

2.882

**System Design and Analysis based on
AD and Complexity Theories**

References

- (1) Nam Pyo Suh, *“Axiomatic Design: Advances and Applications*, Oxford University Press, New York, 2001
- (2) Nam Pyo Suh, *“Complexity: Theory and Applications”*, Oxford University Press, New York, 2005
- (3) Nam P. Suh, *The Principles of Design*, Oxford University Press, 1990

Your name

Your field

Why?

Format/Assumptions

1. Active Learning

2. Project execution

**3. Will assume no prior knowledge of
Axiomatic Design and Complexity
Theory.**

Lecture 1

Introduction to Axiomatic Design

Major Topics to be covered

1. Axiomatic Design

Theory

Applications

Many industrial examples

Actual design exercise

2. Complexity Theory

Theory

Applications

Today's Lecture

- 1. Introduction -- Read Chapter 1 of AD**
- 2. Will email Homework Problems**

Why Axiomatic Design

- 1. Engineering deals with design and manufacture of complex systems**
- 2. Examples: Space Shuttle**
 - Microsoft Operating Systems**
 - Manufacturing Systems**
 - Materials**
 - Organizations**

Demands in Industry

Industrial competitiveness demands that

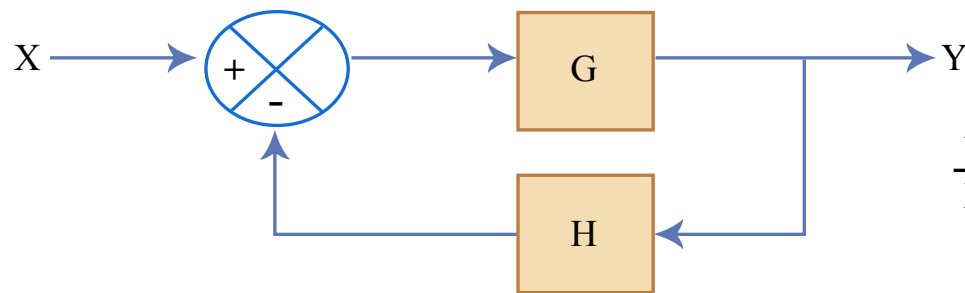
- 1. Shorten the lead-time for the introduction of new products,**
- 2. Lower manufacturing cost,**
- 3. Improve the quality and reliability of products,**
- 4. Satisfy the required functions most effectively.**

Hardware, software, and systems must be designed right to be controllable, reliable, manufacturable, productive, and otherwise achieve their goals. The performance of poorly designed hardware, software and systems cannot be improved through subsequent corrective actions.

Relationship between design and analysis

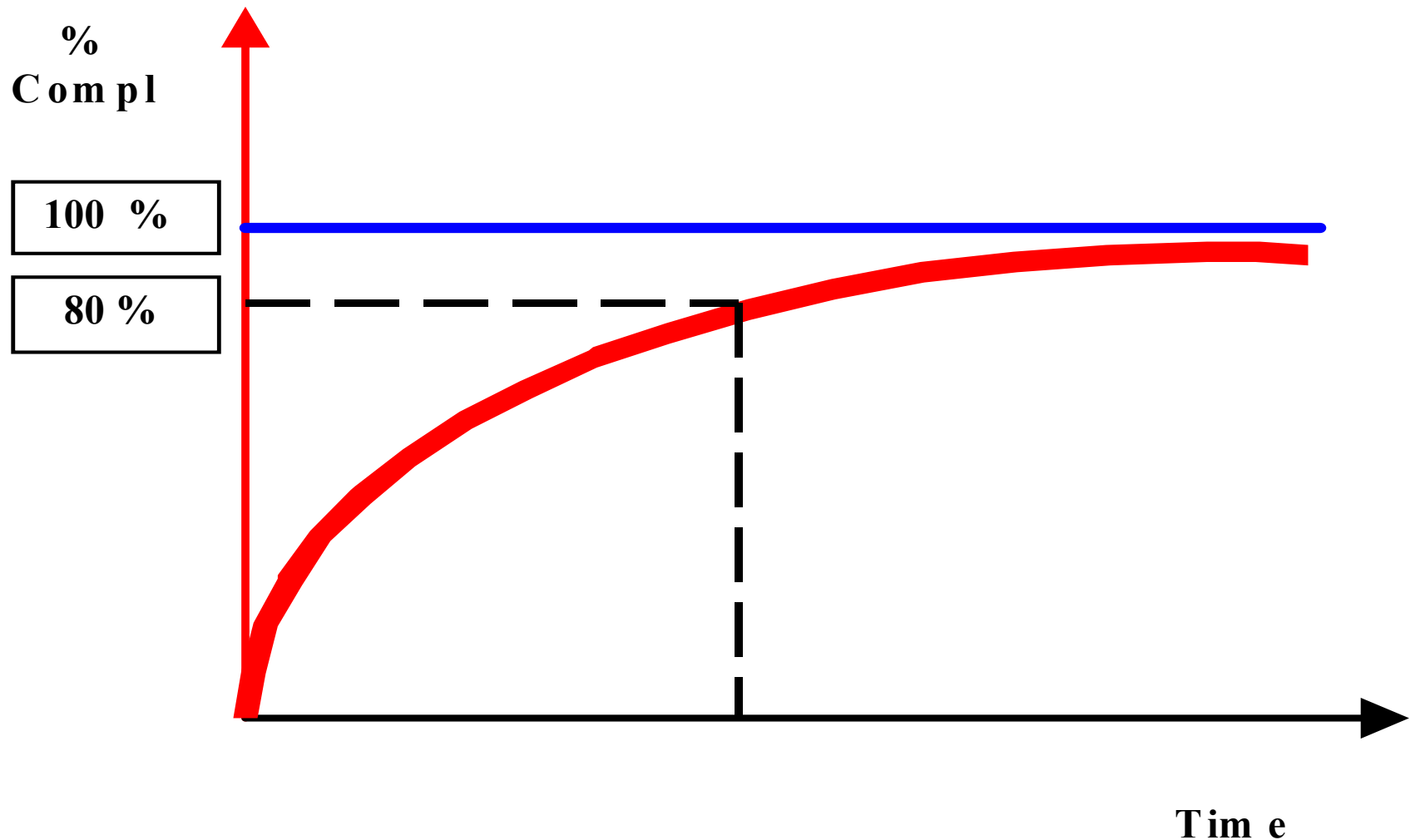
1. **Feedback loop between analysis and synthesis**
2. **Scientific paradigm -- reductionism**
3. **Synthesis -- Many FRs**

