MAS.160 / MAS.510 / MAS.511 Signals, Systems and Information for Media Technology
Fall 2007

Causal FIR filter

$$y[n] = \sum_{k=0}^{M} b_k x[n-k]$$

$$y[n] = b_0 x[n] + b_1 x[n-1] + \ldots + b_M x[n-M]$$

The output y at each sample n is a weighted sum of the present input, x[n], and past inputs, x[n-1], x[n-2],…, x[n-M].

3 point average

causal running average or backward average

$$y[n] = \sum_{k=0}^{M} b_k x[n-k]$$

$$b_0 = \frac{1}{3} \quad b_1 = \frac{1}{3} \quad b_2 = \frac{1}{3}$$

$$y[n] = \frac{1}{3}x[n] + \frac{1}{3}x[n-1] + \frac{1}{3}x[n-2]$$

difference equation

L=3 M=L-1=2

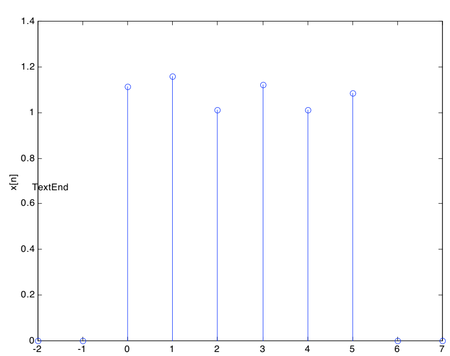
Length 3 2nd order

3 point average

$$y[n] = \frac{1}{3}x[n] + \frac{1}{3}x[n-1] + \frac{1}{3}x[n-2]$$

$$x[n] = \{0 \quad 0 \quad 1.11 \quad 1.16 \quad 1.01 \quad 1.12 \quad 1.01 \quad 1.08 \quad 0 \quad 0\}$$

n=-2 -1 0 1 2 3 4 5 6 7

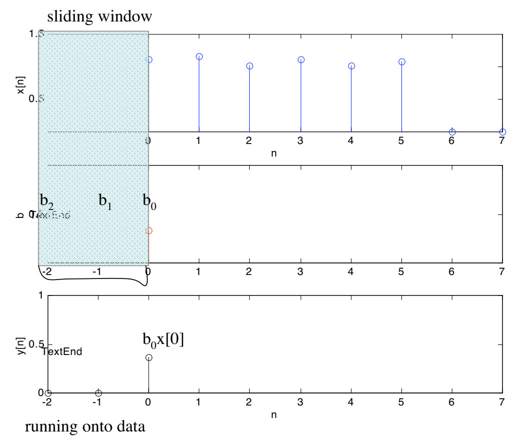


$$y[n] = \frac{1}{3}x[n] + \frac{1}{3}x[n-1] + \frac{1}{3}x[n-2]$$

$$x[n] = \{\mathbf{0} \quad \mathbf{0} \quad \mathbf{1.11} \quad 1.16 \quad 1.01 \quad 1.12 \quad 1.01 \quad 1.08 \quad 0 \quad 0\}$$

n= -2 -1 0 1 2 3 4 5 6 7

$$y[0] = \frac{1}{3}x[0] + \frac{1}{3}x[-1] + \frac{1}{3}x[-2] = \frac{1}{3}1.11 + \frac{1}{3}0 + \frac{1}{3}0 = 0.36$$

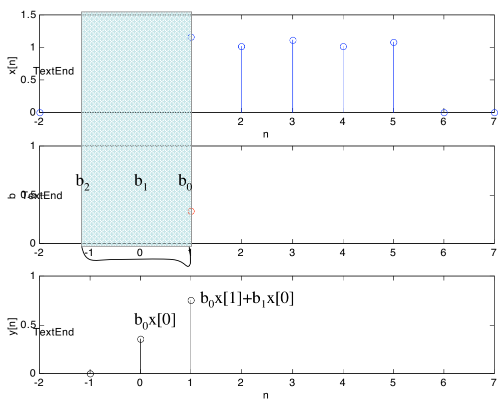


$$y[n]=\frac{1}{3}x[n]+\frac{1}{3}x[n-1]+\frac{1}{3}x[n-2]$$

$x[n]=\{$0 **0** **1.11** **1.16** 1.01 1.12 1.01 1.08 0 0$\}$

n= -2 -1 0 1 2 3 4 5 6 7

$$y[1]=\frac{1}{3}x[1]+\frac{1}{3}x[0]+\frac{1}{3}x[-1]=\frac{1}{3}1.16+\frac{1}{3}1.11+\frac{1}{3}0=0.76$$

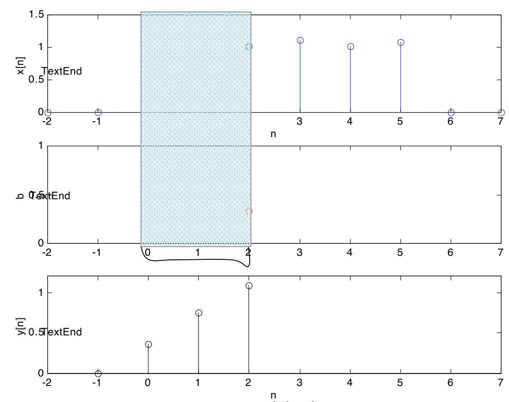


$$y[n]=\frac{1}{3}x[n]+\frac{1}{3}x[n-1]+\frac{1}{3}x[n-2]$$

$x[n]=\{$0 0 **1.11** **1.16** **1.01** 1.12 1.01 1.08 0 0$\}$

n= -2 -1 0 1 2 3 4 5 6 7

$$y[2]=\frac{1}{3}x[2]+\frac{1}{3}x[1]+\frac{1}{3}x[0]=\frac{1}{3}1.01+\frac{1}{3}1.16+\frac{1}{3}1.11=1.09$$

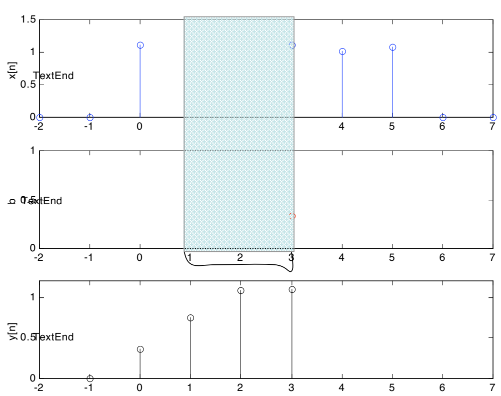


$$y[n]=\frac{1}{3}x[n]+\frac{1}{3}x[n-1]+\frac{1}{3}x[n-2]$$

$x[n]=\{$0 0 1.11 **1.16** **1.01** **1.12** 1.01 1.08 0 0$\}$

n= -2 -1 0 1 2 3 4 5 6 7

$$y[3]=\frac{1}{3}x[3]+\frac{1}{3}x[2]+\frac{1}{3}x[1]=\frac{1}{3}1.12+\frac{1}{3}1.01+\frac{1}{3}1.16=1.10$$

