-- a semi-empirical formula to get the rough binding energy of any particular assembly of protons and neutrons. And it follows experimental calculations pretty well-- surprisingly so.

I want to jump back to here, because I've mentioned cross sections, and I want to actually define what a cross section is, because this is a quantity that you're going to be using everywhere. Let's say that we fired a beam of particles-- it doesn't matter what it is-- at a target of other particles. Let's say, the beam particles are atom A, and the target particles are atom B. And once these A particles pass through the target B, a little bit fewer of them come out the other side reacted, or unscathed, or unscattered. And some of them are absorbed, or scattered, or bounced off, or scattered backwards, or what have you.

We can write the sort of proportionality constant between the change in intensity of our A beam and the thickness of our slab. And we give that proportionality constant this symbol, little sigma. We'll get something going up here. Little sigma, which we call the microscopic cross section. It's in effect, a constant of proportionality that relates the probability of absorbing an atom from this beam I-- or from this beam of atoms A through a slab of B.

And then if you take this formula, you divide by that delta X-- so I'm going to take what's on there and say delta I over delta X equals minus cross section ABn-- which refers here to the number density. So I'll keep our table of symbols altogether so it's a little easier to follow. n is our number density, which means the number of atoms per unit volume. Usually, in nuclear quantities, we use centimeters because these are things that are actually fairly measurable, and cross sections are actually in units of centimeters squared.

And let me finish that expression. We had the number density of our target B. We had our initial intensity, and that's it. Anyone know how to solve this differential equation? If we take the limit of small deltas, it should start to look like a differential equation. The final answer is up there on the screen. Does anyone remember the method to actually solve this differential equation? This is the easy one-- separate the variables.

So in this case, we can divide each side by I of of X, multiply each side by X. I'm going to bring this up so I'm not bending down. So we have dI over I equals minus sigma ab n of B times dX. Integrate both sides and we get log of I equals minus sigma ab n of BX, and some integration constant.

You can apply an initial boundary condition to say at X equals zero, the intensity of the being x was some intensity I naught. Whatever intensity of the beam that we initially fired at the target. And by combining these two, you end up with the expression you get right there, which is that the intensity of the beam coming out is the initial intensity times e to the minus sigma ab nbx.

And we've kind of derived the idea of exponential attenuation. For those who haven't seen that word before, attenuation or the gradual removal of the beam of incident particles by whatever the target happens to be. This quantity right here, we actually have another symbol for it, which we give big sigma. And in this case, big sigma we call the macroscopic cross section.

I'll draw a box around these so we know these are our symbols that we're keeping defined here. And so you may see that the microscopic cross section just depends on single reactions between the incoming atoms A and the target atoms B. The macroscopic cross section depends on how much B is there. So if you want to get per atom probabilities of absorption scattering, whatever thing you're looking at, you use the microscopic cross section.

And if you have a finite amount of stuff there, and you know the number density of your substance B, you can use the macroscopic cross section to get actual total probabilities of beam attenuation-- or to calculate exponential attenuation. We're going to see this again in another form when we talk about designing shielding, and how much shielding do you need to remove how much of the beam?

Well, this quantity right here, there's actually tabulated values for a lot of this stuff at the-- on the NIST website. And I have links to that as well on the Stellar website, so you can-- instead of having to look these all up on JANIS and multiply number densities, there are some easier graphical functions you can just find the value for. But we'll get back to that in a few days.

So anyway, on reading the KAERI table, there's a few quantities right there. We've already defined what the excess mass and the binding energy is. And I want to note right here, if you want to actually calculate binding energies by hand, which I'm going to ask you to do a bit on problem set 2, you'll need to know what the mass of the proton, and the neutron, and the electron are to, again, usually like six or seven digits is the idea behind this course. Notice that they're not exactly one atomic mass unit, because one atomic mass unit, again, was set with that carbon-12 standard. I'm not going to use the word gold standard because that's a misnomer in this field.

And so like I said, what does excess mass really mean, physically? Not much, because it's the comparison to an arbitrary standard or a rather poor approximation of the mass. The binding energy actually does represent the conversion of mass to energy when you assemble a nucleus like Voltron-style from its constituent nucleons.

So let's try a few examples in class right here. I'd like you guys to follow around and try and calculate the binding energy of each of these three nuclei of sulfur. Let me get a better blank board so we can follow along. And there's a few different ways of calculating that binding energy. You can do it by the excess mass. You can do it by-- let's go back to the table of nuclides so I can show you how I would do it.