Relationship between design and analysis



$$\frac{Y}{X} = \frac{G}{1+GH} \approx \frac{G}{GH} = H^{-1} \text{ for } GH \gg 1$$

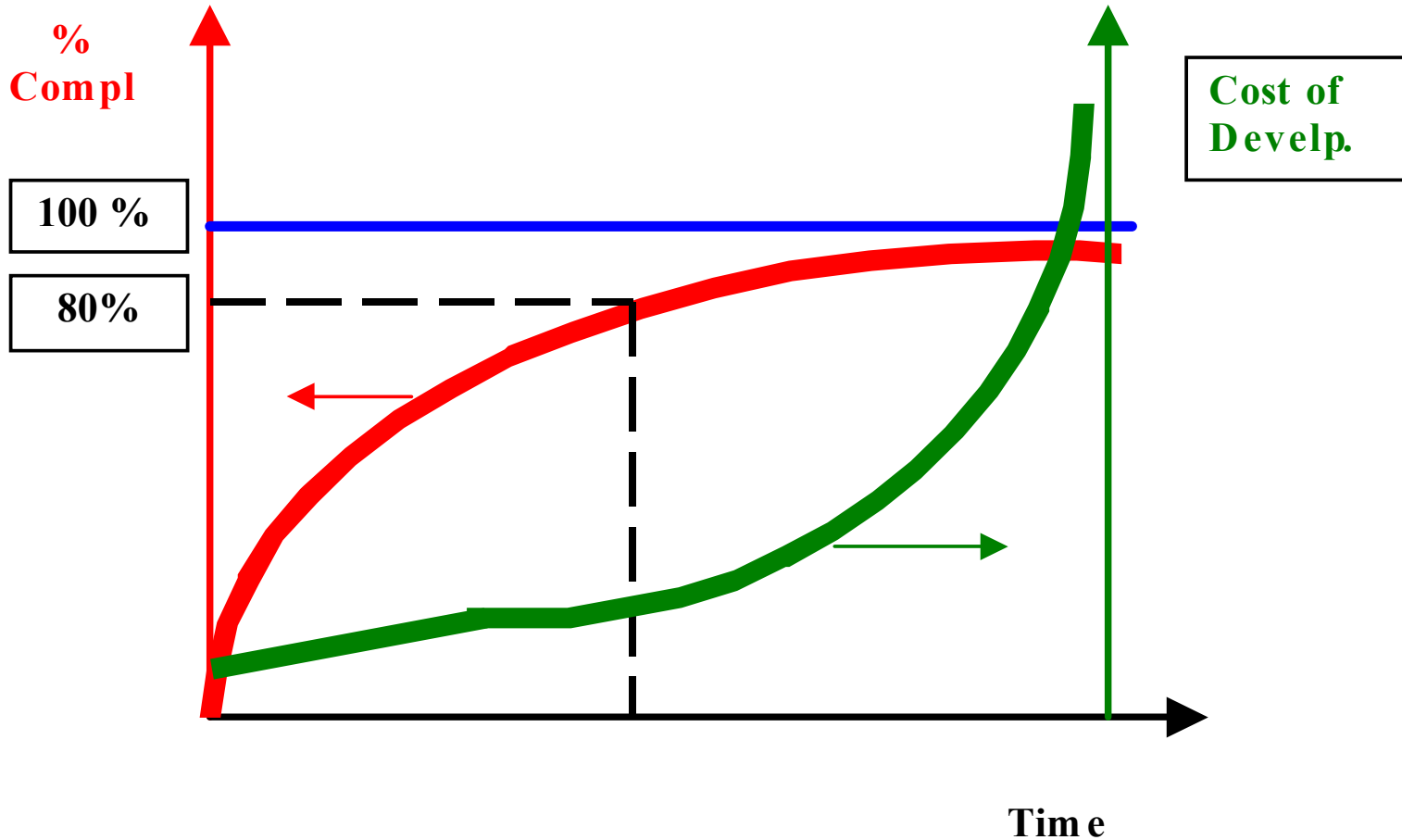
Typical Approach to “Realization” and “Implementation” of New Products