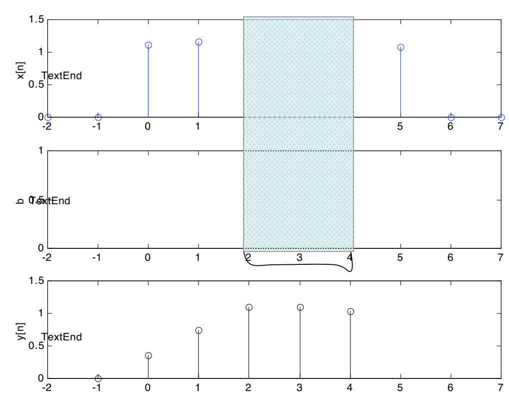


$$y[n]=\frac{1}{3}x[n]+\frac{1}{3}x[n-1]+\frac{1}{3}x[n-2]$$

$x[n]=\{$0 0 1.11 1.16 **1.01** **1.12** **1.01** 1.08 0 0$\}$

n= -2 -1 0 1 2 3 4 5 6 7

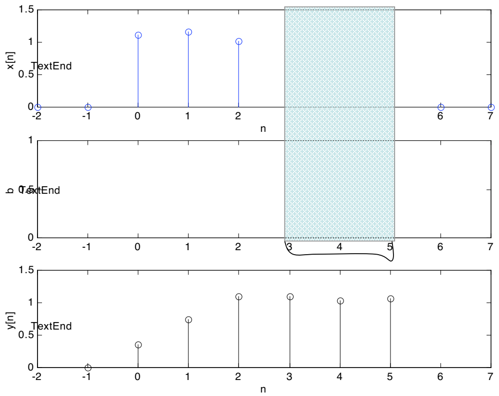
$$y[4]=\frac{1}{3}x[4]+\frac{1}{3}x[3]+\frac{1}{3}x[2]=\frac{1}{3}1.01+\frac{1}{3}1.12+\frac{1}{3}1.01=1.05$$



$y[n] = \frac{1}{3}x[n] + \frac{1}{3}x[n-1] + \frac{1}{3}x[n-2]$

$x[n] = \{0 \quad 0 \quad 1.11 \quad 1.16 \quad 1.01 \quad \mathbf{1.12} \quad \mathbf{1.01} \quad \mathbf{1.08} \quad 0 \quad 0\}$

$$y[5] = \frac{1}{3}x[5] + \frac{1}{3}x[4] + \frac{1}{3}x[3] = \frac{1}{3}1.08 + \frac{1}{3}1.01 + \frac{1}{3}1.12 = 1.07$$

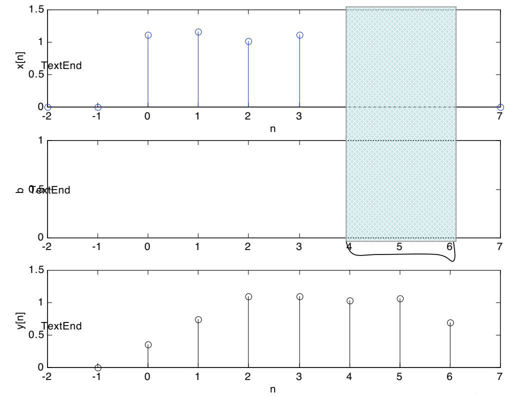


$y[n] = \frac{1}{3}x[n] + \frac{1}{3}x[n-1] + \frac{1}{3}x[n-2]$

$x[n] = \{0 \quad 0 \quad 1.11 \quad 1.16 \quad 1.01 \quad 1.12 \quad \mathbf{1.01} \quad \mathbf{1.08} \quad \mathbf{0} \quad 0\}$

n= -2 -1 0 1 2 3 4 5 6 7

$$y[6] = \frac{1}{3}x[6] + \frac{1}{3}x[5] + \frac{1}{3}x[4] = \frac{1}{3}0 + \frac{1}{3}1.08 + \frac{1}{3}1.01 = 0.70$$



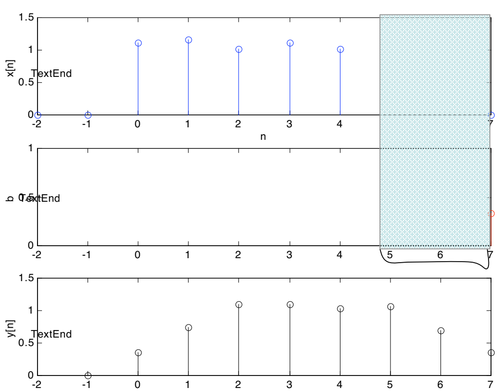
running off data

$y[n] = \frac{1}{3}x[n] + \frac{1}{3}x[n-1] + \frac{1}{3}x[n-2]$

$x[n] = \{0 \quad 0 \quad 1.11 \quad 1.16 \quad 1.01 \quad 1.12 \quad 1.01 \quad \mathbf{1.08} \quad \mathbf{0} \quad \mathbf{0}\}$

n= -2 -1 0 1 2 3 4 5 6 7

$$y[7] = \frac{1}{3}x[7] + \frac{1}{3}x[6] + \frac{1}{3}x[5] = \frac{1}{3}0 + \frac{1}{3}0 + \frac{1}{3}1.08 = 0.36$$



running off data

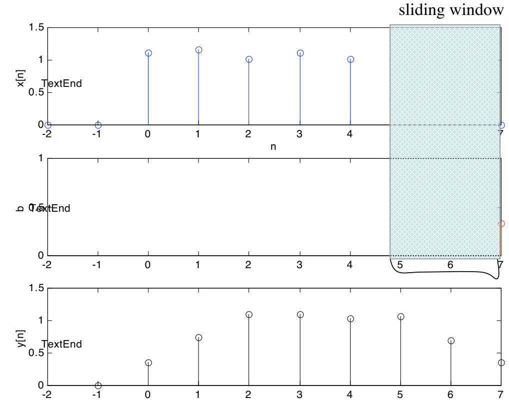
$y[n] = \frac{1}{3}x[n] + \frac{1}{3}x[n-1] + \frac{1}{3}x[n-2]$

$x[n] = \{0 \quad 0 \quad 1.11 \quad 1.16 \quad 1.01 \quad 1.12 \quad 1.01 \quad 1.08 \quad 0.0 \quad 0.0\}$

$y[n] = \{0 \quad 0 \quad 0.36 \quad 0.75 \quad 1.09 \quad 1.10 \quad 1.05 \quad 1.07 \quad 0.7 \quad 0.36\}$

n= -2 -1 0 1 2 3 4 5 6 7

sliding window



running onto data   weighted sum   running off data

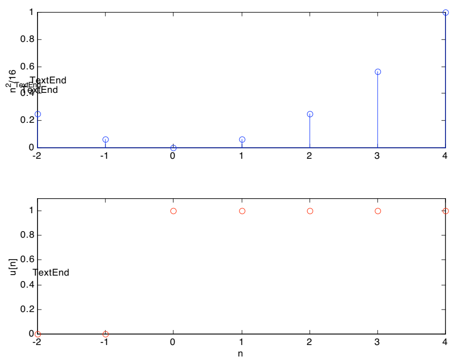
Notice:
y[n] is longer than x[n].
x[n] is M/2 delayed

2 point difference

$$y[n] = x[n] - x[n-1]$$

$$x[n] = \frac{n^2}{16} \cdot u[n] \qquad u[n] = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases}$$

unit step function

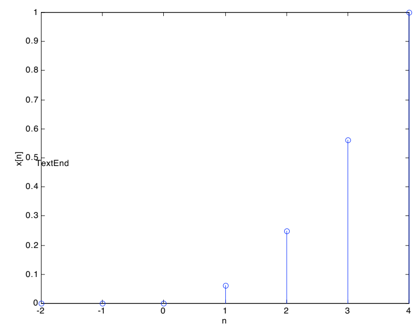


2 point difference

$$y[n] = x[n] - x[n-1]$$

$$x[n] = \frac{n^2}{16} \cdot u[n] \qquad u[n] = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases}$$

unit step function



$$y[n] = x[n] - x[n-1]$$

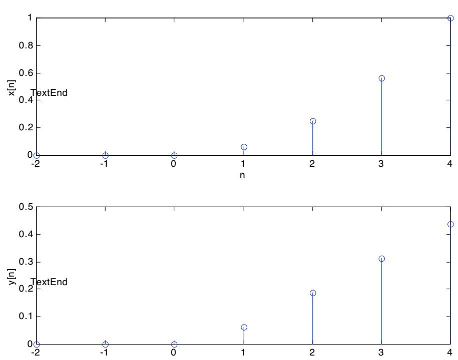
$$x[n] = \frac{n^2}{16} \cdot u[n]$$

$$x[n] = \{0 \quad 0 \quad 0 \quad \tfrac{1}{16} \quad \tfrac{4}{16} \quad \tfrac{9}{16} \quad \tfrac{16}{16}\}$$

$$y[n] = \{0 \quad 0 \quad 0 \quad \tfrac{1}{16} \quad \tfrac{3}{16} \quad \tfrac{5}{16} \quad \tfrac{7}{16}\} = \frac{2n-1}{16} u[n-1]$$

finite difference
approximation to
a derivative.

