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**PROFESSOR:** OK, guys. We're actually slightly ahead of where I thought we'd be at this point, so I'm only going to spend about half of today's lecture finishing up some new material on mass parabolas and stability. I also got a comment in through the anonymous box that said please leave a little bit of time after class for questions. So you can get them out right away, because I'm usually running off to teach some other class at, like, the IDC or some other building.

So from now on, I'll try and leave about five minutes at the end of class for questions on today's material, and we'll make up with the second half-hour or 25 minutes of this class being for all the questions on the material so far in the first two weeks. But first I wanted to give a quick review of where we were Wednesday and launch back into mass parabolas, which are ways of looking at nuclear stability in relative numbers and even or oddness of nuclei.

So you saw last time we intuitively derived the semi-empirical mass formula as a sum of volume, surface, coulomb, asymmetry, and pairing, or whether things are even-even or odd-even terms with the coefficients in MeV gleaned from data, and the forms of the-- and the exponents right here gleaned from intuition. Here we assume that the nucleus can be thought of like a big drop of liquid with some charged particles in it.

And so the droplet should become more stable the more nuclei there are-- or the more nucleons there are. But then you have some more outside on the surface that aren't bonded to the others. All of the protons are repelling each other over linear length scales, because the radius of this liquid drop would scale like  $A^{1/3}$ . There's an asymmetry term, which means if the neutrons and protons are out of balance, there's going to be some less binding energy. And then there's this extra part that tells you whether the nuclei are even-even or odd-odd.

And this works pretty well. If you remember, we looked at theory versus experiment where all the red points here are theoretical predictions, and all the black points are experimental predictions. And for the most part, they look spot-on. It generates the classic binding energy

per nucleon curve that you see in the textbook and can predict from the semi-empirical mass formula.

Zooming in and correcting for, let's say, just getting absolute values of errors, you can see that, except for the very small nuclei and a few peaks, which we explained by looking even closer, the formula, well, it predicts nuclear stability quite well on average. Again, this line right here, if there's a dot that lies on this blue line, it means that theory and experiment agree.

And a deviation by a few MeV here and there, not too bad. But we started also looking at different nuclear stability trends, and we noticed that for odd mass number nuclei, there's usually only one or sometimes none stable isotopes per  $Z$ , whereas for even ones, there's quite a few more. And we're going to be now linking up the stability of nuclei versus what mode of decay they will take in order to find a more stable configuration.

We looked quickly at the number of stable nuclei with even and odd  $Z$  and noted that these places right here where there are no stable nuclei correspond to technetium and promethium. There's no periodic table on the back of this wall, but behind my back on the other wall, there's that periodic table where you can see the two elements that are fairly light with no stable isotopes. That's what those correspond to.

And the peaks correspond to what we call magic numbers or numbers of protons or neutrons where all available states at some energy are pretty much filled. And this goes for both protons and neutrons. So something with a magic number for both  $n$ , number of neutrons, and  $Z$ , number of protons, is going to be exceptionally stable. And we'll see how that's used as a tool to synthesize the super heavy elements that we believe should exist.

And finally we got into these mass parabolas. I found this to be a particularly difficult concept to just get mathematically. If you remember, we wrote out the semi-empirical mass formula and said if you take the derivative with respect to  $Z$ , as we did it, you would get the most stable  $z$  for a given  $A$ .

And we started graphing for  $A$  equals 93 where niobium is stable. That was just the one I had on the brain from some failures in lab earlier this week. We started plotting where those nuclei - what is it - the relative masses are for a fixed  $A$ . So let's regenerate that one right now, because we were a little fast at the end of last lecture.

Then I want to generate one for  $A$  equals 40. And you'll see something kind of curious. I'm

going to leave this up here for a sec.

If you notice, for odd  $A$  nuclei, there's only one parabola, whereas for even  $A$ , there are two. Why that is, we're going to see when we look at the table of nuclides. But notice this nucleus right here can decay by either positron emission or beta emission to get to a more stable form. And there are many real examples, and I'm going to show you how to find them.

So let's start off by going back to the table of nuclides, finding niobium-93. Just go up one more chunk. And there we are. Niobium-93 is a stable isotope. And if you want to see where it came from, you can scroll down a little bit and see its possible parent nuclides right here.

So let's say that niobium-- we'll draw it right there-- is stable. We'll put it at the bottom of this parabola. And let's work down in  $Z$ . So we'll move to zirconium. Zirconium-93 ostensibly has a very similar atomic mass. But if you remember that 93 AMU is a rather poor approximation for the actual mass of all nuclei with  $A$  equals 93.

In fact, if you look very closely at the atomic masses, zirconium-93 is 92.90. Niobium-93, well, it looks like we have to go all the way to another digit there. 92.906. You have to go to, like, even more digits 92.906375, 92.906475. So we go down in  $Z$ , and we've actually gone up in mass by looks like the sixth or seventh digit in AMU.

If we go up in mass, we go down in binding energy. That tells us that there's something that's less stable. And if you notice, we went down by a very small-- or we went up by a very small amount of mass. Notice also that its beta decay energy is really, really small, 91 kiloelectron volts.