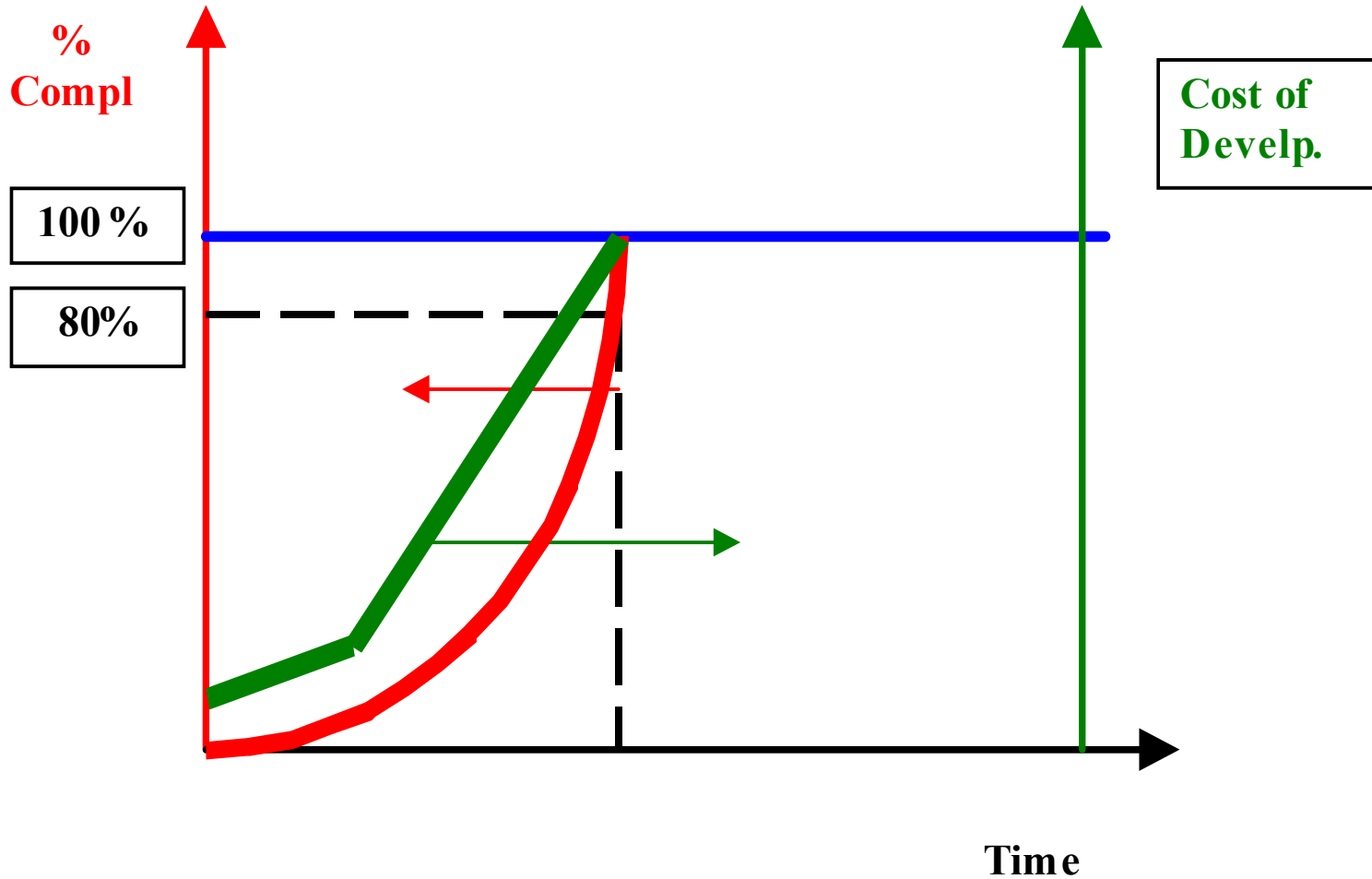
Poor Planning, purely experience-based design decisions, and trial-and-error based design practice may lead to the following consequences:

- 1. Project failure**
- 2. Missed schedule**
- 3. Cost over-run**
- 4. High warranty cost**
- 5. Frequent maintenance**
- 6. “Me, too” product**
- 7. Unhappy customer**

Product Development: Typical Approach



Product Development: Axiomatic Approach



TMA Projection System

Several slides describing TMA projection removed for copyright reasons.

The MIT CMP machine

Our attempt to teach systems design

4 S.M. students designed and manufactured the machine and the control system (including software for system integration) in 2 years. The system operated -- as designed -- when turned on with minimal modification.

1 Ph.D. student studied the CMP process.

Spent \$2 million -- Funded by an industrial firm.

What we taught them was the principles of design, so no debugging or testing of prototypes was needed.

Copper Damascene Process

Photo removed for
copyright reasons.

— Cu 6

— Cu 5

— Cu 4

— Cu 3

— Cu 2

— Cu 1

— W 1

Reference: D. Edelstein et al., *Tech. Dig. IEEE Int. Elec. Dev. Mtg.*, Washington D.C., pp. 773-776 (1997).

History

Goal

To establish the *science base* for areas such as design and manufacturing

How do you establish science base in design?

Axiomatic approach

Algorithmic approach

Axiomatic Design

Axiomatic Design applies to all designs:

- **Hardware**
- **Software**
- **Materials**
- **Manufacturing**
- **Organizations**

Axiomatic Design

Axiomatic Design helps the design decision making process.

- Correct decisions**
- Shorten lead time**
- Improves the quality of products**
- Deal with complex systems**
- Simplify service and maintenance**
- Enhances creativity**

