Lecture 6 Thermionic Engines

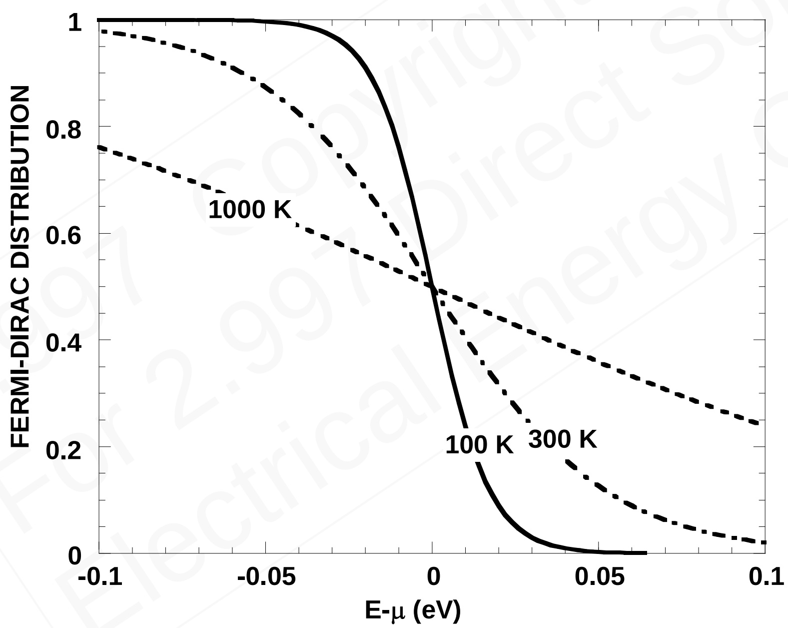
- Review
- Richardson formula
- Thermionic engines
- Schottky barrier and diode
- pn junction and diode
- discussion

Review: Fermi-Dirac Distribution

- **Average number of electrons in the state**

$$f = \sum_{n=0,1} nAe^{-(E-\mu)/(k_B T)} = \frac{1}{\exp\left(\frac{E-\mu}{k_B T}\right)+1}$$

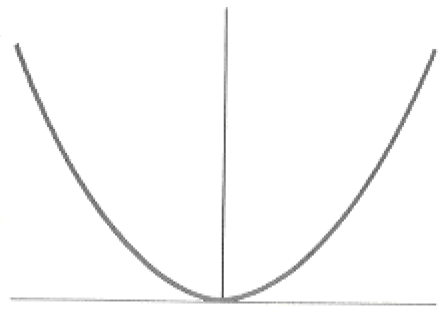
Fermi-Dirac Distribution



At T=0K, μ is called Fermi level, E_f

f=1 for E<μ
f=0 for E>μ

Review: Electron Density



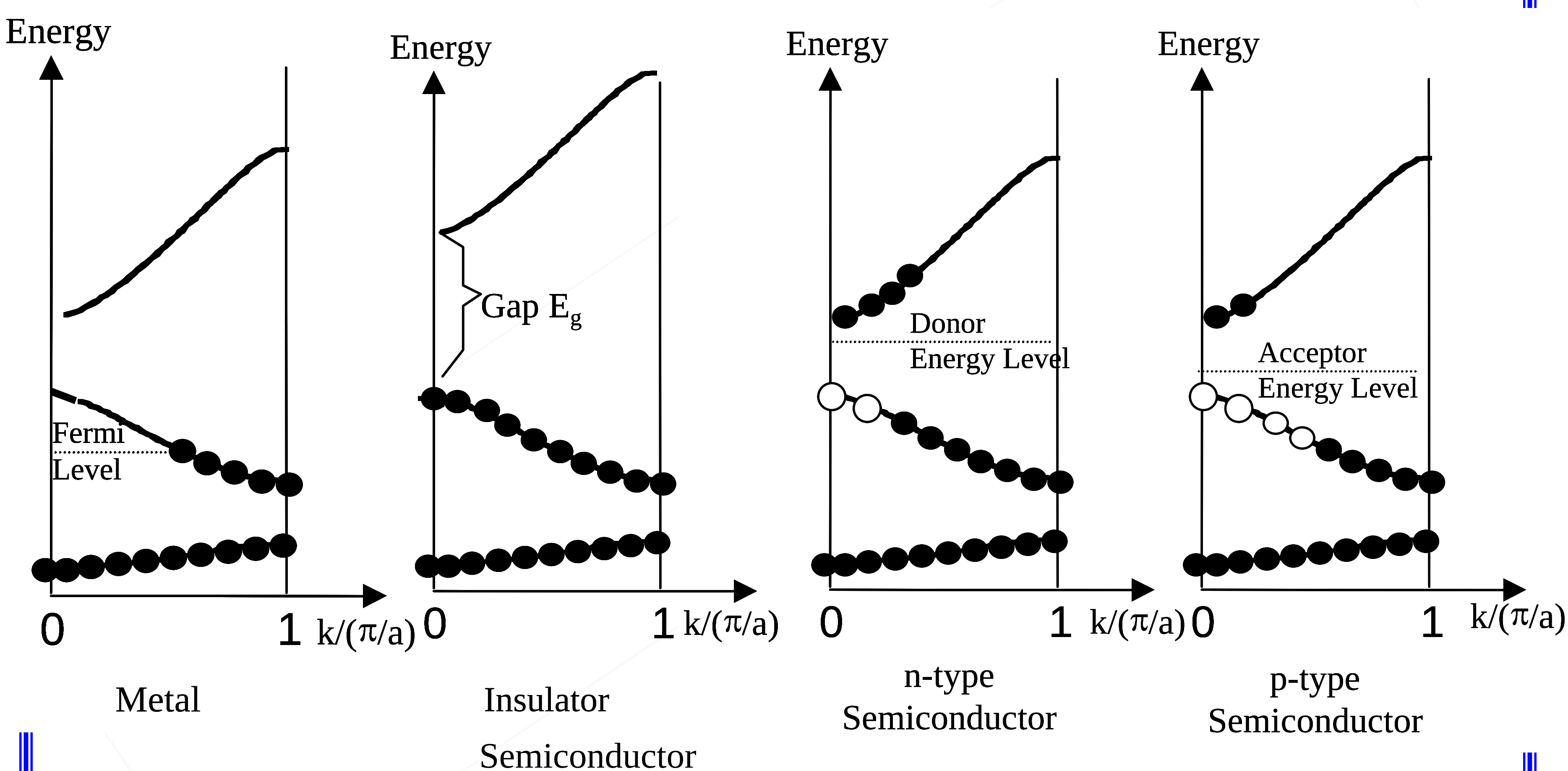
$$E - E_c = \frac{\hbar^2\left(k_x^2 + k_y^2 + k_z^2\right)}{2m}$$

$$N = 2\sum_{-N_x/2}^{N_x/2}\sum_{-N_y/2}^{N_y/2}\sum_{-N_z/2}^{N_z/2} f(E,T) = \frac{2V}{8\pi^3}\int_{-\pi/a}^{\pi/a}\int_{-\pi/a}^{\pi/a}\int_{-\pi/a}^{\pi/a} dk_x dk_y dk_z \exp\left[-\frac{E-\mu}{k_B T}\right]$$

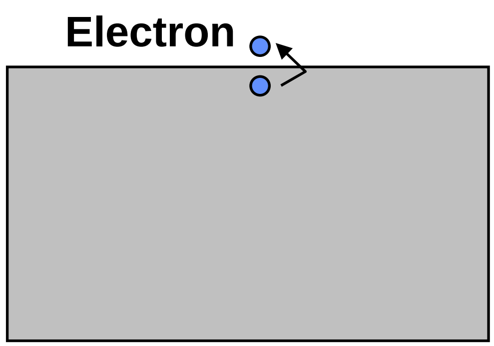
$$n = \int_{E_c}^{\infty} D(E) f(E,\mu,T) dE$$

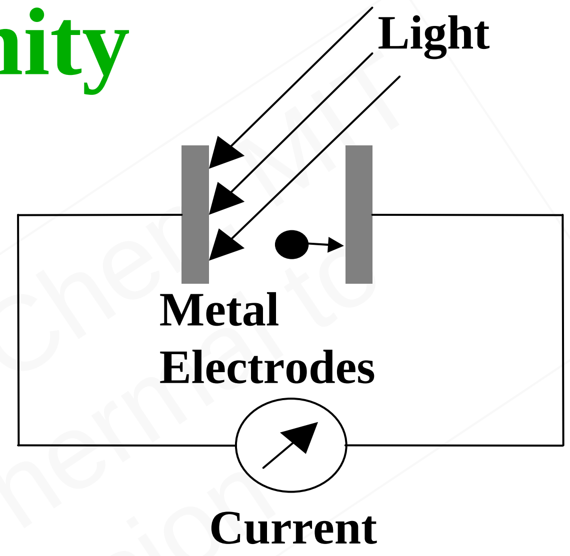
$$n = 2\left(\frac{2\pi m^* \kappa_B T}{h^2}\right)^{3/2} \exp\left(-\frac{E_c-\mu}{k_B T}\right) = N_c \exp\left(-\frac{E_c-\mu}{k_B T}\right)$$