Let's start with sulfur 32. And we'll write up the quantities that we're-- that we know. Let's say the excess mass is the actual mass minus the mass number. The binding energy is Z times mass of hydrogen plus A minus Z mass of a neutron minus the actual mass of that nucleus with AZ c squared. And then what we can do is rearrange this excess mass, isolating the mass term right here, and make a substitution.

So we can say the mass is actually the excess mass plus A. Stick that in right there, and now we can calculate and confirm the binding energies that we see right here from tabulated excess mass values, atomic number, mass number, and the masses of a hydrogen atom and a neutron, which, for reference, I'll write up here as well.

So the mass of a hydrogen is the mass of a proton plus an electron. So 1.0072-- 007276 plus 0.000-- make that a little easier to read-- how many zeros-- 00054858 AMU. Mass of a neutron, surprisingly close to Chadwick's prediction. 8664 AMU. So now I'll head back to the table of nuclides. And let's see if you guys can follow along.

What we want to do is try to confirm this binding energy using the atomic mass, the excess mass, or if we don't even know the atomic mass, we can use the excess mass plus A right there. So let's see-- Z, in this case, for sulfur, is 16 times the mass of hydrogen. This is definitely a calculator moment, because like I said, I don't know about you guys, but I can't do eight significant digits in my head.

0054858-- 1.007855-- probably enough digits-- plus 16, because there's-- mass number here is 32. The atomic number is 16. That leaves us with 16 neutrons times the mass of a neutron, 08664-- minus the excess mass, which in this case is 26.015 MeV-- 015 MeV per c squared. So thanks for that-- thank Jared for that question because, indeed, the excess mass, if you want to write it in terms of a mass, should be in MeV or keV per c squared minus A, which is 32 times c squared.

So let's do all this out-- shouldn't take too long. 007825 plus 16-- 1.008664 minus 32 minus 26.015 divided by c squared. It's basically nothing. Gives us on the order of-- let's see-- times c squared. What did we get right here?

AUDIENCE: Is the 26 negative?

MICHAEL SHORT: Ah, let's see. I believe it is, because we have to subtract the mass, and we're substituting in this delta--

AUDIENCE: Isn't the delta negative?

MICHAEL SHORT: Oh. Good point. There is a negative there. So that's minus negative that. And A is 32. Thank you. Yeah, good point. Let me try this again.

Ah, I know what I'm doing wrong. This part right here, we want to convert to AMU. So we can take our minus-- thank you-- 26.015 MeV per c squared and divide by our conversion factor, 931.49-- let's see-- MeV per c squared per AMU. What does that give us? 26.015 over that. 0.027928 AMU negative. Let's put that in and see how we do.

So plus 0.027928 minus 32. And then we get 271.-- I'll just say 764 MeV. I think six digits is enough. The actual binding energy, 271.780 MeV, so we're off by 16 electron volts-- close enough. Also note that I used a five-digit accurate conversion factor. That might be part of the source of it. Does someone have a question?

AUDIENCE: Yeah. In the equation on top, you did the atomic number times the mass of the proton, but in the one on the bottom, you used atomic mass times the mass of the hydrogen including the electron. Is there--

MICHAEL SHORT: Oh yeah. I actually added the two. So the mass of an electron, since it's got that extra zero, makes so much of a-- so the mass of-- oh-- yeah. The mass of hydrogen would be the proton plus the electron right there.

AUDIENCE: Right. But why do hydrogen, though, if [INAUDIBLE] just the proton?

MICHAEL SHORT: Oh, because there's an electron there, too. Now this can usually be neglected because it's such a small fraction compared to everything else. So now we're talking about-- what-- the fifth or sixth decimal place. But just for exactness, I stuck on it. Yeah, in your calculations, you can try with and without, and I think you'll find that it doesn't matter that much, because in the end we get the binding energy that we see on the table to within 16 electron volts for a total of-- yeah-- 271 MeV. That's pretty accurate. Yeah.

AUDIENCE: If you wanted to calculate the like energy released from a reaction, would you do the binding energy for [INAUDIBLE] reactants that's trapped products for the reactants?

MICHAEL SHORT: That's the next slide. We'll get right there. Yeah, so you're catching on to where we're going. So once you can calculate either the excess mass, or the binding energy, or the total mass of any nucleus, you can start to put them together into nuclear reactions. So since you asked, let's take a quick look at them. Where is our nuclear reaction board? Anyone mind if I hide this board for now, so we can go back to our original? OK.