derivatives enhance
noise (and high frequencies)



Impulse response

$$y[n] = \sum_{k=0}^{M} b_k x[n-k]$$ FIR filter

$$x[n] = \delta[n] = \begin{cases} 1 & n = 0 \\ 0 & otherwise \end{cases}$$ Kronecker delta function

$$y[n] = h[n] = \sum_{k=0}^{M} b_k \delta[n-k] \qquad \delta[n-k] = \begin{cases} 1 & n = k \\ 0 & otherwise \end{cases}$$

impulse response

The impulse response is the output of the system when the input is a delta function.

Impulse response

$$h[n] = y[n] = \sum_{k=0}^{M} b_k \delta[n-k] \qquad \delta[n-k] = \begin{cases} 1 & n = k \\ 0 & otherwise \end{cases}$$

$$h[n] = b_0\delta[\overset{k=0}{n-0}] + b_1\delta[n-1] + \ldots + b_n\delta[\overset{k=n}{n-n}] + \ldots + b_M\delta[M-1]$$

$$= b_0\delta[n-0] + b_1\delta[n-1] + \ldots + b_n\delta[0] + \ldots + b_M\delta[M-1]$$

$$= b_0 0 + b_1 0 + \ldots + b_n 1 + \ldots + b_M 0 \qquad \delta[z] = \begin{cases} 1 & z = 0 \\ 0 & otherwise \end{cases}$$

$= b_n$ impulse response

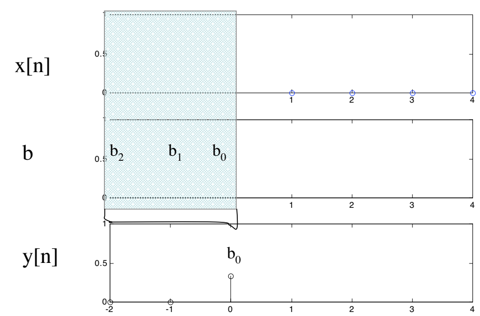
The impulse response is just the filter coefficients.
Finite length filter, finite impulse response (FIR).

Impulse response of 3 pt. average

$$y[n] = \frac{1}{3}x[n] + \frac{1}{3}x[n-1] + \frac{1}{3}x[n-2]$$

$$x[n] = \delta[n] = \begin{cases} 1 & n = 0 \\ 0 & otherwise \end{cases}$$ Delta function

$$y[n] = \frac{1}{3}\delta[n] + \frac{1}{3}\delta[n-1] + \frac{1}{3}\delta[n-2]$$



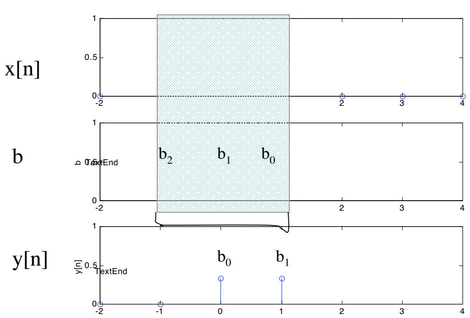
$$y[0] = \frac{1}{3}\delta[0] + \frac{1}{3}\delta[-1] + \frac{1}{3}\delta[-2]$$
$$= \frac{1}{3}1 + \frac{1}{3}0 + \frac{1}{3}0 = \frac{1}{3}$$

Impulse response of 3 pt. average

$$y[n] = \frac{1}{3}x[n] + \frac{1}{3}x[n-1] + \frac{1}{3}x[n-2]$$

$$x[n] = \delta[n] = \begin{cases} 1 & n = 0 \\ 0 & otherwise \end{cases}$$ Delta function

$$y[n] = \frac{1}{3}\delta[n] + \frac{1}{3}\delta[n-1] + \frac{1}{3}\delta[n-2]$$



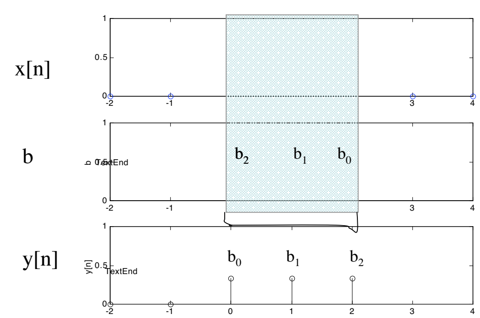
$$y[1] = \frac{1}{3}\delta[1] + \frac{1}{3}\delta[0] + \frac{1}{3}\delta[-1]$$
$$= \frac{1}{3}0 + \frac{1}{3}1 + \frac{1}{3}0 = \frac{1}{3}$$

Impulse response of 3 pt. average

$$y[n] = \frac{1}{3}x[n] + \frac{1}{3}x[n-1] + \frac{1}{3}x[n-2]$$

$$x[n] = \delta[n] = \begin{cases} 1 & n = 0 \\ 0 & otherwise \end{cases}$$ Delta function

$$y[n] = \frac{1}{3}\delta[n] + \frac{1}{3}\delta[n-1] + \frac{1}{3}\delta[n-2]$$



$$y[2] = \frac{1}{3}\delta[2] + \frac{1}{3}\delta[1] + \frac{1}{3}\delta[0]$$
$$= \frac{1}{3}0 + \frac{1}{3}0 + \frac{1}{3}1 = \frac{1}{3}$$

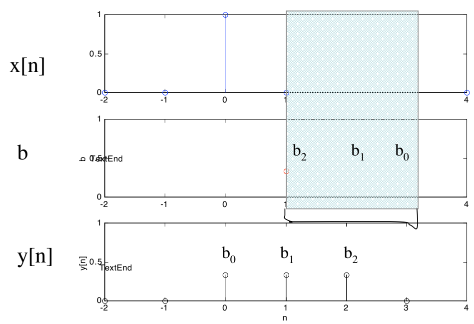
Impulse response of 3 pt. average

$$y[n] = \frac{1}{3}x[n] + \frac{1}{3}x[n-1] + \frac{1}{3}x[n-2]$$

$$x[n] = \delta[n] = \begin{cases} 1 & n = 0 \\ 0 & otherwise \end{cases}$$

Delta function

$$y[n] = \frac{1}{3}\delta[n] + \frac{1}{3}\delta[n-1] + \frac{1}{3}\delta[n-2]$$



$$y[3] = \frac{1}{3}\delta[3] + \frac{1}{3}\delta[2] + \frac{1}{3}\delta[1]$$

$$= \frac{1}{3}0 + \frac{1}{3}0 + \frac{1}{3}0 = 0$$

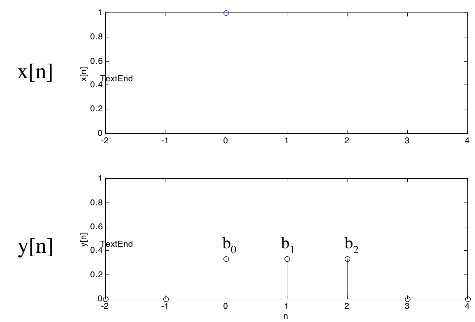
Impulse response of 3 pt. average

$$n = -2 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4$$

$$x[n] = \delta[n] = \{0 \quad 0 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0\}$$