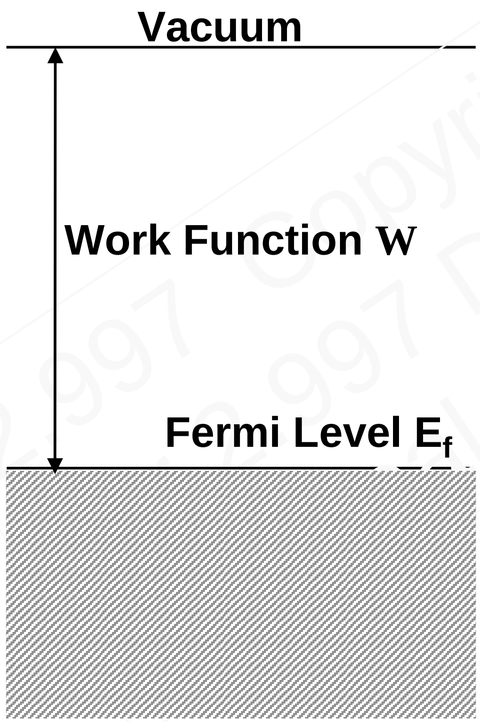
Different Solids

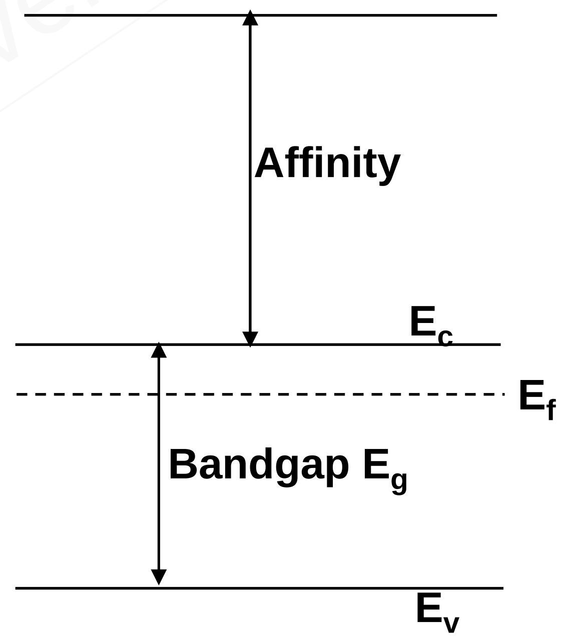


Work Function and Affinity

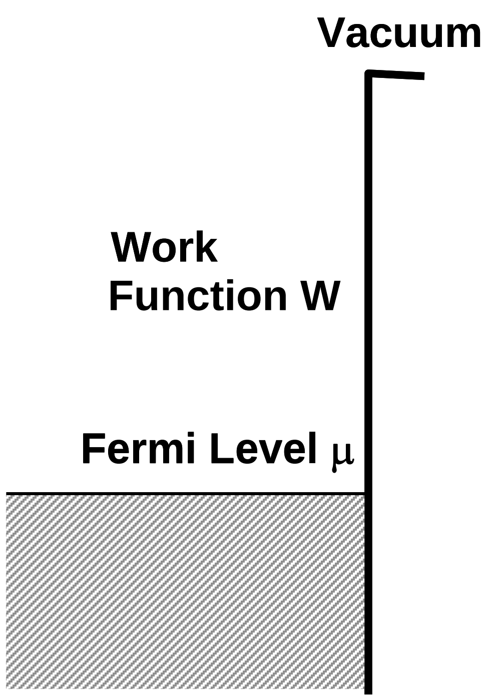








Electrons Just Outside the Metal

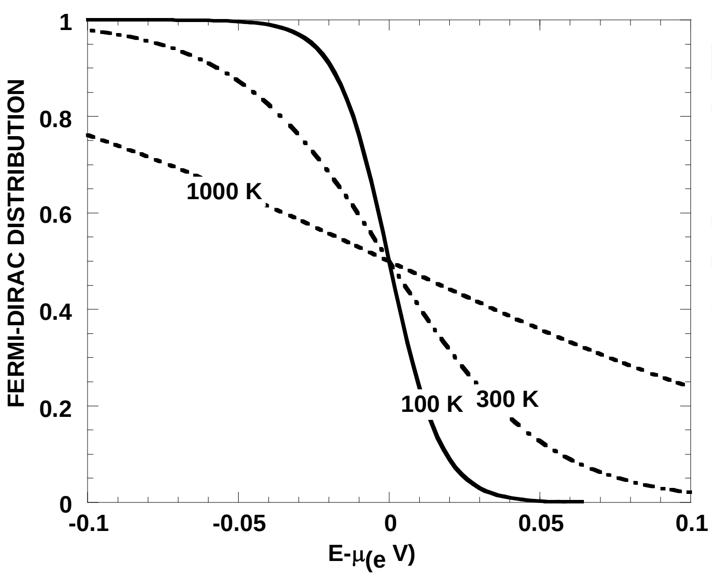


$$E-\mu-W=\frac{\hbar^2\left(k_x^2+k_y^2+k_z^2\right)}{2m}$$

Electron particle Flux Leaving Surface

$$J_p=\sum_{k_x>0}\sum_{k_y=-\pi/a}^{k_y=\pi/a}\sum_{k_z=-\pi/a}^{k_z=\pi/a}v_x f$$

$$=\frac{2V}{8\pi^3}\int_0^{\pi/a}\int_{-\pi/a}^{\pi/a}\int_{-\pi/a}^{\pi/a}dk_x dk_y dk_z\frac{\hbar k_x}{m}f$$



Electron Flux Out of Surface

$$J_p = \frac{2}{8\pi^3}\int_0^{\pi/a}\int_{-\pi/a}^{\pi/a}\int_{-\pi/a}^{\pi/a} dk_x dk_y dk_z \frac{\hbar k_x}{m}\frac{1}{\exp\left(\frac{E-\mu}{k_B T}\right)+1}$$

$$\approx \frac{2}{8\pi^3}\int_0^{\pi/a}\int_{-\pi/a}^{\pi/a}\int_{-\pi/a}^{\pi/a} dk_x dk_y dk_z \frac{\hbar k_x}{m}\exp\left(-\left[W+\frac{\hbar^2\left(k_x^2+k_y^2+k_z^2\right)}{2m}\right]\frac{1}{k_B T}\right)$$

$$= \frac{m}{2\pi^2\hbar^3}\left(k_B T\right)^2\exp\left(-\frac{W}{k_B T}\right)$$

Electrical Current Density

$$J_e = \frac{em}{2\pi^2\hbar^3}(k_B T)^2 \exp\left(-\frac{W}{k_B T}\right) = AT^2 \exp\left(-\frac{W}{k_B T}\right)$$