So why don't we put zirconium just above? And we note that that decay will happen by beta, where the beta, let's say if we have an isotope with mass number  $A$ , protons  $Z$ , and let's just call it symbol question mark. In beta decay, we have the same  $A$ . We'll have a different  $Z$ . We're going to have to give these symbols. Let's call this parent, and we'll call that daughter. Plus a beta, plus an electron antineutrino.

And what has to happen to that  $Z$  in order for everything to be conserved? It's the same reaction that we've got here for-- I'm sorry-- for zirconium. So you'd have to have one fewer proton to release one electron. And so that becomes same  $A$  but different  $Z$ . And this is the beta decay reaction.

Let's go back a little farther. We'll look at the possible parent nuclide for-- did anyone have a

question?

**AUDIENCE:** Don't you need one more proton [INAUDIBLE]?

**PROFESSOR:** Let's see. Which direction are we going in Z here? That goes to yttrium. It actually looks like it's going down. Oh, yeah, for beta decay. I'm thinking-- I have the reaction backwards. Sorry. I need one more proton to account for the extra negative charge. You're right. OK. Yep, I was thinking backwards, because we're now climbing up the decay chain in reverse order.

So this could have come from yttrium-93 with a much higher energy of three MeV. So let's put yttrium right here. That gives a beta decay. And we'll just go one more back to strontium-93. Has an even higher beta decay energy. So let's put that up here.

And let's take a look at its mass real quick. The mass of strontium-93, 92.914 AMU. If we go back to niobium-93, now it's noticeably different to, like, four significant digits instead of six. 92.914 versus 92.906. And so that shows you that a tiny bit of mass and AMU corresponds to a pretty significant change in binding energy by that same conversion factor that we've been using everywhere. 931.49 AMU per big MeV per C squared.

Let's see. Yeah. OK. So let's go now in the other direction, in the positron direction. Niobium can also be made by electron capture from molybdenum. So let's put molybdenum right here.

Let's say that around half an-- what did we have here? It was like half an MeV. Like that. And let's see. Molybdenum-93 could have been made by electron capture from technetium-93 with an energy of 3.201 MeV, even more extreme.

We'll go back one more, because there's a trend that I want you guys to be able to see. And this could have come from electron capture from ruthenium. I think I may have said rubidium last time, but Ru is ruthenium-93. And that 6.3 MeV, something like that.

And this is where we got to yesterday. Now I'd like us to take a closer look at the decay diagrams, which tells us what possible decay reactions can happen in each of these reactions. Since we're right here on the chart, let's take a look at ruthenium turning into technetium by what it says, electron capture. So note that on the table, you can click on electron capture, and if it's highlighted, then the decay diagrams are known. It's not known for every isotope, but for a lot of the ones you'll be dealing with, it is.

And you get something I have to zoom out for-- a lot, a lot of different decays. What I want you

to look at is this one here on the bottom that I'll zoom in to. That should be a little more visible. So notice that if you want to go down the entire 6.4-something MeV, it usually proceeds by B-plus or positron decay, by either method.

And as you go up the chain, as these energy differences get smaller, look what happens to the probability of getting positron decay. It shrinks lower and lower and lower. So there's a trend that the larger the decay energy for this type of reaction, the more likely you're going to get positron decay.

And in fact, where we left off last time is in order to get positron decay, the Q value of the reaction has to be at least 1.022 MeV, better known as at least two times the rest mass of the electron, because in this case, to conserve charge and energy, you shoot out a positron, and you also have to eject an electron in order to conserve all the charge going on here. So there you have it.

Now let's look at the lower energy decay of technetium to molybdenum, which had something like 3 MeV associated with it. So we'll click on technetium, and its energy is 3.2 MeV.

Let's take a look at its electron capture. Significantly simpler. Already what do you notice about these positron to electron capture ratios? Anyone call it out.

**AUDIENCE:** Electron capture is much more likely.

**PROFESSOR:** Indeed. When the energy of the decay goes down-- notice that only these decays are allowed-- The electron capture suddenly becomes much more likely. But notice that it does not let you go directly from 3.2 MeV to 0. There is no allowable decay here.

So this is probably a change of-- that's 3.2. That's 1.3. A little less than 2 MeV. All of a sudden, electron capture becomes much more likely, but positron decay is not disallowed yet. So we can say electron capture or positron decay right there. Everyone with me so far?

So let's go to the really low energy one. We'll click on molybdenum-93 and see how it decays with an energy of 0.405 MeV to niobium. Anyone want to guess what's allowed?

**AUDIENCE:** Electron capture only.

**PROFESSOR:** Electron capture only. There's not enough energy for positron decay. And, indeed, it draws funny, because there's a metastable state. But if you scroll down here, there are two pathways

allowed, both of which by electron capture. Decay diagram's quite a bit simpler. So we leave this one here by saying it can only decay by electron capture.