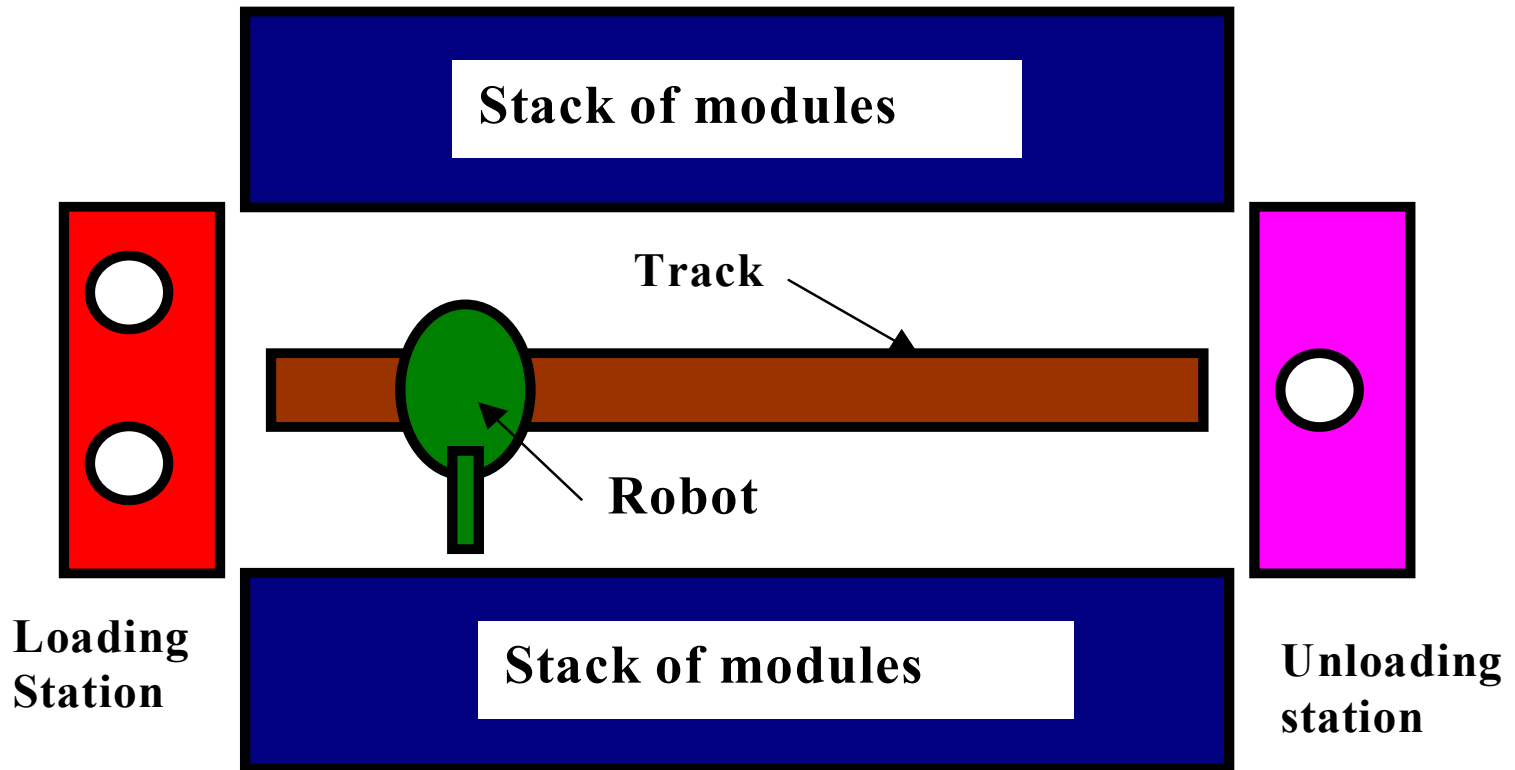
Axiomatic Design

- **Axioms**
- **Corollaries**
- **Theorems**
- **Applications** -- hardware, software, manufacturing, materials, etc.
- **System design**
- **Complexity**

LCD Projector Design

Several slides removed for copyright reasons.
See Example 3.4 in Suh, *Axiomatic Design* (2001).

Introduction



System integration

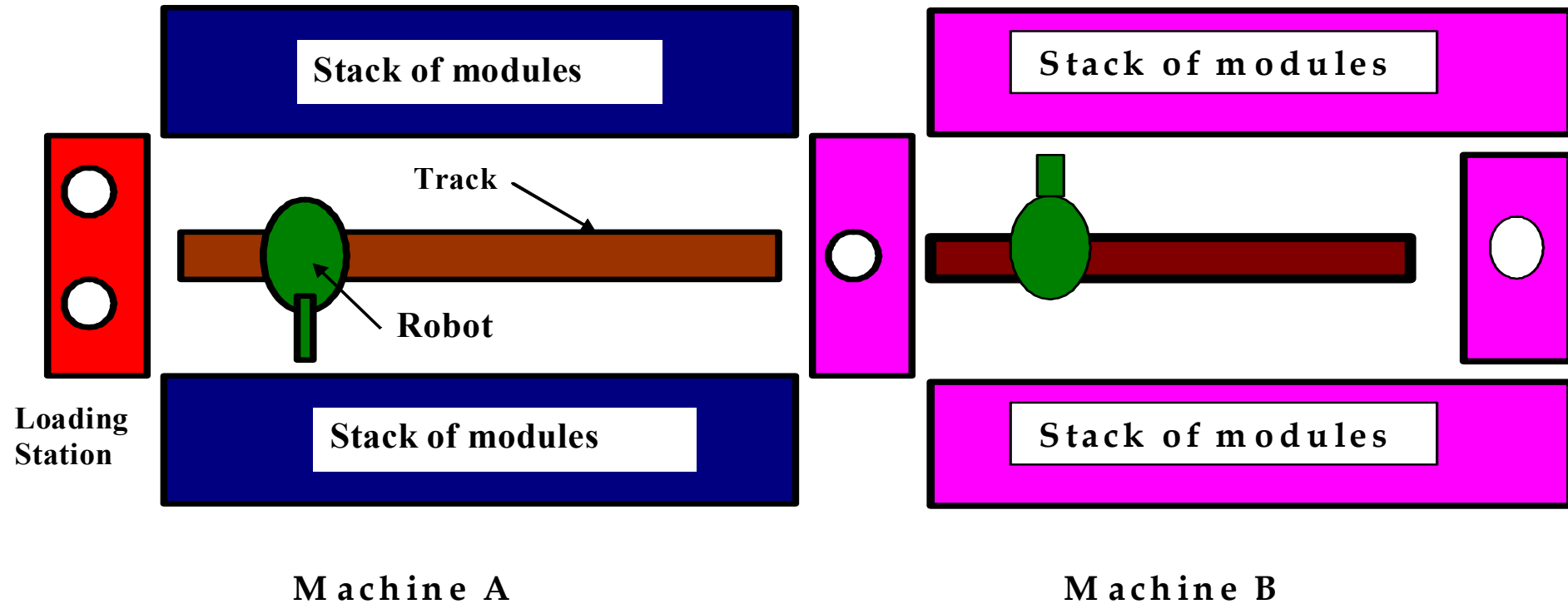


Figure 3 A Cluster of two machines that are physically coupled to manufacture a part.

Engine Design

Consider spark-ignition internal combustion engine used in passenger cars.