O.W. Richardson
1928 Nobel Prize

Richardson Formula or
Richardson-Dushman Equation
For Thermionic Emission

$$A = \frac{emk_B^2}{2\pi^2\hbar^3} = 120\frac{A}{cm^2K^2}$$

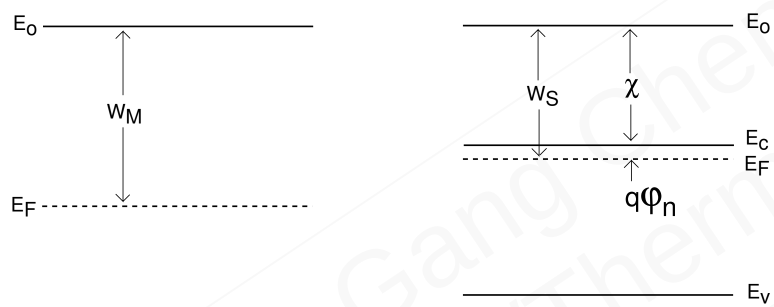
Richardson Constant

$$J_e = (1-r)AT^2 \exp\left(-\frac{\varphi}{k_B T}\right)$$

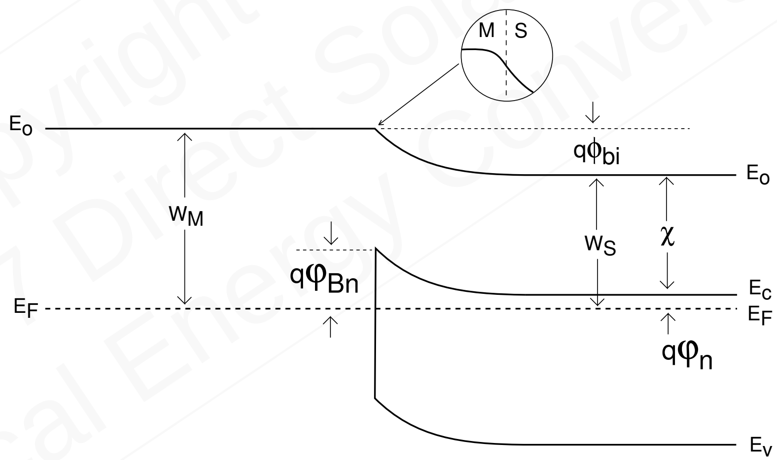
More General

Contact Potential

$$W_S = \chi + q\varphi_n$$



a) metal and semiconductor far apart

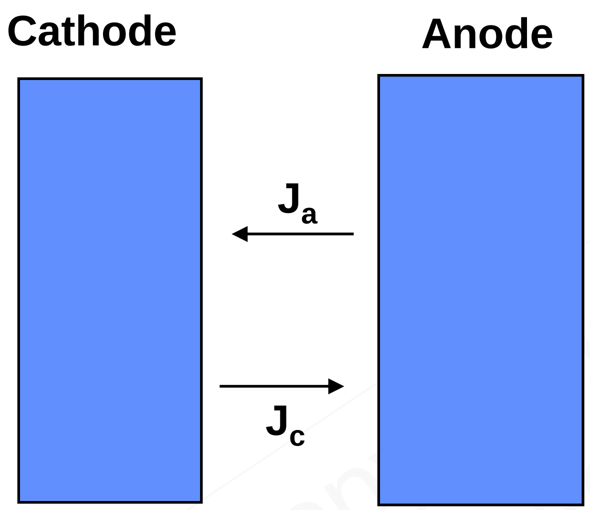


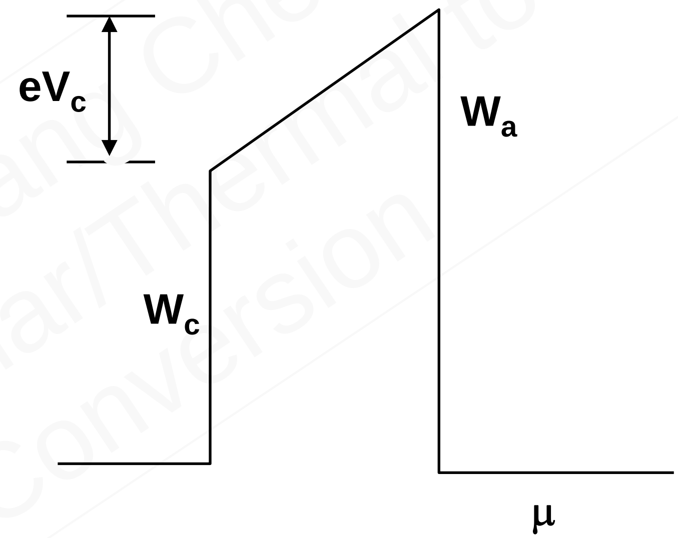
b) metal and semiconductor in intimate contact

$$-e(\varphi - \varphi') = W - W' = -eV_c$$

Courtesy of Jesús del Alamo. Used with permission.

Vacuum Diode





$$J_c = AT^2 \exp\left(-\frac{W_c + eV_c}{k_B T}\right)$$

$$J_a = AT^2 \exp\left(-\frac{W_a}{k_B T}\right)$$

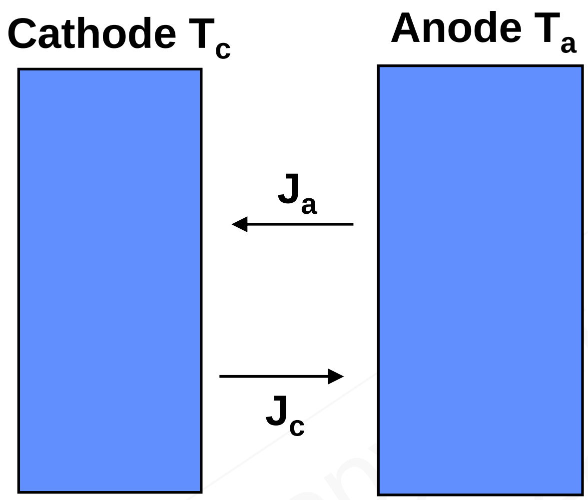
$-eV_c=(W_a-W_c)$

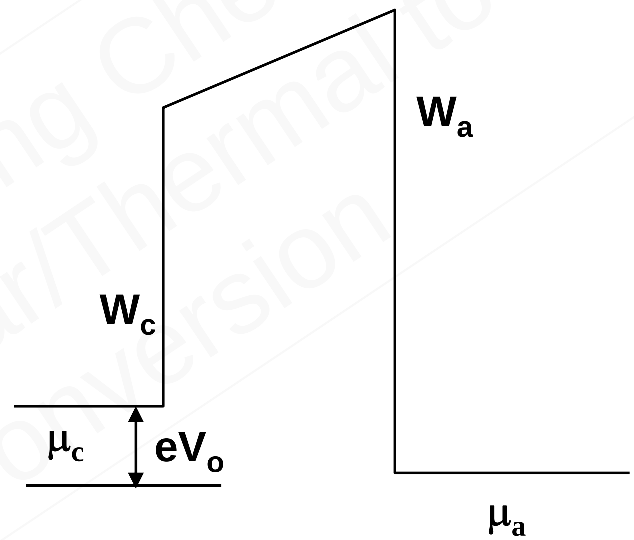
At equilibrium,

$J=J_c-J_a=0$

Thermionic Generator

$T_c > T_a$





$$J_c = AT_c^{\,2} \exp\left(-\frac{W_c + eV_c + eV_o}{k_B T_c}\right)$$

$$J_a = AT_a^{\,2} \exp\left(-\frac{W_a}{k_B T_a}\right)$$

$-eV_o = \mu_a - \mu_c$

At equilibrium,

$J = J_c - J_a = 0$

Thermionic Generator

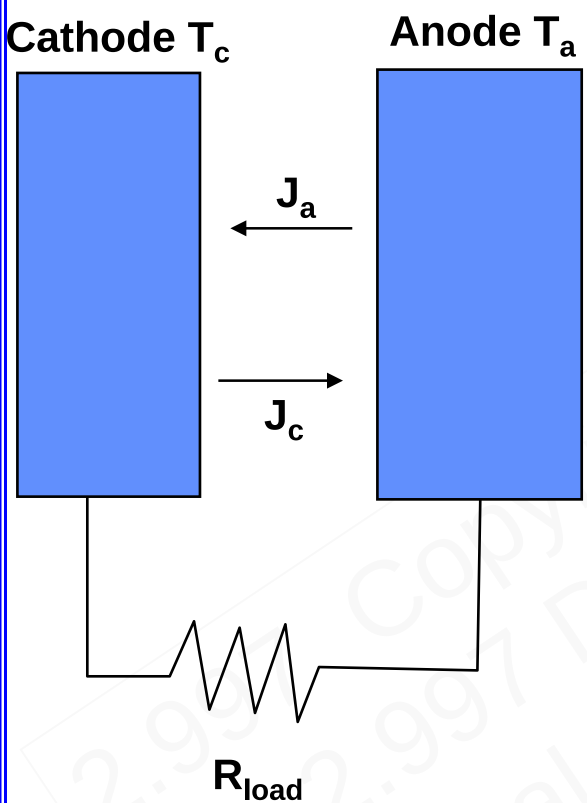
$$AT_c^{\,2}\exp\left(-\frac{W_c+eV_c+eV_o}{k_BT_c}\right)-AT_a^{\,2}\exp\left(-\frac{W_a}{k_BT_a}\right)=0$$

$$2\ln\left(\frac{T_c}{T_a}\right)=\frac{W_c+eV_c+eV_o}{k_BT_c}-\frac{W_a}{k_BT_a}$$

Open Circuit Voltage:

$$V_o=\frac{W_a}{e}\left(\frac{T_c}{T_a}-1\right)+2\frac{k_BT_c}{e}\ln\left(\frac{T_c}{T_a}\right)$$

Under Operation



$$J = AT_c^{\,2}\exp\left(-\frac{W_c + eV_c + eV}{k_B T_c}\right) - AT_a^{\,2}\exp\left(-\frac{W_a}{k_B T_a}\right)$$

$$= AT_c^{\,2}\exp\left(-\frac{W_a + eV}{k_B T_c}\right) - AT_a^{\,2}\exp\left(-\frac{W_a}{k_B T_a}\right)$$

Power Output $\quad P = JV$

Heat Transfer: Electron Heat Flux

Kinetic Energy:

$$J_{k,c} = \frac{2}{8\pi^3} \int_0^{\pi/a} \int_{-\pi/a}^{\pi/a} \int_{-\pi/a}^{\pi/a} dk_x dk_y dk_z \left(\frac{\hbar^2 k_x^2}{2m} \right) \frac{1}{\exp\left(\frac{E-\mu}{k_B T} \right) + 1}$$