Any questions on the odd A before we move on to the even, which is a little more interesting? Cool. OK. Let's move on to the even case.

So for here, I'm going to go back to the overall picture of the table of nuclides. Click on around where I think potassium-40 is. Looks like I got there.

And I want to point out one of these features. If you wanted to undergo decay change and maintain the same mass number, that's diagonally from upper left to lower right. See how all the isotopes here have a 40 in front of them. The really interesting part is as you cross this line, you go from stable to unstable to stable to unstable again. The colors here is dark blue represents stable, and dark gray represents long lifetimes of over 100,000 years.

So this is one of the reasons you find potassium-40 in the environment. In fact, 0.011% of all potassium in you and everything is potassium-40. It's what's known as a primordial nuclide. It's not stable, but its half-life is so long that there's still some left since the universe began or whatever supernova that formed Earth got accumulated into the earth.

But notice it can come from-- it can decay by a couple of different methods. So let's pick one of those stable isotopes, calcium-40, and put that as the bottom of the parabola on this diagram. So we'll put calcium here. And in a relative sense, we'll put a calcium point right there for its total mass. And it could have come from beta decay from potassium-40 or electron capture from scandium-40 40.

So let's look at potassium-40. It can beta decay for about, oh, 1.3 MeV. So potassium is right here. Let's say it could beta decay with about 1.3 MeV.

And we'll trace potassium back a little bit, figure out where would it have come from. Interesting. Doesn't tell us. OK, forget that.

Let's trace calcium back and say there's scandium-40. And scandium-40 can decay with-- wow-- an enormous 14.32 MeV. Let's put that like here. Anyone want to guess which mode, electron capture or positrons, much more likely?

**AUDIENCE:** I think positrons.

**PROFESSOR:** Probably positron. Let's take a look. Oh boy. Another complicated one. But the whole way down, positron, positron, positron for all the most likely decays. You won't find a drawing to every single line. I believe that they know that at some point, drawing extra lines is futile, and they just all overlap each other.

So I don't know exactly how the algorithm works, but it does draw up to some number of possible decay chains. If you want to see every single one, they are tabulated in a very, very long list down below. I'm never going to ask you to do something with all of these, because that would be insane unless it's a relatively simple decay, like that has two or three possibilities.

And let's see. This could have come from electron capture from titanium-40 with 11.68 MeV. Wow. OK. Up here. And there's titanium.

And let's go in the other direction. So I do know that potassium-40 can decay into argon-40. So let's jump there. Argon is a stable isotope too.

So potassium-40 can decay into argon-40 by electron capture. OK, good. A more respectable 1.505 MeV. Is positron decay allowed?

**AUDIENCE:** Yes.

**PROFESSOR:** Yes. Why is that?

**AUDIENCE:** [INAUDIBLE]

**PROFESSOR:** Over 1.022 MeV. Yeah. Anyone have a question? No? OK. So we've got kind of a kink in our mass parabola. Yeah?

**AUDIENCE:** Actually, yeah, so it's possible if it's over 1.022, but it's still very unlikely.

**PROFESSOR:** That's correct.

**AUDIENCE:** Until we get to these higher orders, like 10.

**PROFESSOR:** Yep, so once the Q value's satisfied, it is technically possible. But if you had something with the decay energy of, like, 1.023 MeV, it would be exceedingly unlikely. So in fact, we can take a look at this. This, I would say, is also going to be on the exceedingly unlikely level, and we can take a look.

So if we look at the decay diagram, we know it makes positrons. They're not even really listed. Interesting.

So that process would not be allowed, but this one, because that's about 1.5 MeV, should be allowed. But since that branch ratio or the probability of that happening is already so low, I wonder if it even says. Yep, beta ray with a max or average energy of 482.8 MeV. We're going to go over why that energy is so low when we talk about decay next week. With the relative intensity of something with a lot of zeros before the decimal place.

So there you go. Like you said, energies near 1.022 MeV, slightly above it, are extremely unlikely but possible and measurable. Cool.

And then let's see what could have made argon 40. Could have been beta decay from chlorine-40. So chlorine maybe was here. And I don't think I have to draw any more.

So we've got a funny-looking parabola with a kink in it, because really, you have two mass parabolas overlapping. I'm going to go back to the screen so that the diagram from the notes makes a little more sense. What we've kind of traced out here is that there's two overlapping parabolas here. There's the one with the-- what is it-- the odd Z and the even Z.

So there you go. Just like the one on here, which I think is for a different mass number. Yep. 102. We get the same kind of behavior where things will mostly follow the lower mass parabola, but sometimes if something gets stuck here, it can go either way to get more stable.

So I want to stop here for new stuff, because this is precisely where I thought we'd be at the end of the week. And in the next half an hour, I'd like to open it up to questions or working things out together on the board or anything else you might have had. Yeah?

**AUDIENCE:** I have a question about the parabola things.

**PROFESSOR:** Sure.

**AUDIENCE:** There's-- you said multiple paths, so it doesn't have to do the little peak in the middle? Like, could it follow the lower parabola or the upper, or does it have to jump over?