1. Is the IC engine a good design?
2. What are the functional requirements (FRs) of an IC engine?
3. How would you improve the design?

Functional Requirements of a Spark-Ignition IC Engine

- 1. Maximize fuel efficiency**
- 2. Eliminate hydrocarbon emission**
- 3. Minimize CO emission**
- 4. Minimize NO_x emission**

Conventional Engine is highly coupled!

There is no way we can satisfy the EPA regulation on emission without using catalytic converter.

February 7, 2005 Lecture

Software -- Acclaro

Think functionally first !!

**Review of special homework
problems.**

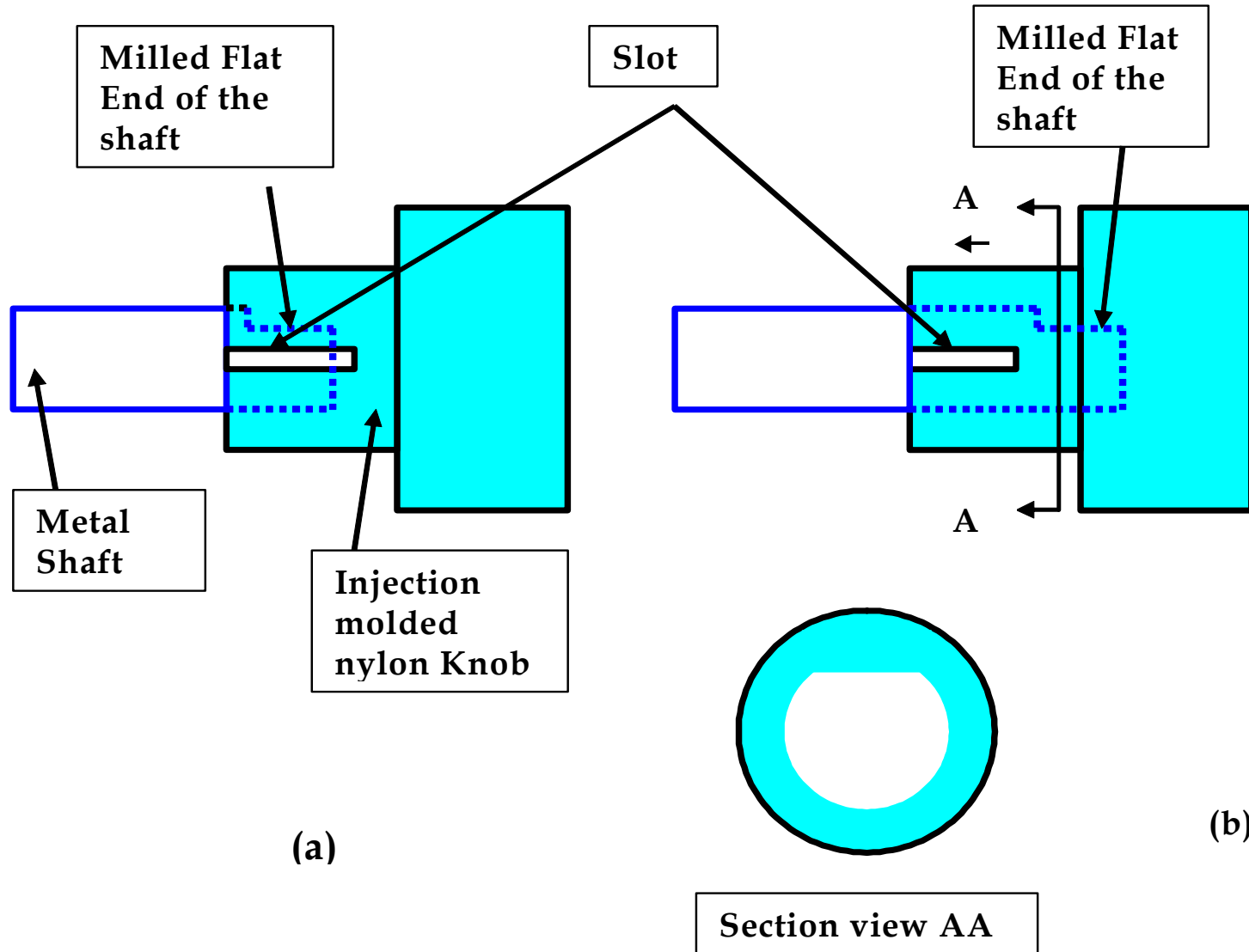
Is this knob a good design or a poor design?

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See Figure 3.1 in Suh, *Axiomatic Design* (2001).

Is this knob a good design or a poor design?

What are the functional requirements of the knob ??

Which is a better design?