$$h[n] = y[n]\Big|_{x[n]=\delta[n]} = \{0 \quad 0 \quad \tfrac{1}{3} \quad \tfrac{1}{3} \quad \tfrac{1}{3} \quad 0 \quad 0\}$$

$$= \{0 \quad 0 \quad b_0 \quad b_1 \quad b_2 \quad 0 \quad 0\}$$



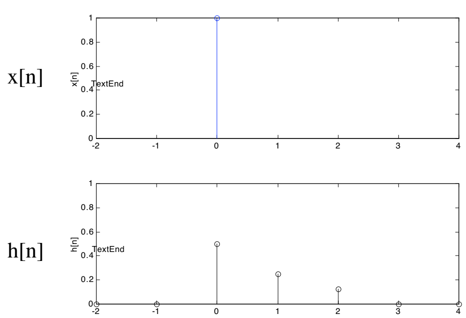
Coefficients from impulse response

$$x[n] = \delta[n] = \{0 \quad 0 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0\}$$

$$n = -2 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4$$

$$\longrightarrow \boxed{\begin{array}{c} y[n] = \sum_{k=0}^{M} b_k x[n-k] \\ b_0, b_1, \ldots b_M = ??? \end{array}} \longrightarrow$$

$$h[n] = y[n]\Big|_{x[n]=\delta[n]} = \{0 \quad 0 \quad \tfrac{4}{8} \quad \tfrac{2}{8} \quad \tfrac{1}{8} \quad 0 \quad 0\}$$



Coefficients from impulse response

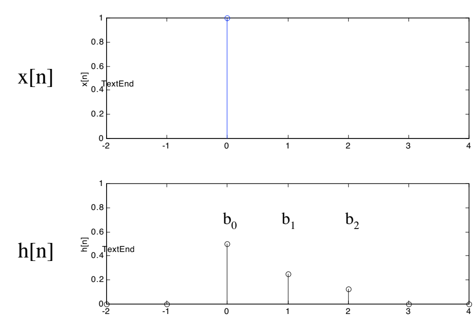
$$x[n] = \delta[n] = \{0 \quad 0 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0\}$$

$$n = -2 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4$$

$$\longrightarrow \boxed{\begin{array}{c} y[n] = \sum_{k=0}^{M} b_k x[n-k] \\ \{b_0, b_1, b_2\} = \{\tfrac{4}{8}, \tfrac{2}{8}, \tfrac{1}{8}\} \end{array}} \longrightarrow$$

$$h[n] = y[n]\Big|_{x[n]=\delta[n]} = \{0 \quad 0 \quad \tfrac{4}{8} \quad \tfrac{2}{8} \quad \tfrac{1}{8} \quad 0 \quad 0\}$$

$$= \{0 \quad 0 \quad b_0 \quad b_1 \quad b_2 \quad 0 \quad 0\}$$



Response from 2 impulses

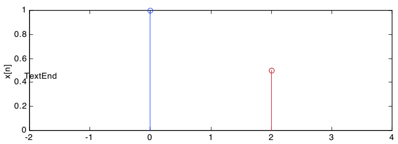
$$x[n]=\delta[n]+0.5\delta[n-2] \longrightarrow \boxed{\begin{array}{c} y[n]=\sum_{k=0}^{M} b_k x[n-k] \\ \{b_0,b_1,b_2\}=\{\frac{4}{8},\frac{2}{8},\frac{1}{8}\}\end{array}} \longrightarrow y[n]=$$

$$=\{0\quad 0\quad 1\quad 0\quad 0.5\quad 0\quad 0\} \qquad \{0\quad 0\quad \tfrac{4}{8}\quad \tfrac{2}{8}\quad \tfrac{1}{8}+\tfrac{4}{16}\quad \tfrac{2}{8}\quad \tfrac{1}{16}\}$$

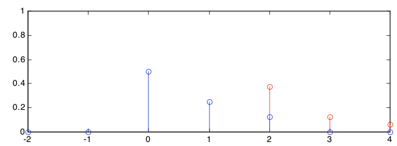
$n=-2\quad -1\quad 0\quad 1\quad 2\quad 3\quad 4$

$$y[n]=\{0\quad 0\quad b_0x[0]\quad b_1x[0]\quad b_2x[0]+b_0x[2]\quad b_1x[3]\quad b_2x[4]\}$$

x[n]



y[n]



Sum the responses of each impulse

Response from 2 impulses

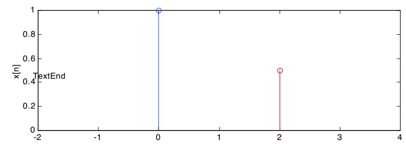
$$x[n]=\delta[n] \longrightarrow \boxed{\begin{array}{c} y[n]=\sum_{k=0}^{M} b_k x[n-k] \\ h[n]=\{b_0,b_1,b_2\}=\{\frac{4}{8},\frac{2}{8},\frac{1}{8}\}\end{array}} \longrightarrow y[n]=$$

$$=\{0\quad 0\quad 1\quad 0\quad 1\quad 0\quad 0\} \qquad \{0\quad 0\quad \tfrac{4}{8}\quad \tfrac{2}{8}\quad \tfrac{1}{8}+\tfrac{4}{16}\quad \tfrac{2}{8}\quad \tfrac{1}{16}\}$$

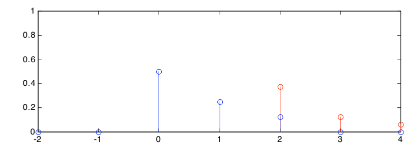
$n=-2\quad -1\quad 0\quad 1\quad 2\quad 3\quad 4$

$$y[n]=\{0\quad 0\quad h[0]x[0]\quad h[1]x[0]\quad h[2]x[0]+h[0]x[2]\quad h[1]x[3]\quad h[2]x[4]\}$$

x[n]



y[n]



$$y[2]=h[0]x[2-0]+h[1]x[2-1]+h[2]x[2-2]$$

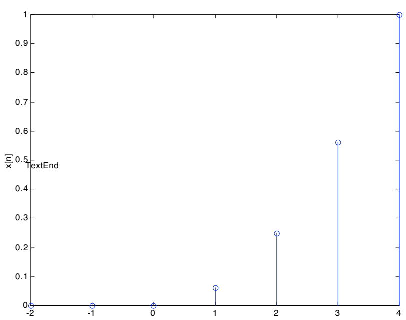
$$y[n]=\sum_{k=0}^{3} h[k]x[n-k]$$

Convolution sum

$$x[n]=\frac{n^2}{16}\cdot u[n] \qquad u[n]=\begin{cases}1 & n\geq 0\\ 0 & n<0\end{cases}$$

$$x[n]=\{0\quad 0\quad 0\quad \tfrac{1}{16}\quad \tfrac{4}{16}\quad \tfrac{9}{16}\quad \tfrac{16}{16}\}$$

$n=-2\quad -1\quad 0\quad 1\quad 2\quad 3\quad 4$



$$x[n]=0\cdot\delta[n]+\tfrac{1}{16}\cdot\delta[n-1]+\tfrac{4}{16}\cdot\delta[n-2]+\tfrac{9}{16}\cdot\delta[n-3]+\tfrac{16}{16}\cdot\delta[n-4]$$

Any discrete signal be thought of a weighted sum of delayed impulses

Convolution

$$y[n]=\sum_{k=0}^{M} b_k x[n-k] \qquad \text{FIR filter}$$

$$h[n]=y[n]\Big|_{x[n]=\delta[n]}=\begin{cases}b_n & n=0,1\ldots M\\ 0 & otherwise\end{cases} \qquad \text{impulse response}$$

$$\downarrow$$

$$y[n]=\sum_{k=0}^{M} h[k]x[n-k] \qquad \text{convolution sum}$$

or

$$y[n]=h*x$$

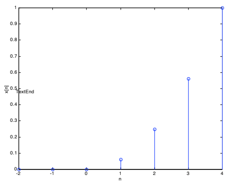
The output y[n] is equal to the input x[n] convolved with the unit impulse response h[n].