**PROFESSOR:** It's going to go in whatever way makes it more stable. So you're never going to have a nucleus that's going to spontaneously gain mass in order to get to a different path. You can only go down on the mass axis.



But let's say you happen to be starting here at potassium-40. You can go down via either mechanism to the next mass parabola down.

**AUDIENCE:** But if you were argon, you would go up. That's what you would do.

**PROFESSOR:** That's right. If you're at argon, you're stuck. And in fact, if you want to take a look, what do I mean scientifically by "stuck"? I mean stable. Argon-40 is a stable nucleus that comprises 99.6% of the argon. So that's what I mean by "stuck" is stable.

And if we look at the rest of the table of nuclides for similar-looking places-- so let's hunt near potassium-40. So notice potassium-40 right here has got stable isotopes to the upper left and the lower right. If we look back over here, manganese-54. Same deal. It's got a stable isotope to the upper left and a stable one to the lower right. How much you want to bet that when we click on manganese-54, it's got two possible parent nuclides-- or I'm sorry, two possible decay methods.

So let's take a look. Manganese-54 can either electron capture and positron decay to chromium-54 or beta decay to iron-54. Let's take a look at one more to hammer the point home, and I think that'll probably be enough. Cobalt-58 right near nickel-58 and iron-58.

Interesting. That one's not allowed unless there's more down here. So you can electron capture to iron-58, but there's no allowed decay to-- what was it? Nickel-58.

OK. Let's look for more. Chlorine-36 has argon and sulfur on either side. There it is. Beta decay and electron capture. And how much you want to bet there's basically never a positron here? But basically, not actually never.

So you get a positron 0.01% of the time and electron capture 1.89% of the time. Where is the other 98-and-change percent? Right here in the beta decay. So in this case, chlorine-36 will preferentially beta decay.

If you also notice, it's a-- let's see. I don't know if that actually matters. But I am going to say it's more likely to beta decay. So when you sum these up, you get 100% of the possible decays.

Let's see how many energy levels there are there too. Hopefully not too many. That qualifies as not too many. Yeah? Sean?

**AUDIENCE:** Are the changes in mass always going to be attributed to beta decays or electron captures or positron [INAUDIBLE]?

**PROFESSOR:** They'll be due to those as well as some other processes, which we're going to cover on decay. But if you notice, I've been giving you a lot of flash-forwards in this class. We've introduced cross-sections as a thing, the proportionality constant between interaction probabilities. We're going to hit them hard later.

I've also been kind of introducing or flash-forwarding different methods of decay. So there's also alpha decay. There's also isomeric transition or gamma emission. There's also spontaneous fission. This is the whole basis behind how fission can get working without some sort of kick-starting element. So maybe now's a good time to show you.

Let's go to uranium-235 and see how it decays. It goes alpha decay to thorium-231 most of the time. And if you look how, it's not terrible. We can make sense of this.

It also undergoes SF, which stands for spontaneous fission. So one out of every seven-- what is it? Seven out of every billion times, it will just spontaneously fission into two fission products. And this is why if you put enough uranium-235 together in one place, you can make a critical reactor.

In reality, you don't tend to want to put enough U-235 together to just spontaneously go critical. We use other isotopes as kickstarters. For example, californium, I think it's 252. Let's take a quick look.

There we go. Californium-252 undergoes spontaneous fission 3% of the time. It's even heavier, even more unstable. So there is a reactor called HFIR, or the high flux isotope reactor at Oak Ridge National Lab. One of its main outputs is californium kickstarters for reactors.

So to get things going, you put a little bit of californium in as a gigantic neutron source, and then you don't really need it anymore once it gets going. So it's one of the safer ways of starting up a reactor is put in a crazy neutron source, and then once it gets going, take it out or leave it in and burn it. I'm not actually sure which one they do. Yep?

**AUDIENCE:** Is the name californium based on California?

**PROFESSOR:** It is. When we get to-- now is a good time to introduce the super heavy elements since you asked. So a lot of these older elements were named after-- actually this is kind of a hobby of

mine. So I don't know if you guys saw the periodic table outside. I collect elements, because if you're going to collect something, you might as well collect everything that everything else is made of.

It's the same reason I went into nuclear energy. I started off course 6, or 6.1, specifically electrical. And I was like, well, I could be designing, like, the next screen for a cell phone, or we could solve the energy problem, which is the problem all others are based off of. So my whole life theme has been go to the source. That's why I came here in high school and never left. That's why I declared course 22. That's why I collect elements and probably is the reason for many other things which only a psychiatrist could diagnose.

But let's look at some of the other elements. For example, yttrium. I think it has a isotope 40. Anyone know-- no, it doesn't have a 40. What about a 50? 60? 100? Whatever. At least it knew that Y was yttrium. Anyone know--

**AUDIENCE:** 89.

**PROFESSOR:** --where this is coming from? 89? Seems high. Oh my god. You're right.

**AUDIENCE:** I work with it.

**PROFESSOR:** OK. Gotcha. You work with it. Awesome. Anyone know what this is all about, yttrium? There's a town called Ytterby in Sweden where large deposits of yttrium and ytterbium, or Yb, tend to be found or Db, named for dubnium.