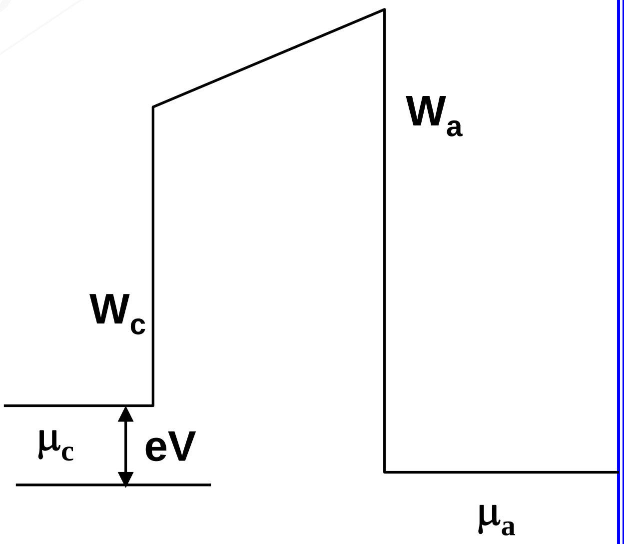
$$= \frac{2k_B T_c}{e} J_c$$

Total Heat from Cathode to Anode:

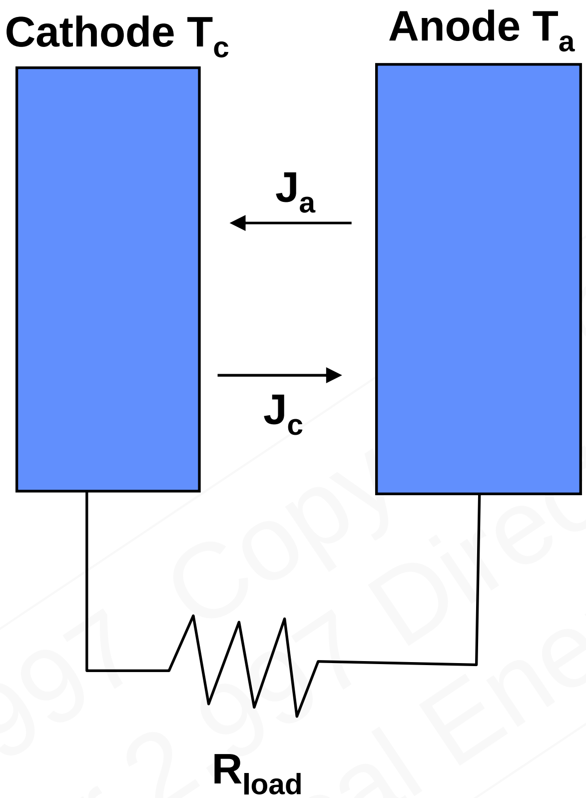
$$J_{q,c} = \left(\frac{W_a}{e} - V + \frac{2k_B T_c}{e} \right) J_c$$

Total Heat from Cathode to Anode:

$$J_{q,a} = \left(\frac{W_a}{e} + \frac{2k_B T_a}{e} \right) J_a$$



Net Heat Input



$$Q_h = J_{q,c} - J_{q,a} + Q_{rad} + Q_{cond}$$

Q_{rad} --- Radiation heat transfer between cathode and anode

Q_{cond}---Conduction heat loss through leads, and gas in between

Efficiency

$$\eta = \frac{Power}{Q_h}$$

When radiation is neglected

$eV_a = W_a$

S.W. Angrist,
Direct Energy Conversion,
Allyn and Bacon, Boston, 1965

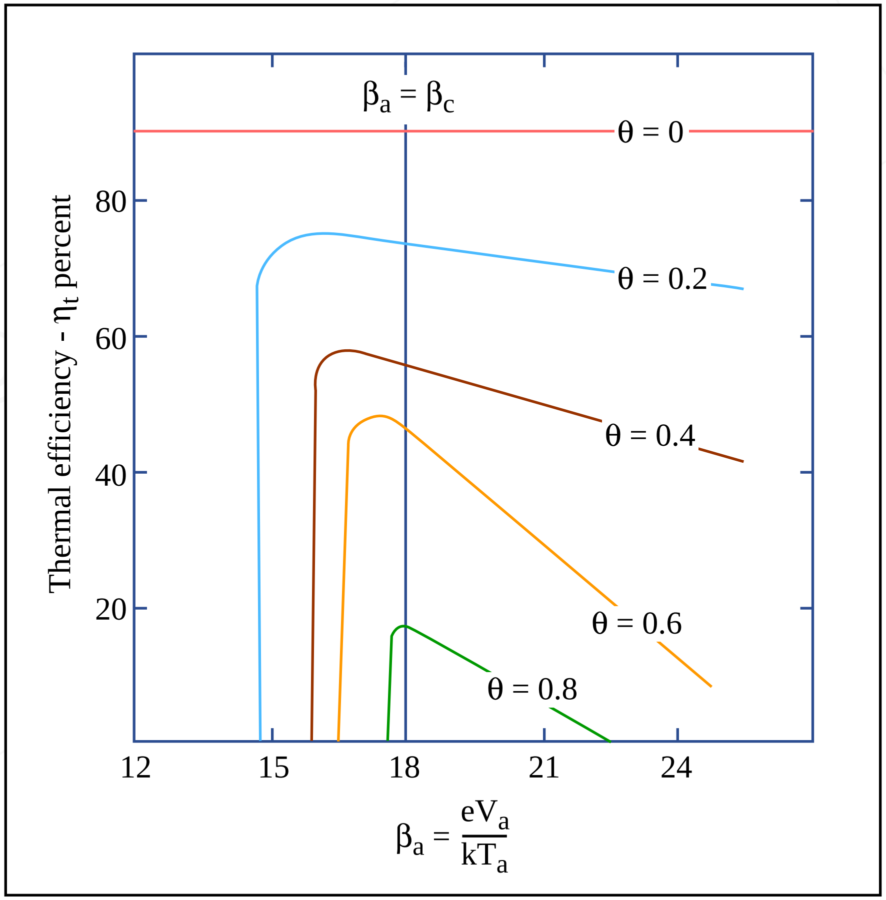


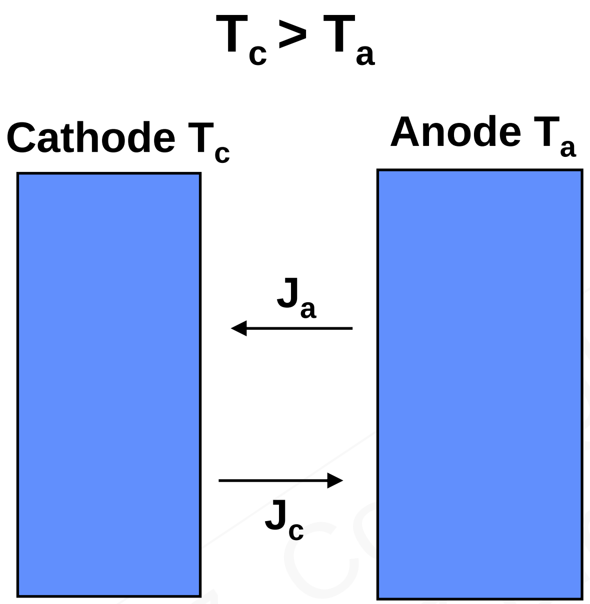
Figure by MIT OpenCourseWare.

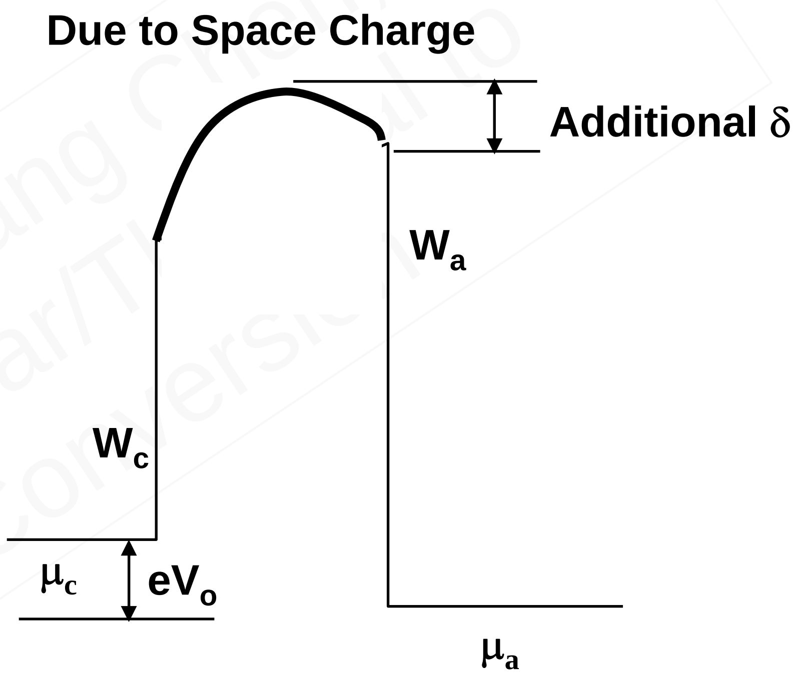
Experimental Demonstration

Image removed due to copyright restrictions.
Please see Fig. 3 in Hatsopoulos, George N., and Joseph Kaye. "Measured Thermal Efficiencies of a Diode Configuration of a Thermo Electron Engine." *Journal of Applied Physics* 29 (1958): 1124-1125.