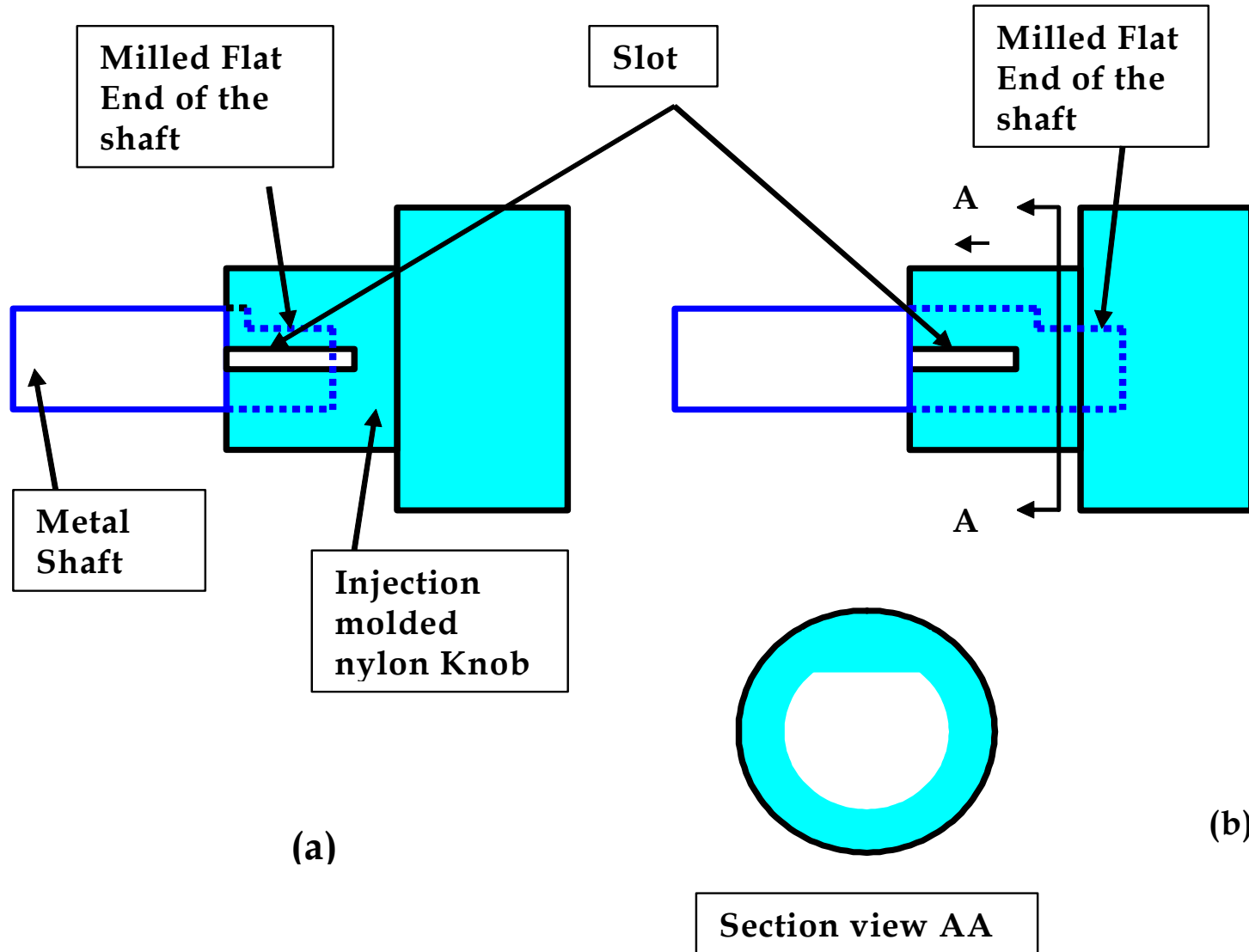


(a)

(b)

Section view AA

Solution: The one on the right. Why?



Typical Design Process

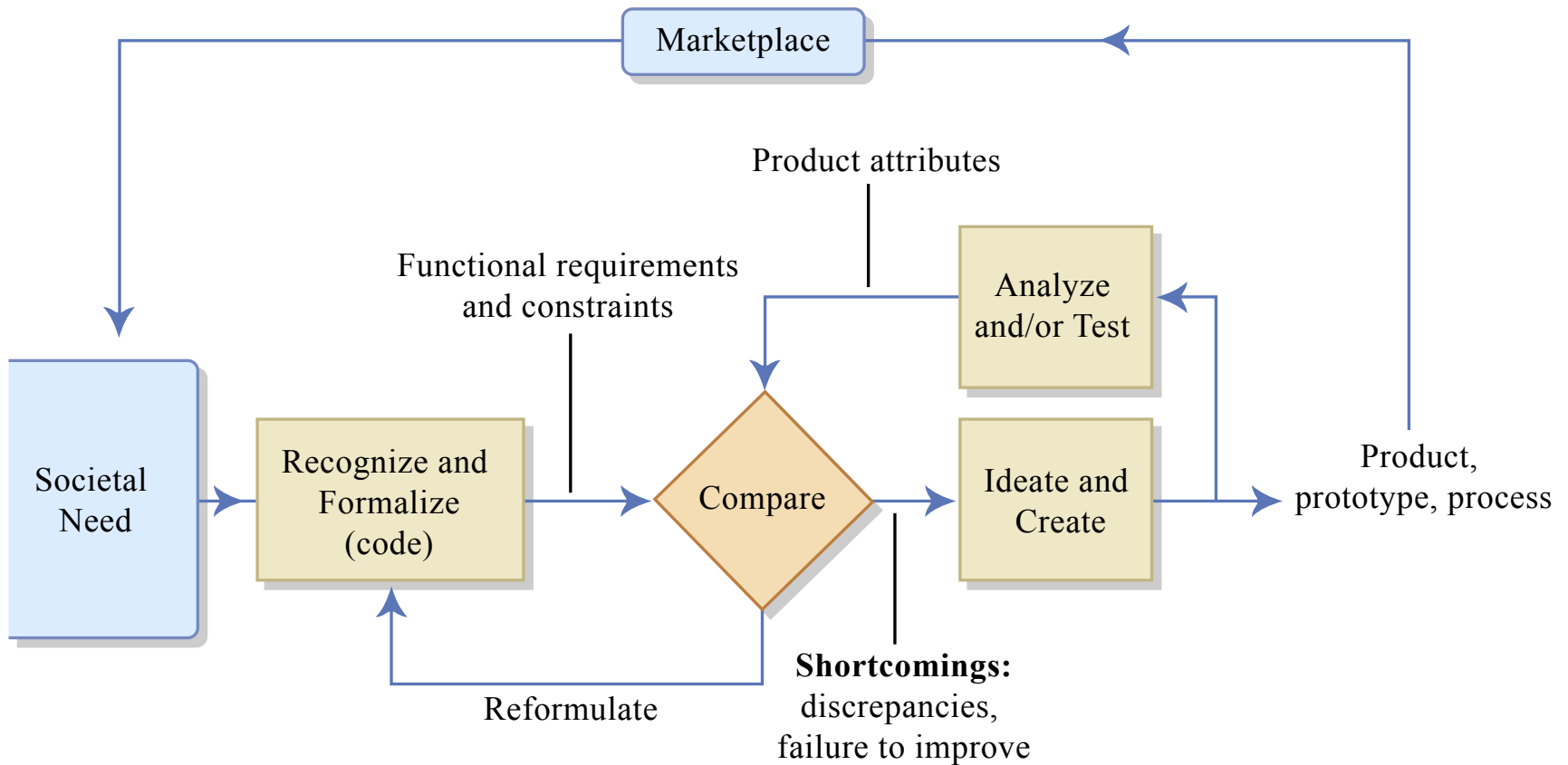


Figure by MIT OCW.