So let's say the really basic elements tend to come from Latin. Fe stands for iron, which actually stands for ferrum. Lead is plumbum. Gold is aurum. Silver, Ag, is argentium. I don't know if I'm saying that right. I never took Latin, and I've never heard it spoken, of course.

And then a lot of the heavier and heavier elements as we go are being named for more and more famous scientists or places where they tend to be made like Db. I'm going to guess 260 for a mass there. Oh, nice. For Dubna in Russia that has got one of the few gigantic super heavy element colliders where they're constantly synthesizing and characterizing these super heavy elements. So finally they said, you know what? They've made enough of these in Dubna. Let's name one of the elements after them.

Or Sg, seaborgium, for Glenn Seaborg. Or No, nobelium, for Alfred Nobel. Yep?

**AUDIENCE:** I just have a question for the actual mass parabola.

**PROFESSOR:** Uh-huh.

**AUDIENCE:** Like, do the parabolas ever, like, reach each other?

**PROFESSOR:** Do they intersect?

**AUDIENCE:** Yeah.

**PROFESSOR:** I've never seen a case where they intersect. That would make for a crazy situation indeed. However, part of what the homework assignment's about is to derive an analytical form for a mass parabola and then check the data to see how well it works. So for any cases where you have an even mass number, and you have either odd-odd or even-even nuclei, you can check those equations analytically to see if they'll intersect.

**AUDIENCE:** And for the case of  $A$  equals 40, I'm not really sure what the top parabola is.

**PROFESSOR:** So the top parabola for potassium-40-- let's take a quick look at how many protons and neutrons it has. Potassium has a proton number of 19, which means it has a neutron number of 21. So the top parabola is odd  $N$  and odd  $Z$ , where the bottom one is even  $N$  and even  $Z$ . Whereas for odd mass number nuclei, it has to be either odd-even or even-odd, else it would be even, which is a funny sentence when you say it all out loud.

Yeah. So that's the idea here is that notice that the even-even parabola tends to be further down. All those nuclear magic numbers, 2, 8, 20, 28-- I'm not going to quote the rest. Those the little ones I know. All even numbers. So any other questions on these mass parabolas before we launch into super heavy elements? Yeah?

**AUDIENCE:** [INAUDIBLE] the bump on the right?

**PROFESSOR:** Uh-huh.

**AUDIENCE:** How do you know that that's where [INAUDIBLE]?

**PROFESSOR:** Analytically or experimentally? Which question?

**AUDIENCE:** Analytically.

**PROFESSOR:** So analytically. Analytically there should be some isotope of-- well, not potassium. That

wouldn't be allowed. So in this case, the stable element positions have got to kind of switch off, shouldn't they?

So if that's potassium-40, that would still have to be potassium. You don't really have another choice. There isn't really a position there, is there, analytically? That's the interesting thing is that you can either be odd-odd or even-even for an even mass number. But you can't just take off one neutron from potassium-40, and then you've got potassium-39. Then you're on a different mass number.

Or if you exchange a proton and a neutron, which you pretty much do in either of these directions. There's no way to get straight down here.

**AUDIENCE:** Right.

**PROFESSOR:** Yeah?

**AUDIENCE:** For odd-odd, delta is negative, right?

**PROFESSOR:** For odd what? For-- sorry?

**AUDIENCE:** For odd-odd, delta is negative?

**PROFESSOR:** Yeah, let's go back to that slide just to make sure. You mean the pairing term in the semi-empirical mass formula?

**AUDIENCE:** Yeah.

**PROFESSOR:** Yeah, so for odd-odd nuclei, indeed, delta's negative, which means lower binding energy, which means higher mass. And that's why we see it bump up on the mass right here. Yeah? And do you have a second part of the question?

**AUDIENCE:** It was more so how to relate [INAUDIBLE] like the binding energy to that mass parabola.

**PROFESSOR:** We can actually relate-- so we can relate the binding energy to the mass and the mass parabola analytically, because the binding energy is equal to  $Z$  times protons plus  $N$  times mass of neutron minus the actual mass of that same nucleus,  $A$  comma  $Z$ . So they're actually directly related, just negatively. So something with a higher mass is going to have a low binding energy, which means it's less bound and less stable. And indeed, the further up the mass scale we go, the higher those beta or electron capture or positron energies are.

And there's another thing you can check too, which is the half-life. Half-life is what we'll be talking about on Tuesday. It's how long before an average amount of a substance has undergone radioactive decay. So let's look at some of these isotopes and start looking at half-life trends as another measure of stability.

So potassium-40 has an exceptionally long half-life. So it's relatively stable. Let's take a look not at either the stable isotopes, but let's go up the mass parabola chain in one direction. Calcium-40, scandium-40. So let's take a look at scandium-40. Scandium-40 has a half-life less than a second. And it's got quite a high decay energy by whatever method you want to use.