Hatsopoulos and Kaye, JAP, 1958.

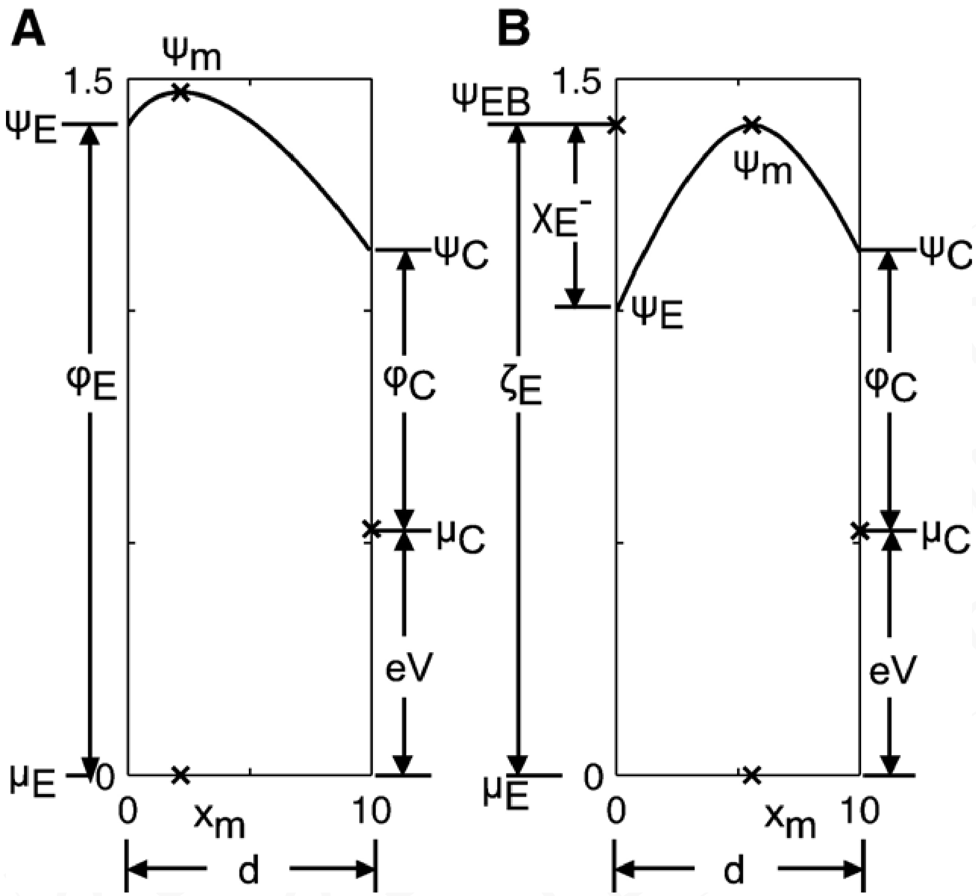
Real Device Issues





- Large work function, high temperature
- Cesium to reduce space charge, but reliability problem
- Vacuum operation or plasma operation (filled with gas)

Recent Trends

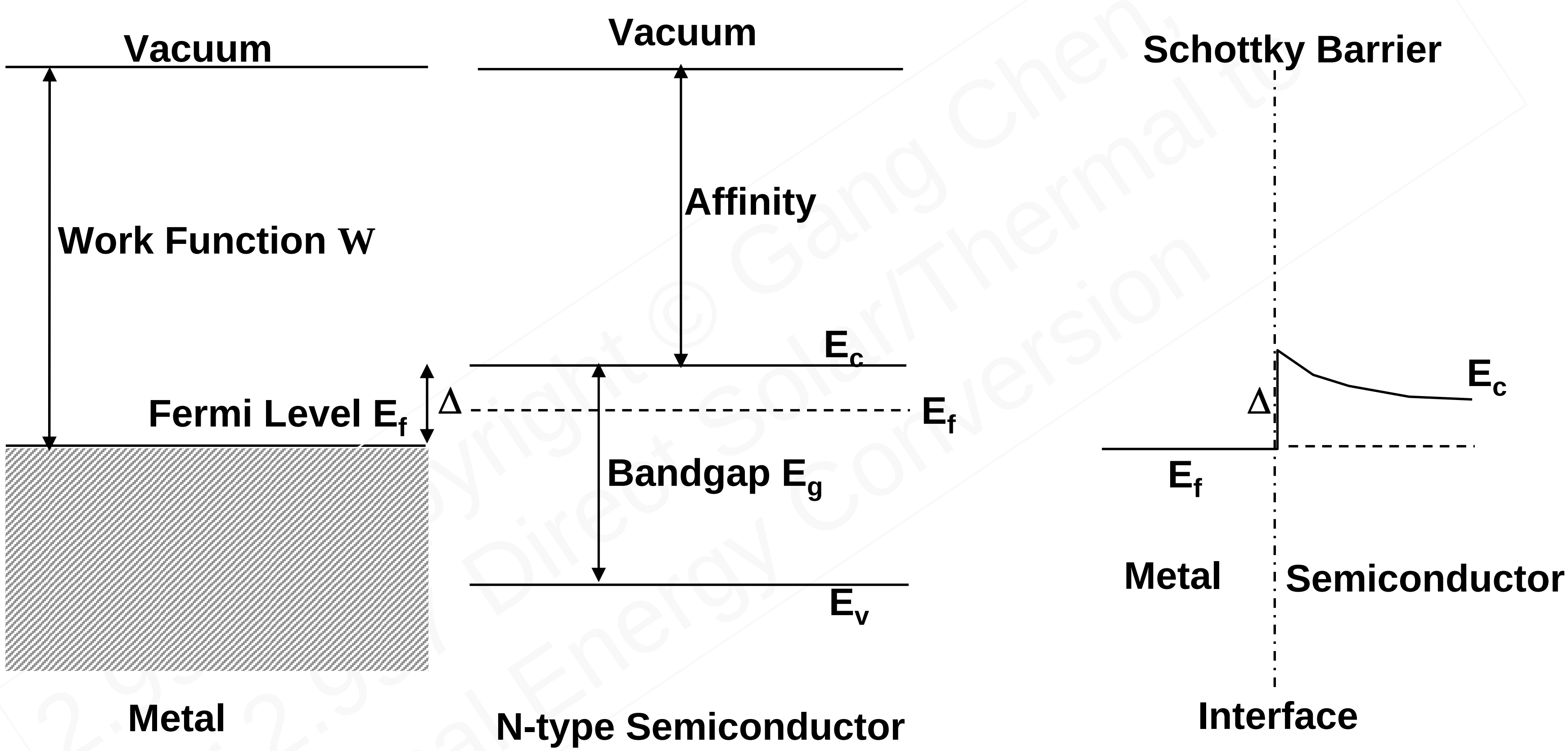


Negative Electron Affinity Materials (Diamond)