Let's take a look at this reaction right here, the actual boron neutron capture therapy reaction. And now we can get towards calculating this Q, what the difference is between the-- the total energies of the products and the reactants, and where does that go? So now we can either look up or calculate the binding energy of each of these nuclei, subtract off the energy of the gamma, which I've looked up already, is about 0.478 MeV. And we can figure out what the total Q of this reaction is.

So in this case-- I'll skip ahead to the slide where I've got it because that way I won't write anything wrong on the board-- got everything right up here. We assume that both boron and the neutron have roughly zero kinetic energy. And at the end, they come out with some other kinetic energies as well as this gamma ray. The sum of this energy differences, we refer to as Q. And we can actually confirm this total Q with a few different methods.

In this case, it's always conserve something. That's the whole theme of this course, is you can conserve total masses, you can conserve total kinetic energies. We may not know those, but tabulated in the KAERI table are the binding energies of each of these nuclei. So let's try that out right now. So let's look at the binding energies of each of these nuclei and see what the difference is, the total energy released.

First of all, what's the binding energy of a lone neutron? Anyone have any idea? I see a lot of these-- zero. Yep. You haven't assembled an nucleus out of a lone neutron, so we'll go with the neutron has a binding energy of zero MeV. Boron, not quite the case, but we can go back to the table of nuclides and punch that in-- boron-10. We can look up its binding energy, which is about 64.7507-- I keep saying about, which is exactly 64.7507 MeV.

And then our other two nuclei, helium-4-- so you can punch in helium-4 here. It's got a binding energy of exactly 28.295673 MeV. And finally, lithium-7, let's punch that in. I think you guys are going to get very familiar with this table. There's a few versions out there. There's a new slick Java version that I found a little hard to use. So I like the text-only version, because it's just as simple and fast as it gets-- 39.244526-- 526.

So any sort of increase in total amount of binding energy between the reactants and the products is going to release or absorb energy. Now because boron does capture a thermal neutron, or a neutron with approximately zero eV of kinetic energy, does anyone have any idea whether this would release or consume energy? In other words, do think this is an exothermic or endothermic reaction? Yeah, Alex?

AUDIENCE: I'm guessing that heat would be released through the material-- the capture material would be heated up.

MICHAEL SHORT: OK. Indeed. If the total Q value is greater than zero, we refer to this as exothermic-- kind of like in chemistry. And if Q is less than zero, we refer to this as endothermic. So let's do our binding energy subtraction now. We want to figure out how much excess binding energy is released. So I'm going to take the reactants-- I'm sorry-- I'm going to take the product. So helium 295673-- add lithium 244526-- subtract boron 0.7507-- subtract the neutron, which is zero, and we're left with 2.79 MeV.

And because it's positive, this is an exothermic reaction, which is what we'd expect, because this reaction actually happens. If this was an endothermic reaction, what could you do to make it occur? Yeah?

AUDIENCE: Heat up the reactants.

MICHAEL SHORT: Like with temperature, or what do you mean?

AUDIENCE: Make them have higher kinetic energy or--

MICHAEL SHORT: There you go. So actually-- yeah-- you kind of said the same thing twice. Heating things up does give them higher kinetic energy. If you rely on temperature, you'll be imparting eV worth of kinetic energy. But if you accelerate them, or get them from a different nuclear reaction, and you get them up to the MeV level, where whatever this Q value could be might be negative, then you can get the reaction to occur. For example, what is the Q of that reaction?

AUDIENCE: Negative 2.79.

MICHAEL SHORT: Negative that. So in this case, if you want lithium to absorb an alpha particle, and make boron and a neutron, you would have to accelerate the alphas to that same amount of energy in order to get this to occur. So nuclear reactions do go both ways, just not as easily. Kind of like chemical reactions, you can drive them in different directions by changing the temperature or changing the concentration of the reactants. Here the concentration doesn't matter. But the kinetic energy related directly to the temperature definitely is.

And so in this case, it's 2.79 MeV. If I tell you the gamma ray takes off 0.478 MeV of that, we're left with 2.31 MeV between the lithium nucleus and the helium nucleus. Now my next question-- my last question for you today-- oh man-- is what's the split? I think I don't want to keep you longer, because it's one minute of 10:00. So this is the question that we're going to pick up with on Thursday, which is how much of the energy is taken off by helium? And how much is taken off by lithium? Sorry, I should have kept better track of the time.