Let's go up to titanium-40. Anyone want to guess? Do you think the half-life is going to go up or down? Let's see if the half-life goes down. We know the decay energy goes up. Indeed. Half-life goes down from 182 milliseconds to 50 milliseconds.

And let's say titanium-40 could have come from-- wow. Two proton decay from Cr-42 with a half-life of 350 nanoseconds. So as we go up the mass ladder and down the stability ladder, the half-life decreases, which kind of follows intuitively. Something that's exceptionally stable should have a half-life of infinity, and something that's exceptionally unstable should just blow apart instantly.

Like, remember the first week of class, we talked about helium-4 grabbing a neutron, becoming helium-5, and instantaneously going back to helium-4. If you look at helium-5, its half-life is measured in MeV, or 7 times 10 to the minus 7 femtoseconds. So if helium-4 absorbs a neutron, it simply doesn't want it and gets rid of it in 10 to the minus 7 femtoseconds, which would tell us that it's exceptionally unstable.

So I hope that's a long-winded answer to that question about what does it mean to be going up in the mass levels. Any other questions on mass parabolas or the liquid drop model or stability in general? Yes.

**AUDIENCE:** For something that goes upwards [INAUDIBLE] just because the mass [INAUDIBLE].

**PROFESSOR:** So if you're changing one neutron to a proton in each case, you're switching back and forth from the odd-odd to the even-even mass parabolas. So if I were to redraw these dots more to scale, this would have to be on the odd-odd. And, well, let me draw them a little better. Yep.

**AUDIENCE:** OK.

**PROFESSOR:** That's on the odd-odd, and that's on the even-even.

**AUDIENCE:** OK.

**PROFESSOR:** Yeah. So excuse my poor drawing skills. But if you're switching one proton to a neutron or vice-versa, by definition, you're jumping back and forth between these parabolas.

**AUDIENCE:** OK, thank you.

**PROFESSOR:** That's a good question for clarification. You had a question too?

**AUDIENCE:** Yeah, about the semi-empirical mass formula. When do you use that to find binding energy as opposed to, like, any of the other ways?

**PROFESSOR:** I'm sorry. The semi-empirical mass formula is a good way to get an analytical guess at most of them. If you want an exact answer, always use the actual binding energy.

**AUDIENCE:** So, like, how often is it used now?

**PROFESSOR:** I would not say it's used much now except-- well, that's going to be one of your homework questions is this formula predicts that as you get heavier and heavier and heavier, nuclei should just continuously get less stable. And that was, as of when this was derived, let's say decades ago, we now know something different is happening.

So if you look at the table of nuclides, you can sort of see some swells in the number of black pixels until it cuts off. And this region actually where we think super heavy elements happen, I want to jump to the actual table of nuclides, which I'll say is our snapshot of knowledge today, and go all the way to the top. And our knowledge kind of cuts off at these elements, which are, for now, temporarily named in a very uncreative way. We don't even know anything about them.

Uun is probably going to have a proton number of what? 110. I don't know what the prefixes are, but UUU would be un-un-un 111. Probably has 111 protons.

Beyond here, off the screen or probably up into the next room, it's predicted that once you approach the next magic number in nuclei, there should be an island of stability where it may not necessarily be totally stable, but the half-lives should go up again. And we should be able

to synthesize super heavy matter. And if you actually graph neutron number versus half-life-- so notice how we were looking at half-life as a measure of stability. It starts to go up, then comes down, and then, to the extent of our knowledge, it is going back up again to the next predicted magic number. So what we think should be happening is half-lives should be continuously going up. And yeah? You had a question?

**AUDIENCE:** Like, what do we do with these weird things?

**PROFESSOR:** Well, whatever you want. It's going to be-- it should be dense as all heck, because nuclear matter is quite a bit denser than ordinary matter, and quite a bit is quite an understatement. So what would you do with super heavy matter? A lot of it could be used to probe the structure of matter. There's a lot about how the nucleus is constructed that we don't know. And beyond the scope of this course would also be an understatement.

There's folks that are making their careers now on figuring out what are the forces between nucleons? Why do things spontaneously fizzle at the rates that they do? You'll even hit a little bit of this in 22.02 when you can calculate the rough half-life for alpha decay using quantum tunneling through the potential barrier in a nucleus. And so the more nuclei we have to mess around with, the more data and real examples we have to study.

But practical applications, well, I could imagine, we might find something denser than osmium. Osmium right now has a density of about 22 grams per cubic centimeter. This stuff, zirconium, is about 6.9 or so. Steel is like 8. Lead's like 11. Mercury is like 19.

Have any of you ever played with liquid mercury before? This is a "don't try this at home, kids" kind of moment. My grandfather happened to be a dentist, so we happened to have a lot of mercury to mess around with. And it's, like, unintuitively heavy. It's unbelievable. A 1-pound jar is about that big. I think it would be cool if we could find something even denser.