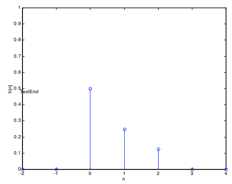
$$x[n] = \frac{n^2}{16} \cdot u[n] \longrightarrow \boxed{y[n] = \sum_{k=0}^{M} h[k]x[n-k]} \longrightarrow y[n]$$

$$x[n] = \{0 \quad \tfrac{1}{16} \quad \tfrac{4}{16} \quad \tfrac{9}{16} \quad \tfrac{16}{16} \quad 0 \quad 0\}$$

$n = 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6$

$$h[n] = \left\{\tfrac{4}{8} \quad \tfrac{2}{8} \quad \tfrac{1}{8}\right\}$$

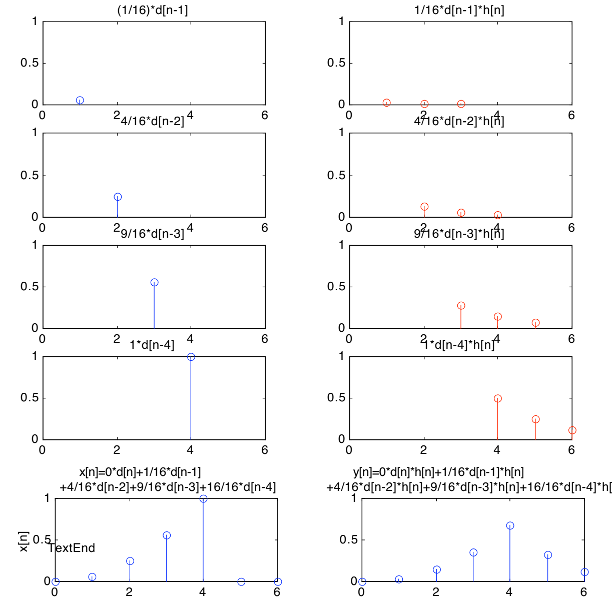




$x[n] \quad * \quad h[n] \quad = \quad y[n]$

$$x[n] = \{0 \quad \tfrac{1}{16} \quad \tfrac{4}{16} \quad \tfrac{9}{16} \quad \tfrac{16}{16} \quad 0 \quad 0\} \qquad h[n] = \left\{\tfrac{4}{8} \quad \tfrac{2}{8} \quad \tfrac{1}{8}\right\}$$

=



$$x[n] = \{0 \quad \tfrac{1}{16} \quad \tfrac{4}{16} \quad \tfrac{9}{16} \quad \tfrac{16}{16} \quad 0 \quad 0\} \qquad h[n] = \left\{\tfrac{4}{8} \quad \tfrac{2}{8} \quad \tfrac{1}{8}\right\}$$

$n = 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6$

x[n]	0	1/16	4/16	9/16	16/16	0	0
h[n]	4/8	2/8	1/8				

==

0	0	0

==

$$x[n] = \{0 \quad \tfrac{1}{16} \quad \tfrac{4}{16} \quad \tfrac{9}{16} \quad \tfrac{16}{16} \quad 0 \quad 0\} \qquad h[n] = \left\{\tfrac{4}{8} \quad \tfrac{2}{8} \quad \tfrac{1}{8}\right\}$$

$n = 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6$

x[n]	0	1/16	4/16	9/16	16/16	0	0
h[n]		4/8	2/8	1/8			

==

0	0	0	
	4/128	2/128	1/128

==

$x[n] = \{0\ \ \frac{1}{16}\ \ \frac{4}{16}\ \ \frac{9}{16}\ \ \frac{16}{16}\ \ 0\ \ 0\}$, $n = 0\ \ 1\ \ 2\ \ 3\ \ 4\ \ 5\ \ 6$ $\qquad h[n] = \{\frac{4}{8}\ \ \frac{2}{8}\ \ \frac{1}{8}\}$

x[n]	0	1/16	4/16	9/16	16/16	0	0
h[n]			4/8	2/8	1/8		
=====							
	0	0	0				
		4/128	2/128	1/128			
			16/128	8/128	4/128		
=====							

$x[n] = \{0\ \ \frac{1}{16}\ \ \frac{4}{16}\ \ \frac{9}{16}\ \ \frac{16}{16}\ \ 0\ \ 0\}$, $n = 0\ \ 1\ \ 2\ \ 3\ \ 4\ \ 5\ \ 6$ $\qquad h[n] = \{\frac{4}{8}\ \ \frac{2}{8}\ \ \frac{1}{8}\}$

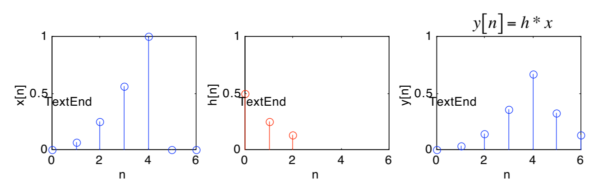
x[n]	0	1/16	4/16	9/16	16/16	0	0
h[n]				4/8	2/8	1/8	
=====							
	0	0	0				
		4/128	2/128	1/128			
			16/128	8/128	4/128		
				36/128	18/128	9/128	
=====							

$x[n] = \{0\ \ \frac{1}{16}\ \ \frac{4}{16}\ \ \frac{9}{16}\ \ \frac{16}{16}\ \ 0\ \ 0\}$, $n = 0\ \ 1\ \ 2\ \ 3\ \ 4\ \ 5\ \ 6$ $\qquad h[n] = \{\frac{4}{8}\ \ \frac{2}{8}\ \ \frac{1}{8}\}$

x[n]	0	1/16	4/16	9/16	16/16	0	0
h[n]					4/8	2/8	1/8
=====							
	0	0	0				
		4/128	2/128	1/128			
			16/128	8/128	4/128		
				36/128	18/128	9/128	
					64/128	32/128	16/128
=====							
y[n]	0	4/128	18/128	45/128	86/128	41/128	16/128

$x[n] = \{0\ \ \frac{1}{16}\ \ \frac{4}{16}\ \ \frac{9}{16}\ \ \frac{16}{16}\ \ 0\ \ 0\}$, $n = 0\ \ 1\ \ 2\ \ 3\ \ 4\ \ 5\ \ 6$ $\qquad h[n] = \{\frac{4}{8}\ \ \frac{2}{8}\ \ \frac{1}{8}\}$

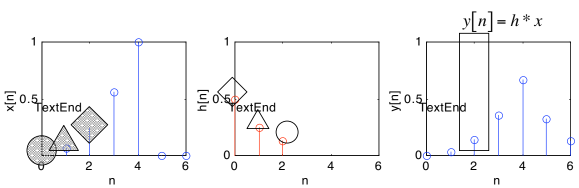
x[n]	0	1/16	4/16	9/16	16/16	0	0
h[n]	4/8	2/8	1/8				
=====							
	0	0	0				
		4/128	2/128	1/128			
			16/128	8/128	4/128		
				36/128	18/128	9/128	
					64/128	32/128	16/128
=====							
y[n]	0	4/128	18/128	45/128	86/128	41/128	16/128



n	0	1	2	3	4	5	6
x[n]	0 ●	1/16 ▲	4/16 ◆	9/16	16/16	0	0
h[n]	4/8 ◇	2/8 △	1/8 ○				
	0 ◇●	0 △●	0 ○●				
		4/128 ◇▲	2/128 △▲	1/128 ○▲			
			16/128 ◇◆	8/128 △◆	4/128 ○◆		
				36/128	18/128	9/128	
					64/128	32/128	16/128
y[n]	0	4/128	18/128	45/128	86/128	41/128	16/128

$$y[2]=h[0]x[2-0]+h[1]x[2-1]+h[2]x[2-2]$$

◇ ◆ △ ▲ ○ ●



$$y[2]=h[0]x[2-0]+h[1]x[2-1]+h[2]x[2-2]$$

$$y[2]=\sum_{k=0}^{2}h[k]x[2-k]$$

$$y[n]=\sum_{k=0}^{M}h[k]x[n-k] \quad \text{convolution sum}$$

$x[n]=\{0 \ \ \tfrac{1}{16} \ \ \tfrac{4}{16} \ \ \tfrac{9}{16} \ \ \tfrac{16}{16} \ \ 0 \ \ 0\}$ $\quad h[n]=\{\tfrac{4}{8} \ \ \tfrac{2}{8} \ \ \tfrac{1}{8}\}$ L=3, M=L-1=2