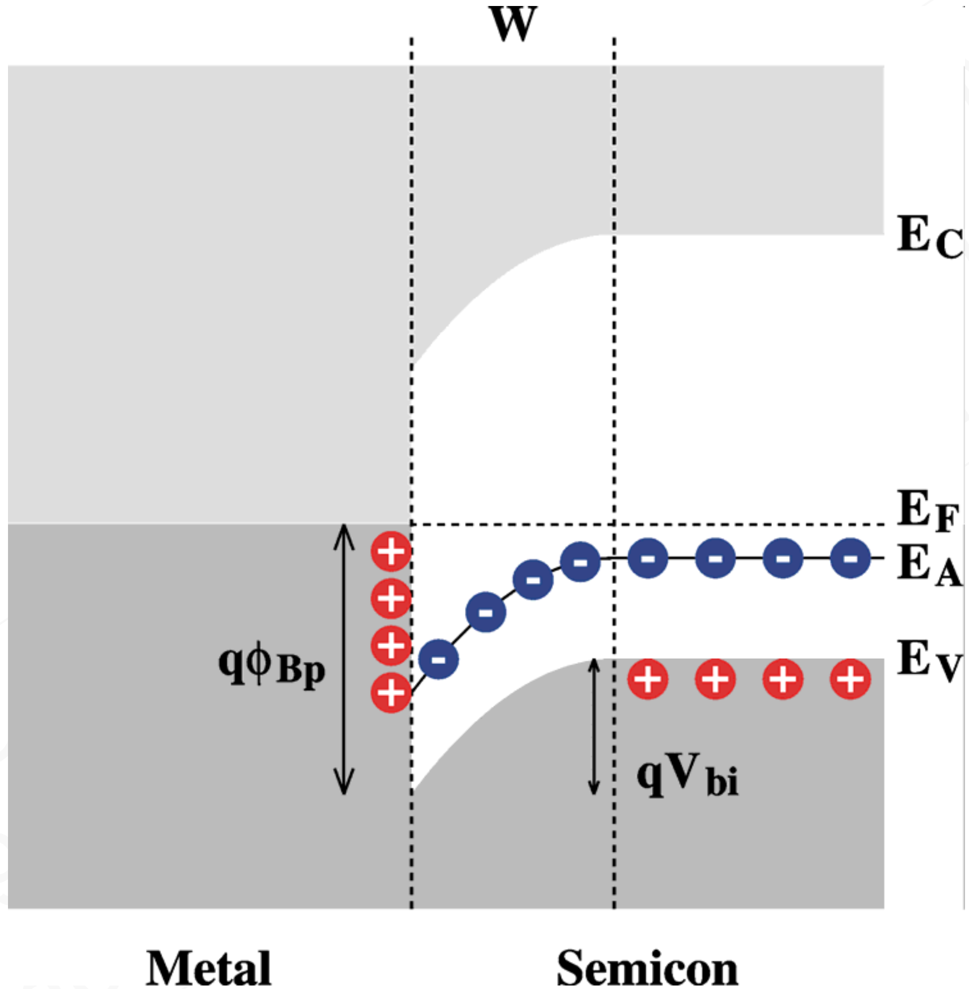
Smith et al., Diamond and related materials, 15, 2082 (2006)

- **Negative electron affinity materials**
- **Small gap devices**
- **Solid-state thermionics**

Schottky Barrier



P-type Schottky Barrier

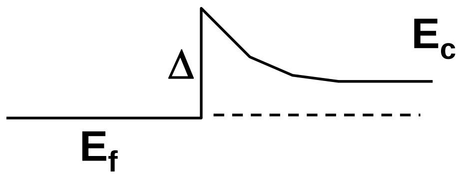


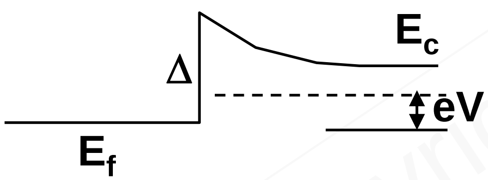
http://w3.ualg.pt/~pjotr/Images/Schottky.png

Schottky Diode

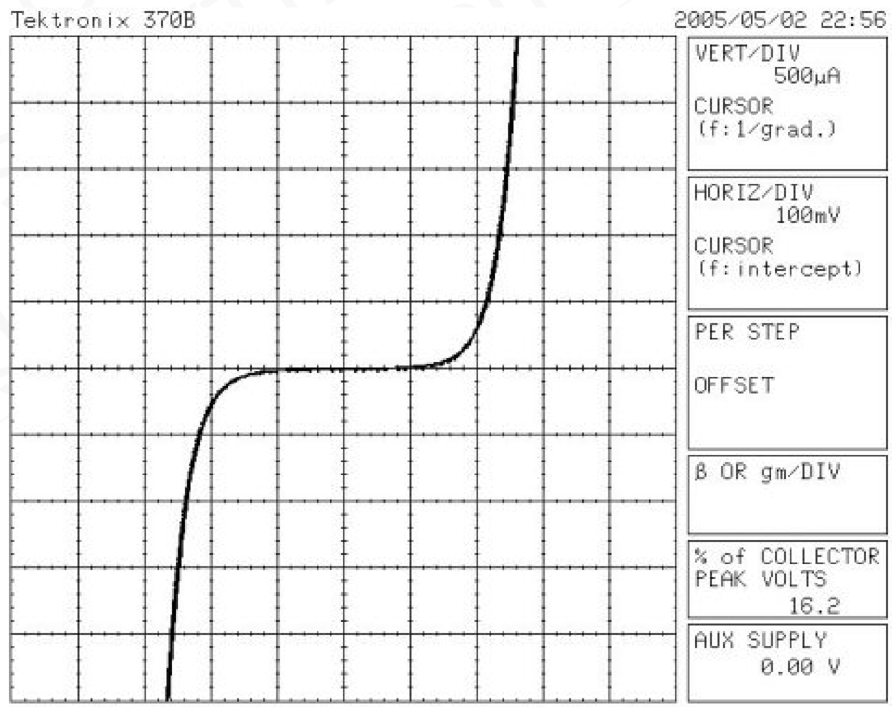
Richardson Formula

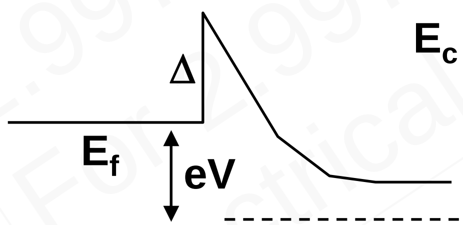
$$J = AT^2 \exp\left(-\frac{\Delta}{k_B T}\right)\left[\exp\left(\frac{eV}{k_B T}\right) - 1\right]$$