Who are the Designers?

How do we design? What is design?

Is the mayor of Boston a designer?

Design Process

1. *Know their "customers' needs".*
2. *Define the problem* they must solve to satisfy the needs.
3. *Conceptualize the solution through synthesis*, which involves the task of satisfying several different functional requirements using a set of inputs such as product design parameters within given constraints.
4. Perform *analysis* to optimize the proposed solution.
5. *Check the resulting design solution* to see if it meets the original customer needs.

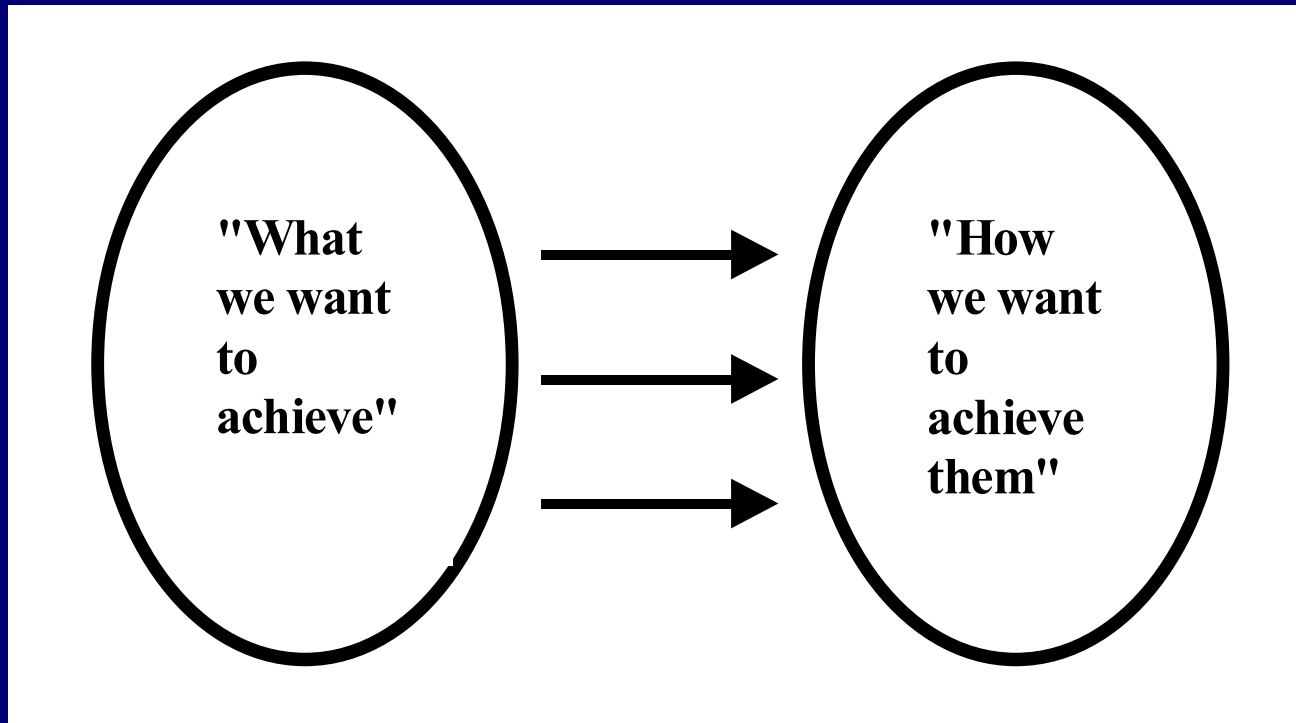
Definition of Design

Design is an interplay between

what we want to achieve and

how we want to achieve them.

Definition of Design



Example: Refrigerator Door Design

Figure removed for copyright reasons.
See E1.1 in Suh, *Axiomatic Design* (2001).

Mapping from Customer Needs to Functional Requirements

Example: Arrow's Impossibility Theorem

Consider the case of having three choices, A, B and C. Three people were asked to indicate their preference among these three choices.

Based on the input from these individuals, can we make a decision as to what the group as a whole prefers?

Example - Solution

The answer is "No. The following table lists the preferences indicated by Smith, Kim and Stein:

Individuals	Preferences	Choices		
		A vs. B	B vs. C	A vs. C
Smith	$A > B > C, A > C$	A	B	A
Kim	$B > C > A, B > A$	B	B	C
Stein	$C > A > B, C > B$	A	C	C
Group preference		$A > B$	$B > C$	$C > A$

The results show that the group is confused as to what it wants. It prefers A over B, and B over C, but it prefers C over A rather than A over C as one might have expected based on the first two choices.

Creativity and Axiomatic Design

Axiomatic design enhances creativity by eliminating bad ideas early and thus, helping to channel the effort of designers .

Historical Perspective on Axiomatic Design

Axioms are truths that cannot be derived but for which there are no counter-examples or exceptions.

Many fields of science and technology owe their advances to the development and existence of axioms.