$n=0 \ \ 1 \ \ 2 \ \ 3 \ \ 4 \ \ 5 \ \ 6$

$$x[n]=0\cdot\delta[n]+\tfrac{1}{16}\cdot\delta[n-1]+\tfrac{4}{16}\cdot\delta[n-2]+\tfrac{9}{16}\cdot\delta[n-3]+\tfrac{16}{16}\cdot\delta[n-4]$$

$$h[n]=\tfrac{4}{8}\cdot\delta[n]+\tfrac{2}{8}\cdot\delta[n-1]+\tfrac{1}{8}\cdot\delta[n-2]$$

$$y[n]=\sum_{k=0}^{M}h[k]x[n-k] \quad \text{convolution sum}$$

$$y[0]=h[0]x[0-0]+h[1]x[0-1]+h[2]x[0-2]=\tfrac{4}{8}\cdot 0+\tfrac{2}{8}\cdot 0+\tfrac{1}{8}\cdot 0=0$$

$$y[1]=h[0]x[1-0]+h[1]x[1-1]+h[2]x[1-2]=\tfrac{4}{8}\cdot\tfrac{1}{16}+\tfrac{2}{8}\cdot 0+\tfrac{1}{8}\cdot 0=\tfrac{4}{128}$$

$x[n]=\{0 \ \ \tfrac{1}{16} \ \ \tfrac{4}{16} \ \ \tfrac{9}{16} \ \ \tfrac{16}{16} \ \ 0 \ \ 0\}$ $\quad h[n]=\{\tfrac{4}{8} \ \ \tfrac{2}{8} \ \ \tfrac{1}{8}\}$ L=3, M=L-1=2

$n=0 \ \ 1 \ \ 2 \ \ 3 \ \ 4 \ \ 5 \ \ 6$

$$x[n]=0\cdot\delta[n]+\tfrac{1}{16}\cdot\delta[n-1]+\tfrac{4}{16}\cdot\delta[n-2]+\tfrac{9}{16}\cdot\delta[n-3]+\tfrac{16}{16}\cdot\delta[n-4]$$

$$h[n]=\tfrac{4}{8}\cdot\delta[n]+\tfrac{2}{8}\cdot\delta[n-1]+\tfrac{1}{8}\cdot\delta[n-2]$$

$$y[n]=\sum_{k=0}^{M}h[k]x[n-k] \quad \text{convolution sum}$$

$$y[0]=h[0]x[0-0]+h[1]x[0-1]+h[2]x[0-2]=\tfrac{4}{8}\cdot 0+\tfrac{2}{8}\cdot 0+\tfrac{1}{8}\cdot 0=0$$

$$y[1]=h[0]x[1-0]+h[1]x[1-1]+h[2]x[1-2]=\tfrac{4}{8}\cdot\tfrac{1}{16}+\tfrac{2}{8}\cdot 0+\tfrac{1}{8}\cdot 0=\tfrac{4}{128}$$

$$y[2]=h[0]x[2-0]+h[1]x[2-1]+h[2]x[2-2]=\tfrac{4}{8}\cdot\tfrac{4}{16}+\tfrac{2}{8}\cdot\tfrac{1}{16}+\tfrac{1}{8}\cdot 0=\tfrac{18}{128}$$

$$y[3]=h[0]x[3-0]+h[1]x[3-1]+h[2]x[3-2]=\tfrac{4}{8}\cdot\tfrac{9}{16}+\tfrac{2}{8}\cdot\tfrac{4}{16}+\tfrac{1}{8}\cdot\tfrac{1}{16}=\tfrac{45}{128}$$

$$y[4]=h[0]x[4-0]+h[1]x[4-1]+h[2]x[4-2]=\tfrac{4}{8}\cdot\tfrac{16}{16}+\tfrac{2}{8}\cdot\tfrac{9}{16}+\tfrac{1}{8}\cdot\tfrac{4}{16}=\tfrac{86}{128}$$

$$y[5]=h[0]x[5-0]+h[1]x[5-1]+h[2]x[5-2]=\tfrac{4}{8}\cdot 0+\tfrac{2}{8}\cdot\tfrac{16}{16}+\tfrac{1}{8}\cdot\tfrac{9}{16}=\tfrac{41}{16}$$

$$y[6]=h[0]x[6-0]+h[1]x[6-1]+h[2]x[6-2]=\tfrac{4}{8}\cdot 0+\tfrac{2}{8}\cdot 0+\tfrac{1}{8}\cdot\tfrac{16}{16}=\tfrac{16}{128}$$

$$x[n] = \{0 \quad \tfrac{1}{16} \quad \tfrac{4}{16} \quad \tfrac{9}{16} \quad \tfrac{16}{16} \quad 0 \quad 0\}$$
$$n = 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6$$

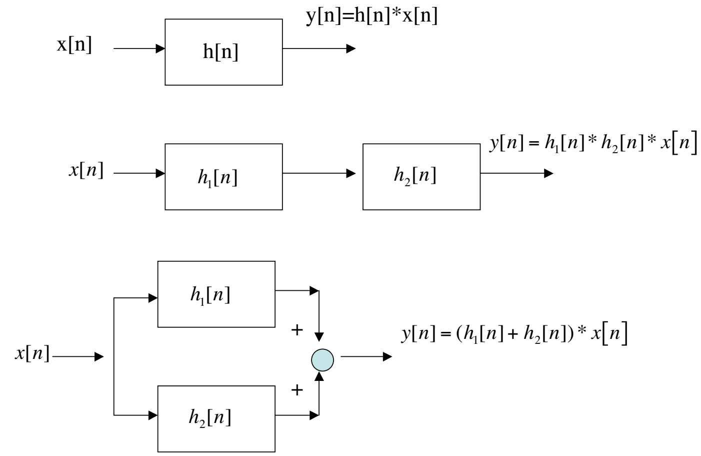
$$h[n] = \left\{\tfrac{4}{8} \quad \tfrac{2}{8} \quad \tfrac{1}{8}\right\}$$

x[n]	0	1/16	4/16	9/16	16/16	0	0
h[n]	4/8	2/8	1/8				
x[n]*h[n]	0	4/128	18/128	45/128	86/128	41/128	16/128
	0	0.3125	0.1406	0.3516	0.6719	0.3203	0.125

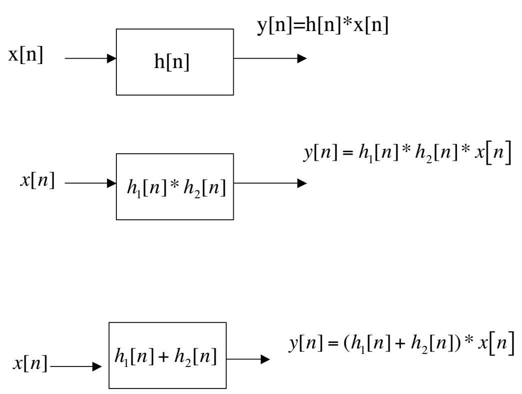
MATLAB

```
»conv([4/8, 2/8, 1/8], [0, 1/16, 4/16, 9/16, 16/16])
ans =
      0    0.0312    0.1406    0.3516    0.6719    0.3203    0.1250
```