And then really, really dense matter happens to make really, really good photon shields and gamma-- not the *Star Trek* thing. I mean this in the actual nuclear physics sense. The best way to stop gamma rays for gamma shielding is just put more matter in front of it. And if we find a denser state of matter that's earth stable, you then have a smaller gamma shield. So there are practical applications too in radiation shielding.

They also might make awesome nuclear fuel, because you better believe they're going to fizzle like crazy. So who knows? Maybe we can-- I don't think that would be cost-effective, but it



would probably work.

So the way they're doing this is actually slamming calcium 48 nuclei into other super heavy elements that have exceptionally long half-lives. So if you can't read what the screen says, this here is berkelium, for Berkeley, with proton number 97, mass number 249. Let's take a look at the Bk-249, which happens to be way beyond uranium.

So it's definitely not a stable isotope, but it has a half-life of 320 days. That means you can make a bunch of it, chemically separate it, make it into a target, and fire calcium-48 nuclei into it. Anyone want to guess, why do we use calcium-48? And I'll give you a hint and write the proton number for calcium. The isotope that we use is calcium-28-- or calcium-48.

Anyone want to take a guess? Why start here? Why not just smash two berkeliums into each other? Calcium 48 happens to be exceptionally stable, because it's got two magic numbers. Its proton number is 20, one of those peaks of stability. And its neutron number is 28. So start with something super stable, something with a lot more binding energy to begin with, and you maximize your chance of making something with more binding energy that won't just spontaneously disappear. So there are reasons calcium 48 was chosen and not something heavier or lighter.

If we go back to that article, you can see what happens here is you make some element 117, which has yet to be made, and it undergoes alpha decay until it reaches some rather stable-- you know, 17 seconds. That's pretty exceptional. And if you notice the trends here, as you decay, the alpha energy steadily goes down, and the half-life steadily goes up.

And so what you do is you make a super, super heavy element, hoping that it will decay and rest in one of these islands of stability beyond the magic numbers that we know right now. Which I thought this was super cool, because this is actually happening now. Like, new elements are made. I think we've been seeing one a year or so for the past few years on average. There might have been a year when there was more than one announced recently.

There's only a few places in the world doing them, but you can start to-- already with two weeks of 22.01, you can start to get a handle for why do they use the nuclei they do, and then what sort of things are you looking for? Decays with lower and lower energy mean you're already starting to get less steep on whatever imaginary mass parabola. Don't quite know how to draw this one, because it's beyond anything we know. And as the half-lives keep going up, you can tell that it's reaching a measure of stability.

However, to get you started on the homework, for the open-ended ended problem-- I think I'll bring it up right now so you guys can take a look. So let's go to the Stellar site. Hopefully it doesn't call me. Good.

And two problem set two. This is the way I know that everyone's seen the P set seven days before it's due, because I'm going to put it up on the screen so you can see it all the way at the end. Predicting the island of stability. Does the semi-empirical mass formula predict the island of stability?

Well, let's start you off with the easier part of the question, which is yes or no. And I'm going to leave you to the why and the how. If we graph binding energy per nucleon versus mass number, the semi-empirical mass formula predicts something like this. What happens as we go beyond the realm of known mass numbers? Anyone? How should I extend this curve? What did you say, Alex?

**AUDIENCE:** I don't know.

**PROFESSOR:** Just keep going. Yeah. Does this predict an island of stability? I don't think so. So that's one of the few questions I'm asking you in this homework. And it's up to you guys. Use your creativity. Again, this is an open-ended problem. I'm not looking for a specific answer. I want to see how you think and how you would change this formula to account and actually predict the island of stability while still satisfying the mostly correct predictions from the elements we know. So-- sorry, go ahead.

**AUDIENCE:** So should it, like, converge a little bit?

**PROFESSOR:** Well, you're on the right track. If you want to show stability, you'd want it to maybe have a higher value right here. Higher binding energy per nucleon would correspond to a lower mass, which would correspond to higher stability. So how would you predict this island of stability? And then more specifically, how would you reconcile the inaccuracies in the semi-empirical mass formula?

Because we know it doesn't work very well for all cases. There are some cases like right around here where it works great, and there's some like right here and right here where it really doesn't. You can get things wrong by like 10 MeV, which is pretty significant. You know, that's like four digits on the mass scale, like, the fourth decimal place. That's huge to a nuclear

engineer. So that's something to get thinking about.

And remember I did tell you that there will be some open-ended problems. I'm going to mark them as open-ended so you actually know. We're not looking for a right or wrong answer. This is one of those kinds of things where we want to see how you think and what do you think is missing.

There's other hard problems where we give you the answer, because I'm not interested in you deriving some insane expression and getting it right. I'm interested in the derivation process. What are the steps you choose? What sort of assumptions do you make? What sort of terms can you neglect and say, that's in the ninth decimal place. I'm going to forget it.

So in this case, we give you the answer, because we're going to grade you on the process. And you can use the answer to check your process and see if you're on the right track or not. For the skill-building questions, we actually do want you to come up with some sort of an answer like explaining the terms in the semi-empirical mass formula or modifying an equation to calculate something else. We will be looking for a right answer there. But those are questions to make sure that you get the basics of the material.