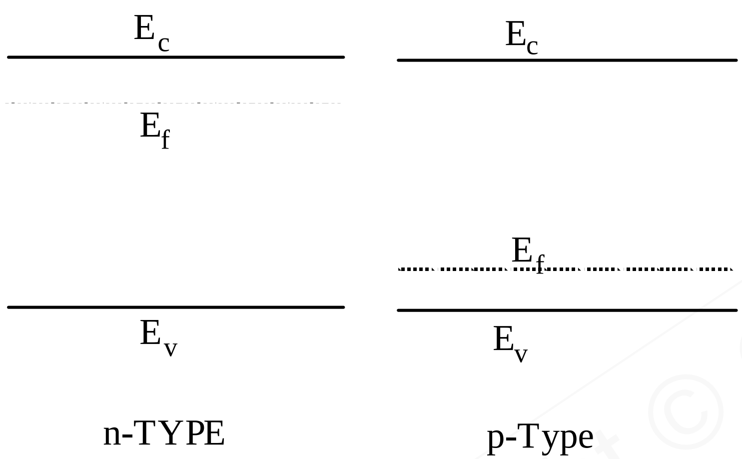
Forward Bias



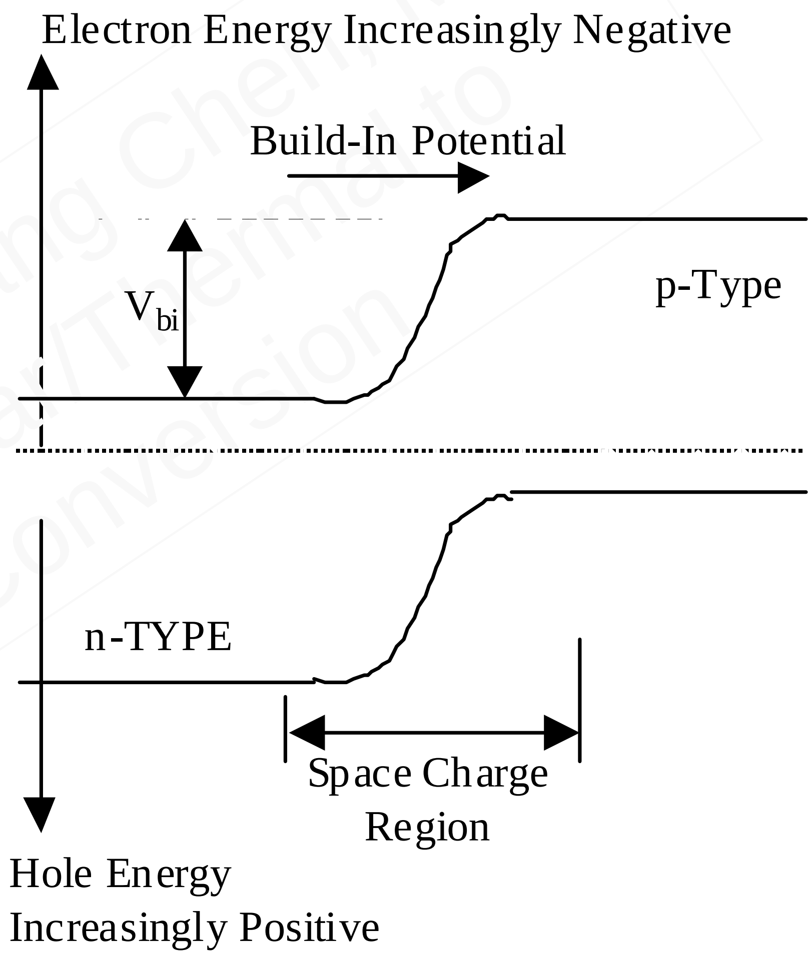


Reverse Bias

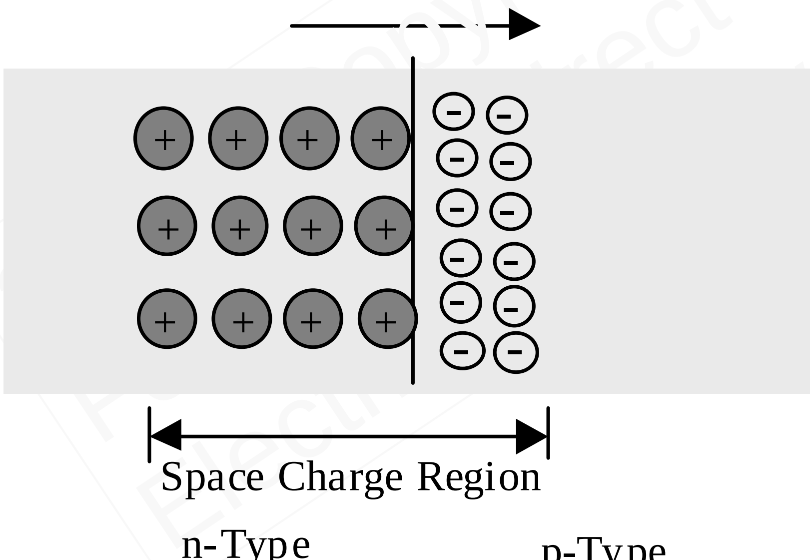
pn Junction



E_c
E_f
E_v
n-TYPE
E_c
E_f
E_v
p-Type



Electron Energy Increasingly Negative
Build-In Potential
V_{bi}
p-Type
n-TYPE
Space Charge Region
Hole Energy Increasingly Positive



Space Charge Region
n-Type
p-Type

pn Junction Basics

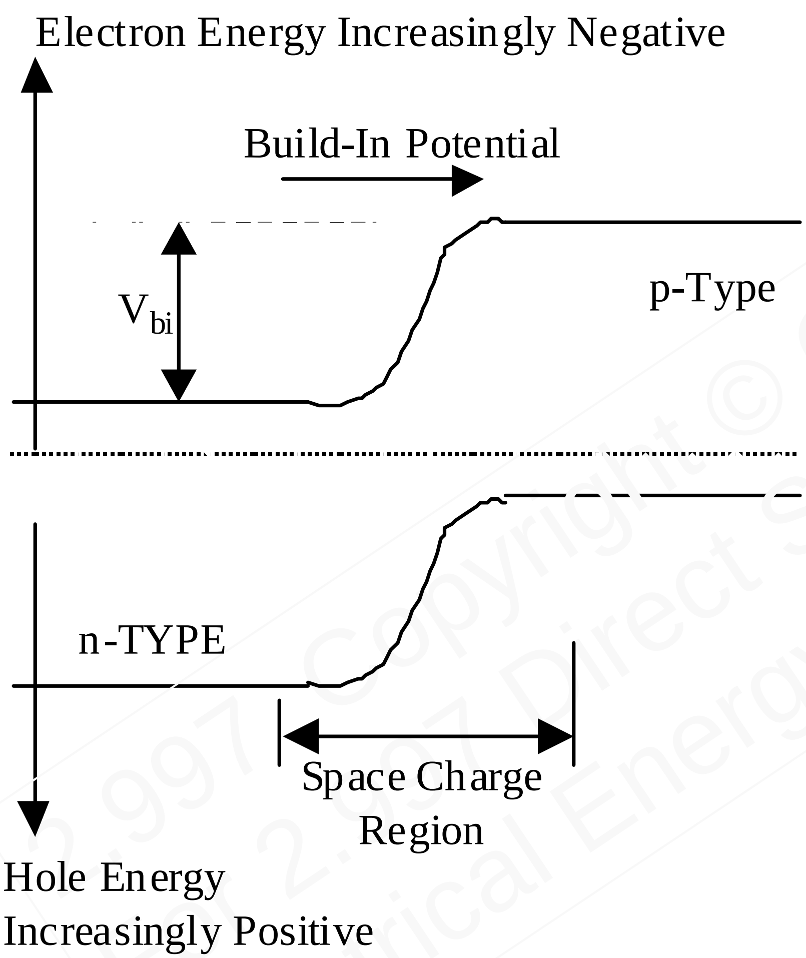
- **For nondegenerate semiconductor**

$$n = \mathrm{N_c} \exp\left(-\frac{E_c - \mu}{k_B T}\right)$$

$$\mathrm{p} = \mathrm{N_v} \exp\left(-\frac{\mu - E_v}{k_B T}\right)$$

$$pn = \mathrm{N_c} N_\mathrm{v} \exp\left(-\frac{E_g}{k_B T}\right) = n_i^2$$

pn Junction Basics: Built-in Potential



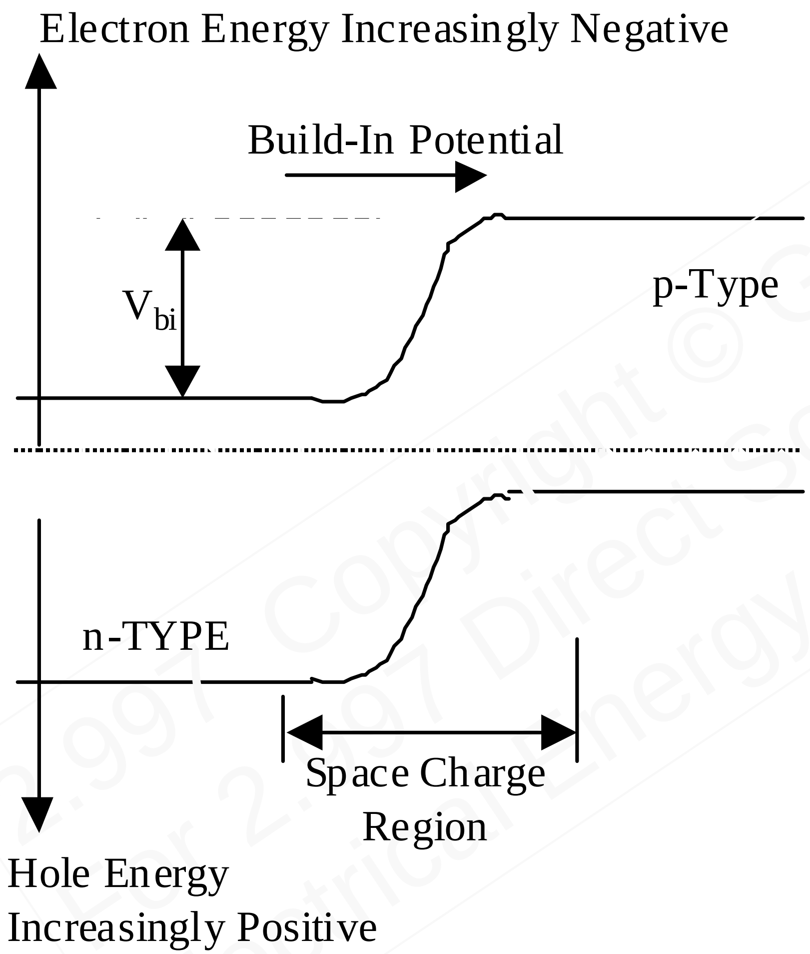
$$N_D = N_c \exp\left(-\frac{E_{c,n}-\mu}{k_B T}\right)$$

$$E_{c,n}-\mu = k_B T \ln\left(\frac{N_c}{N_D}\right)$$

$$E_{c,p}-\mu = E_g - k_B T \ln\left(\frac{N_v}{N_A}\right)$$

$$eV_{bi} = E_{c,p} - E_{c,n} = E_g - k_B T \ln\left(\frac{N_v N_c}{N_A N_D}\right) = k_B T \ln\left(\frac{N_A N_D}{n_i^2}\right)$$

pn Junction Basics: Space Charge Region



$$w = \sqrt{\frac{2\varepsilon_s}{e}\left(\frac{N_A + N_D}{N_A N_D}\right)V_{bi}}$$