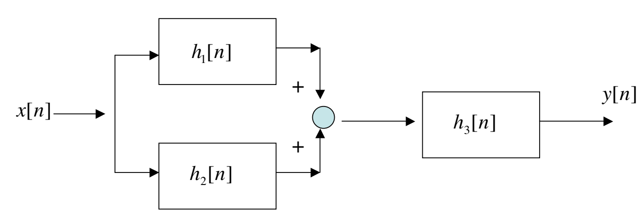
LTI Systems



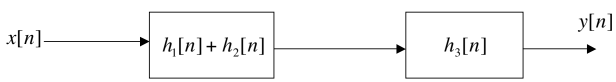
LTI Systems



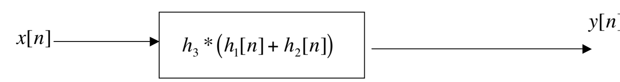
LTI Systems



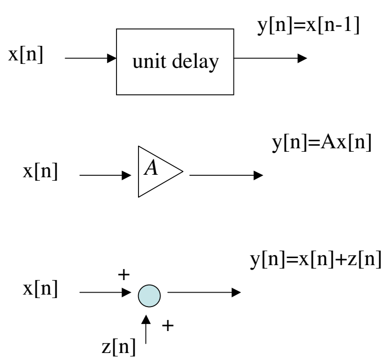
LTI Systems



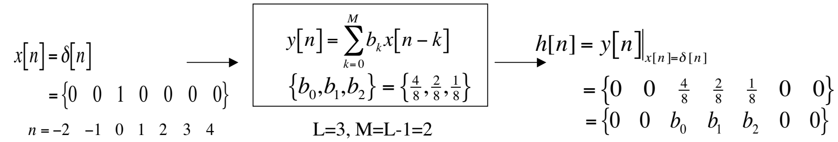
LTI Systems

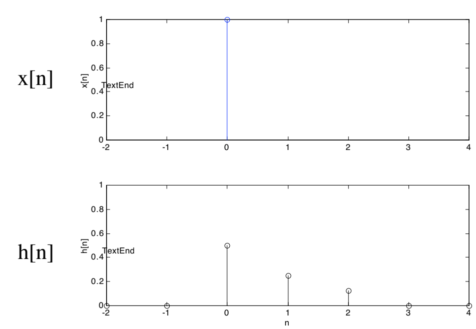


Block Diagrams



Block Diagrams: Direct Form





Block Diagrams: Direct Form

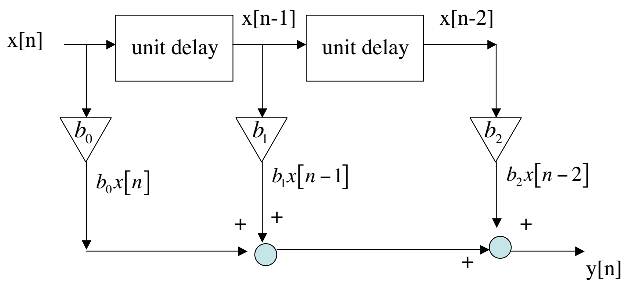
$x[n] = \delta[n]$
$= \{0 \;\; 0 \;\; 1 \;\; 0 \;\; 0 \;\; 0 \;\; 0\}$
$n = -2 \;\; -1 \;\; 0 \;\; 1 \;\; 2 \;\; 3 \;\; 4$

$\longrightarrow$ $y[n] = \sum_{k=0}^{M} b_k x[n-k]$, $\{b_0, b_1, b_2\} = \{\frac{4}{8}, \frac{2}{8}, \frac{1}{8}\}$ $\longrightarrow$

L=3, M=L-1=2

$h[n] = y[n]\big|_{x[n]=\delta[n]}$
$= \{0 \;\; 0 \;\; \frac{4}{8} \;\; \frac{2}{8} \;\; \frac{1}{8} \;\; 0 \;\; 0\}$
$= \{0 \;\; 0 \;\; b_0 \;\; b_1 \;\; b_2 \;\; 0 \;\; 0\}$

$y[n] = b_0 x[n] + b_1 x[n-1] + b_2 x[n-2]$



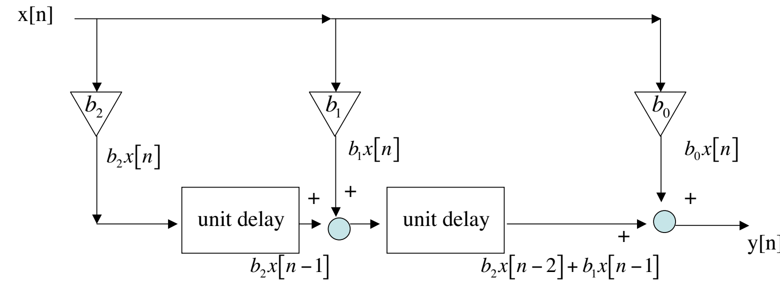
Block Diagrams: Transpose Form

$x[n] = \delta[n]$
$= \{0 \;\; 0 \;\; 1 \;\; 0 \;\; 0 \;\; 0 \;\; 0\}$
$n = -2 \;\; -1 \;\; 0 \;\; 1 \;\; 2 \;\; 3 \;\; 4$

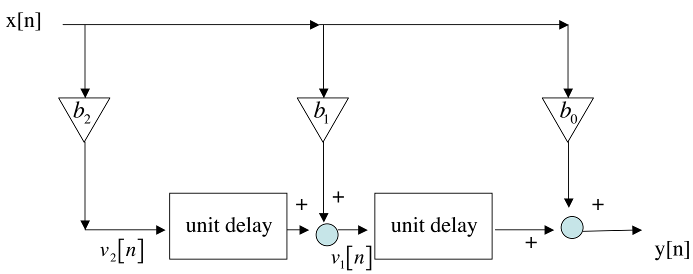
$\longrightarrow$ $y[n] = \sum_{k=0}^{M} b_k x[n-k]$, $\{b_0, b_1, b_2\} = \{\frac{4}{8}, \frac{2}{8}, \frac{1}{8}\}$ $\longrightarrow$

$h[n] = y[n]\big|_{x[n]=\delta[n]}$
$= \{0 \;\; 0 \;\; \frac{4}{8} \;\; \frac{2}{8} \;\; \frac{1}{8} \;\; 0 \;\; 0\}$
$= \{0 \;\; 0 \;\; b_0 \;\; b_1 \;\; b_2 \;\; 0 \;\; 0\}$

$y[n] = b_0 x[n] + b_1 x[n-1] + b_2 x[n-2]$



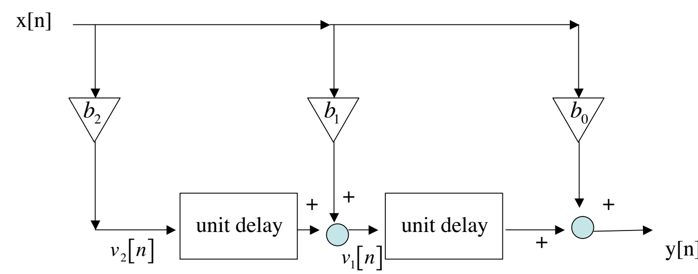
Block Diagrams to Difference Equations



$v_2[n] = b_2 x[n]$ $\quad$ $v_1[n] = b_1 x[n] + v_2[n-1]$ $\quad$ $y[n] = b_0 x[n] + v_1[n-1]$