If you can answer all of the questions in the first half of these P sets fairly quickly, let's say in three or four hours, you're totally on the right track. The hard ones is because this is MIT. And we want you to think beyond just knowing what's in the Turner book or the Yit book. Like I said, you guys are the leaders of this field.

So any other questions on stability in general? Yes?

**AUDIENCE:** Just a real quick reminder. When you say, like, even-even, are you talking protons, neutrons?

**PROFESSOR:** Correct. So that would be even N and even Z or odd N and odd Z like in the reading and like on these mass parabolas. Yep. Any other questions? Yes.

**AUDIENCE:** Is the only proof or reason that we say that there's an island of stability because the mass increases up to the point of unknown?

**PROFESSOR:** There's a few-- so the question was, is the only reason people think there will be super heavy elements because the mass increases, right?

**AUDIENCE:** Yeah.

**PROFESSOR:** So in this case, the mass will always-- are you talking about now the total mass or--

**AUDIENCE:** Why is the idea that there is this island of stability?

**PROFESSOR:** Ah, OK.

**AUDIENCE:** If this doesn't prove it, do we have other reasons [INAUDIBLE]?

**PROFESSOR:** We have a few things to go on. There are a number of different aspects of nuclear stability that are all pointing to the same conclusion. One of them, you can see on this graph here. If you look at the alpha decay half-life as a function of neutron number, it doesn't just increase or decrease monotonically. It swells up and down.

And it reaches a relative maximum near certain magic numbers. We can confirm that with the lower mass nuclei. It doesn't work for really low mass, because tiny things don't tend to undergo alpha decay. But there are patterns that we're simply recognizing and saying, well, if this is the next magic number, it should continue to increase.

And I should mention too, this scale is logarithmic. So the top right here is like 10 to the 4 seconds. Just so you know, there are 86,400 seconds in-- what is it-- a day. And 3 times 10 to the 7 seconds in a year.

So if this graph-- let's say for Z 111-- were to continue on its track, it should reach like 10 to the 9 or 10 to the 10, which could be like 100-year lifetimes or 100-year half-lives, which means definitely you can chemically separate them and do things with them. I don't know if they would be safe enough to deal with.

But we also don't really know what's going to happen. You can see that there is some uncertainty, and things don't always follow the trend. Even the error bars are outside the dashed lines. But so we have this to go on. We have the alpha decay half-life.

We also have the alpha decay energy. As you approach an island of stability, something that's more stable won't give off as much kinetic energy to its alpha particle. There is also-- for the ones that you can actually measure that live long enough, you can measure their mass to charge ratio and actually get a good picture of their actual mass.

So we would expect the mass defect to follow a certain trend as we go up. The mass is always going to increase. If you add more nucleons, it's going to increase. But the mass defect, which

is the real mass minus the atomic number mass-- if stability were to increase, do you think the mass defect would increase or decrease with more stability?

Let's take a quick look at this. If  $A$  were to stay the same, a shrinking real mass-- and remember, lower mass means more stability-- would mean a higher or a low mass defect? It would mean a lower mass defect or a lower excess mass as you'd call it. So in this case, you would expect the mass of the nucleus to be smaller than its  $A$  if it was going more stable.

And all of these trends work in the same direction, which is saying, OK, so we have the alpha energy. We have the mass defect. We have the half-life all pointing to the same thing that something should be more stable. And we have some patterns to go on, but our understanding is kind of incomplete. So-- yeah?

**AUDIENCE:** So if there's super heavy elements, do they exist somewhere in space, or do stars make them, possibly?

**PROFESSOR:** Ooh.

**AUDIENCE:** Or are they--

**PROFESSOR:** Good question.

**AUDIENCE:** --currently made?

**PROFESSOR:** So the question is, if super heavy elements exist, do they exist out there in space? I think there would be a couple places they would exist. The source of most of the elements beyond iron is supernovas, where regular old fusion doesn't cut it anymore. When you hit the maximum of this binding energy per nucleon curve, you're at about iron 56. That's why stars tend to form a core of iron before it goes really bad in whatever way it does for a star. There are multiple ways.

When you get a supernova, you have an insane explosion, and the core gets compressed from the outside, forcing fusion of heavy elements to happen. That's because you're putting in extra kinetic energy. So it's like you have an endothermic reaction where if  $Q$  is less than zero, how do you make that reaction happen? Add kinetic energy, which can come from a tremendous explosion outside of the outer regions of the star.

So who's to say that some of these super heavy elements aren't formed in supernovas? I think

they would be. But would they actually make it out to be part of Earth and then, let's say, live the 5 billion years that Earth's been around? We don't know if their half-lives are long enough. There very well may have been some 5 billion years ago or when the supernova was made. But we haven't detected any here on Earth. So we know that they're not 5 billion years stable. Rather, I wouldn't even say we know that, but we have a pretty good idea.

That's a great question is like, are they naturally made? Probably. Yeah. Any other questions? I like these outside the material ones. We can take things beyond our known universe, start to explain them.