One Side Only

$$w = \sqrt{\frac{2\varepsilon_s}{eN_B}V_{bi}}$$

Debye Length

$$w = \sqrt{\frac{2\varepsilon_s}{e^2 N_B}k_B T}$$

pn Junction I-V

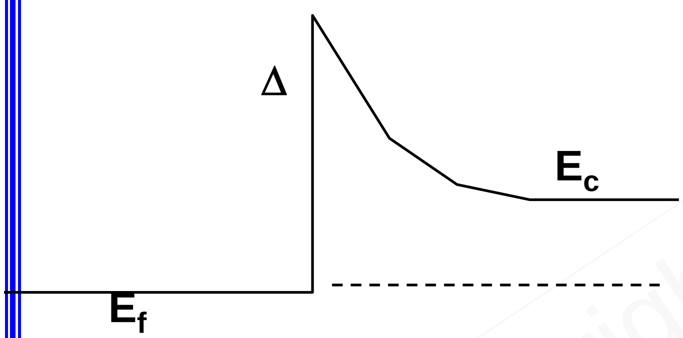
$$J = J_s\left(e^{eV/k_BT} - 1\right)$$

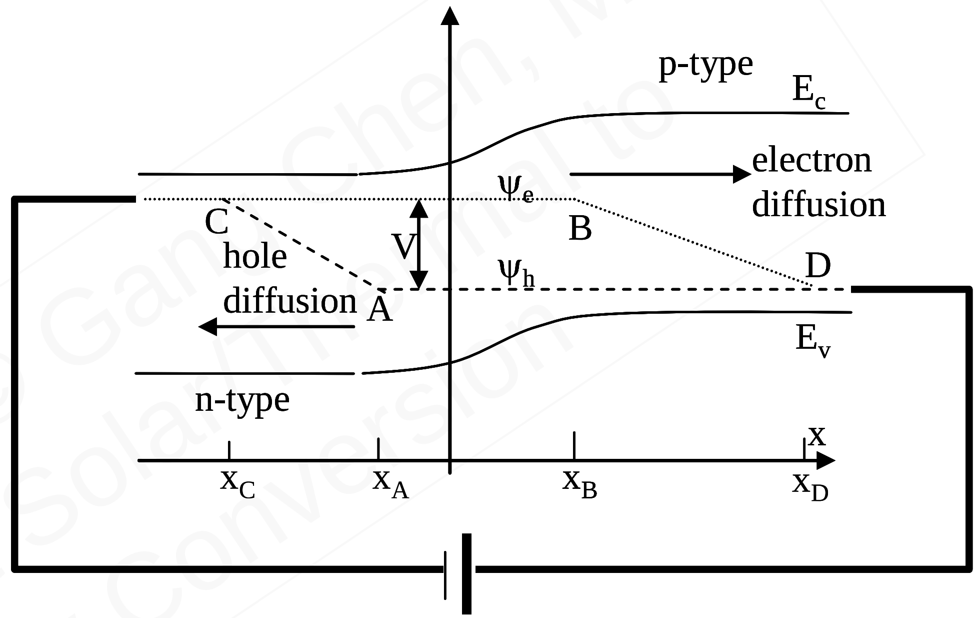
Saturation Current

$$J_s = eN_cN_v\left(\frac{1}{N_A}\sqrt{\frac{a_h}{\tau_h}} + \frac{1}{N_D}\sqrt{\frac{a_e}{\tau_e}}\right)\exp\left(-\frac{E_G}{\kappa_B T}\right)$$

a_h---hole diffusivity (m^2/s)
a_e---electron diffusivity
τ_h---hole recombination time
τ_e---electron recombination time

Compare Schottky diode and pn diode





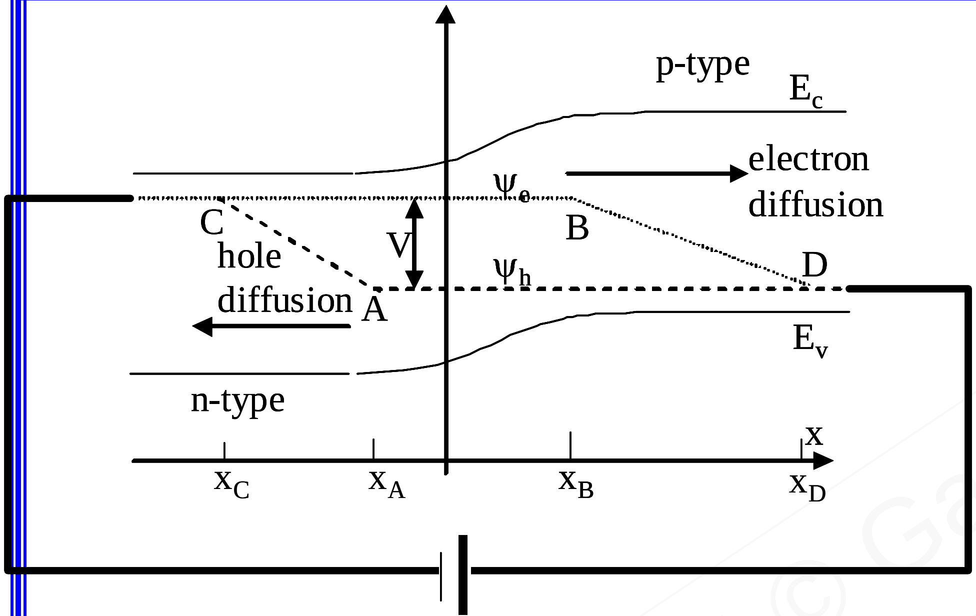
$$J = J_s\left[\exp\left(\frac{eV}{k_B T}\right) - 1\right]$$

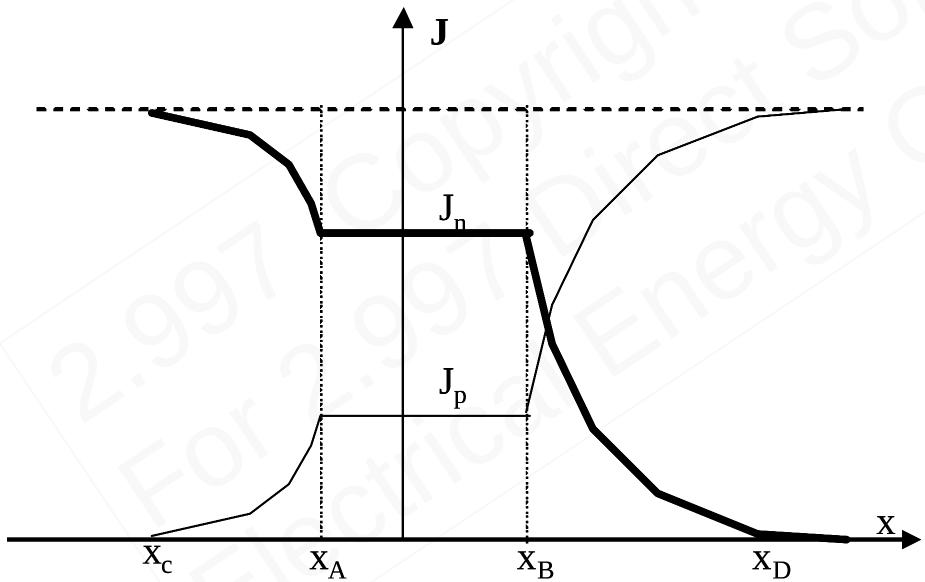
$$J = J_s\left(e^{eV/k_B T} - 1\right)$$

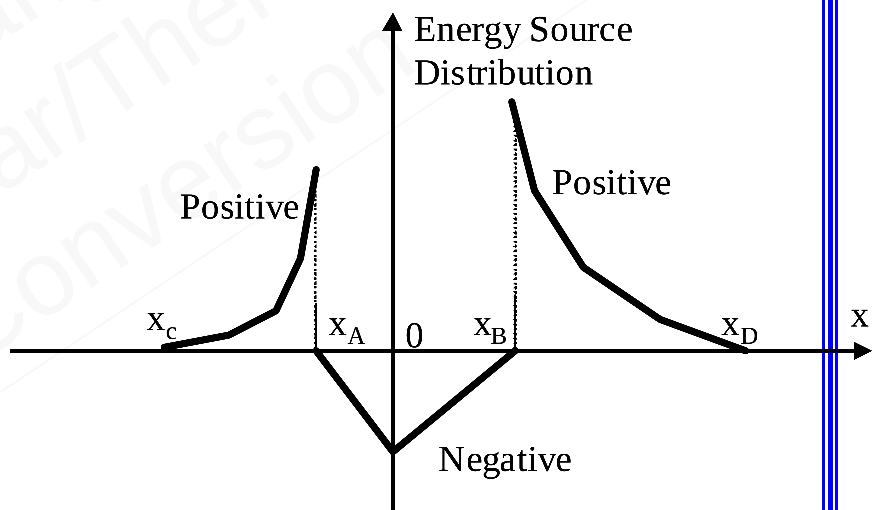
$$J_s = AT^2 \exp\left(-\frac{\Delta}{k_B T}\right)$$

$$J_s = eN_c N_v\left(\frac{1}{N_A}\sqrt{\frac{a_h}{\tau_h}} + \frac{1}{N_D}\sqrt{\frac{a_e}{\tau_e}}\right)\exp\left(-\frac{E_G}{\kappa_B T}\right)$$

Current and Energy Distribution







2.997 Direct Solar/Thermal to Electrical Energy Conversion Technologies
Fall 2009