$y[n] = b_0 x[n] + v_1[n-1]$
$v_1[n] = b_1 x[n] + v_2[n-1]$
$v_2[n] = b_2 x[n]$

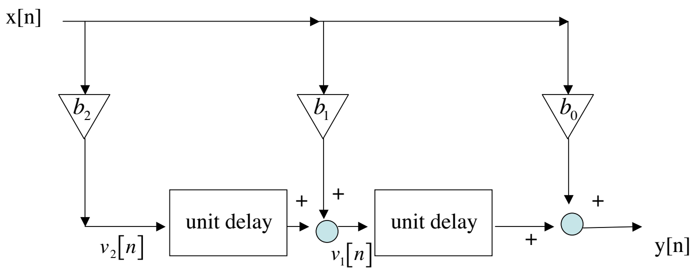
Block Diagrams to Difference Equations



$v_2[n] = b_2 x[n]$ $\quad$ $v_1[n] = b_1 x[n] + v_2[n-1]$ $\quad$ $y[n] = b_0 x[n] + v_1[n-1]$

$y[n] = b_0 x[n] + v_1[n-1]$
$v_1[n-1] = b_1 x[n-1] + v_2[n-2]$
$v_2[n-2] = b_2 x[n-2]$

Block Diagrams to Difference Equations

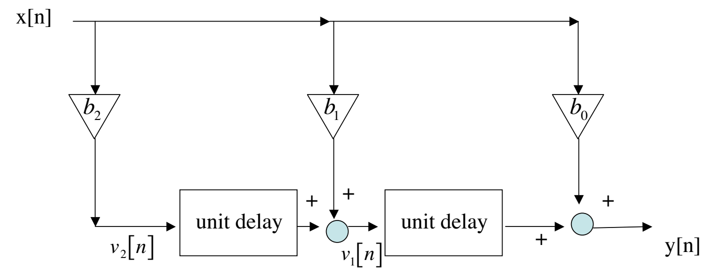


$v_2[n] = b_2 x[n]$ $v_1[n] = b_1 x[n] + v_2[n-1]$ $y[n] = b_0 x[n] + v_1[n-1]$

$$y[n] = b_0 x[n] + v_1[n-1]$$

$$v_1[n-1] = b_1 x[n-1] + b_2 x[n-2]$$

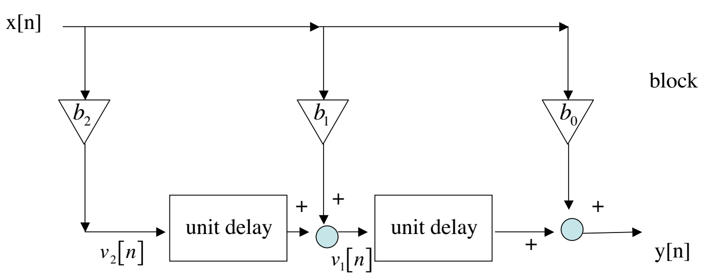
Block Diagrams to Difference Equations



$v_2[n] = b_2 x[n]$ $v_1[n] = b_1 x[n] + v_2[n-1]$ $y[n] = b_0 x[n] + v_1[n-1]$

$$y[n] = b_0 x[n] + b_1 x[n-1] + b_2 x[n-2]$$

Block Diagrams to Difference Equations



block diagram

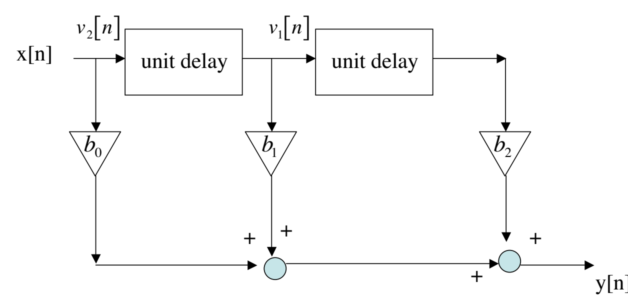
$v_2[n] = b_2 x[n]$ $v_1[n] = b_1 x[n] + v_2[n-1]$ $y[n] = b_0 x[n] + v_1[n-1]$

$y[n] = b_0 x[n] + b_1 x[n-1] + b_2 x[n-2]$ difference equation

$h[n] = b_0 \delta[n] + b_1 \delta[n-1] + b_2 \delta[n-2]$ impulse response

equivalent ways
of describing system

Block Diagrams to Difference Equations



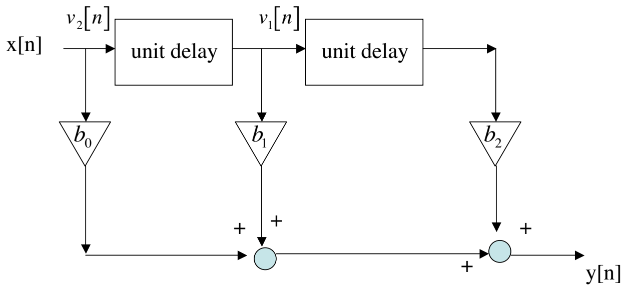
$v_2[n] = x[n]$ $v_1[n] = v_2[n-1]$ $y[n] = b_0 v_2[n] + b_1 v_1[n] + b_2 v_1[n-1]$

$$y[n] = b_0 v_2[n] + b_1 v_1[n] + b_2 v_1[n-1]$$

$$v_1[n] = v_2[n-1]$$

$$v_2[n] = x[n]$$

Block Diagrams to Difference Equations



$v_2[n] = x[n]$ $\quad$ $v_1[n] = v_2[n-1]$ $\quad$ $y[n] = b_0 v_2[n] + b_1 v_1[n] + b_2 v_1[n-1]$

$$y[n] = b_0 v_2[n] + b_1 v_1[n] + b_2 v_1[n-1]$$

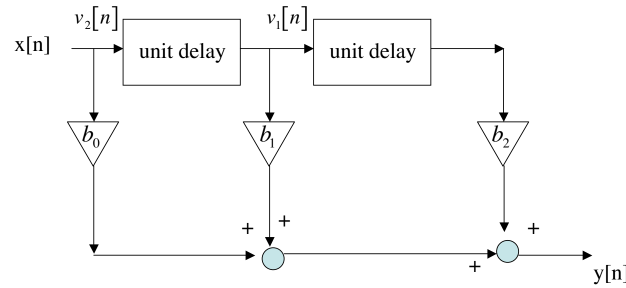
$$v_2[n] = x[n] \qquad v_1[n-1] = v_2[n-2]$$

$$v_2[n-2] = x[n-2]$$

$$v_1[n] = v_2[n-1]$$

$$v_2[n-1] = x[n-1]$$

Block Diagrams to Difference Equations



$v_2[n] = x[n]$ $\quad$ $v_1[n] = v_2[n-1]$ $\quad$ $y[n] = b_0 v_2[n] + b_1 v_1[n] + b_2 v_1[n-1]$

$$y[n] = b_0 x[n] + b_1 x[n-1] + b_2 x[n-2]$$

Homework:

p5_1: $y(n) := \frac{1}{L} \cdot \left[\sum_{k=0}^{L-1} a^{n-k} \cdot u(n-k) \right]$

L-point running average for input sequence $x[n]=a^n u[n]$, $n \geq 0$

hint: $\sum_{k=M}^{N} \alpha^k = \frac{\alpha^M - \alpha^{N+1}}{1 - \alpha}$

let z=n-k
k=n-z

$$\frac{1}{L} \cdot \left[\sum_{n-z=0}^{L-1} a^z \cdot u(z) \right]$$

$$\frac{1}{L} \cdot \left[\sum_{z=n}^{n-(L-1)} a^z \cdot u(z) \right]$$

remember $n \geq 0$

p5_6: FIR & delays
FIR and single delay

y[n]=ax[n]+bx[n-1]

3 point average

$y[n] = \frac{1}{3} x[n] + \frac{1}{3} x[n-1] + \frac{1}{3} x[n-2]$ $\quad$ $x[n] = \sin(2\pi n/15) \cdot u[n]$ $\quad$ $u[n] = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases}$