(1) Euclid's geometry

**(2) The first and second laws of thermodynamics
are axioms**

(3) Newtonian mechanics

Axiomatic Design Framework

The Concept of Domains

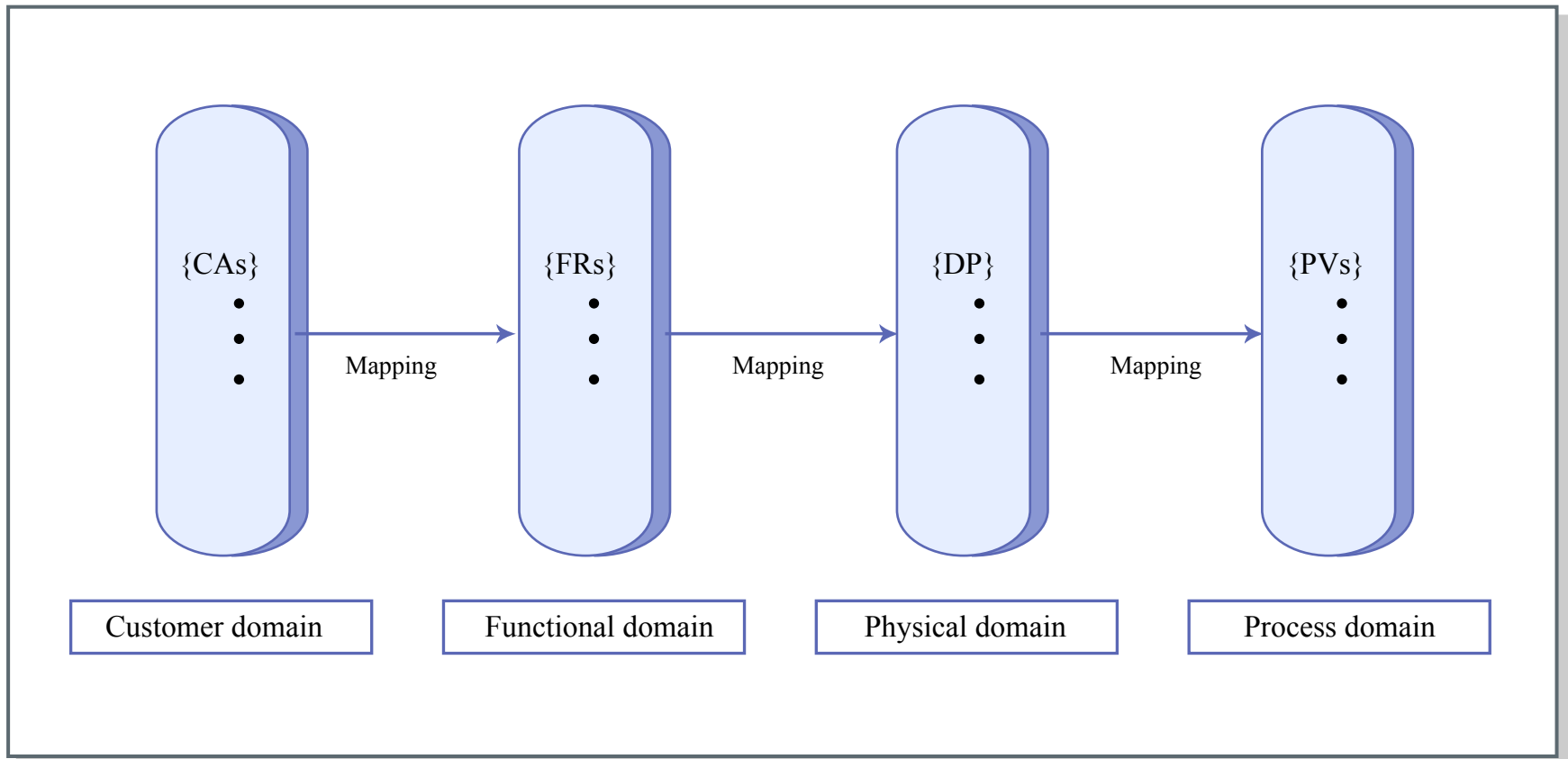


Fig. 1.1 Four Domains of the Design World. $\{x\}$ are characteristic vectors of each domain

Characteristics of the four domains of the design world

Domains Character Vectors	Customer Domain {CAs}	Functional Domain {FRs}	Physical Domain {DPs}	Process Domain {PVs}
Manufacturing	Attributes which consumers desire	Functional requirements specified for the product	Physical variables which can satisfy the functional requirements	Process variables that can control design parameters (DPs)
Materials	Desired performance	Required Properties	Micro-structure	Processes
Software	Attributes desired in the software	Output Spec of Program codes	Input Variables or Algorithms Modules Program codes	Sub-routines machine codes compilers modules
Organization	Customer satisfaction	Functions of the organization	Programs or Offices or Activities	People and other resources that can support the programs
Systems	Attribute desired of the overall system	Functional requirements of the system	Machines or components, sub-components	Resources (human, financial, materials, etc.)
Business	ROI	Business goals	Business structure	Human and financial resource

Definitions

□ *Axiom:*

An axiom is a self-evident truth or fundamental truth for which there is no counter examples or exceptions. It cannot be derived from other laws of nature or principles.

Corollary:

A corollary is an inference derived from axioms or propositions that follow from axioms or other proven propositions.

Definitions - cont'd

Functional Requirement:

Functional requirements (FRs) are a minimum set of independent requirements that completely characterize the functional needs of the product (or software, organizations, systems, etc.) in the functional domain. By definition, each FR is independent of every other FR at the time the FRs are established.

Constraint:

Constraints (Cs) are bounds on acceptable solutions. There are two kinds of constraints: input constraints and system constraints. Input constraints are imposed as part of the design specifications. System constraints are constraints imposed by the system in which the design solution must function.

Definitions - cont'd

Design parameter:

Design parameters (DPs) are the key physical (or other equivalent terms in the case of software design, etc.) variables in the physical domain that characterize the design that satisfies the specified FRs.

Process variable:

Process variables (PVs) are the key variables (or other equivalent term in the case of software design, etc.) in the process domain that characterizes the process that can generate the specified DPs.

The Design Axioms

Axiom 1: The Independence Axiom

Maintain the independence of the functional requirements (FRs).

Axiom 2: The Information Axiom

Minimize the information content of the design.

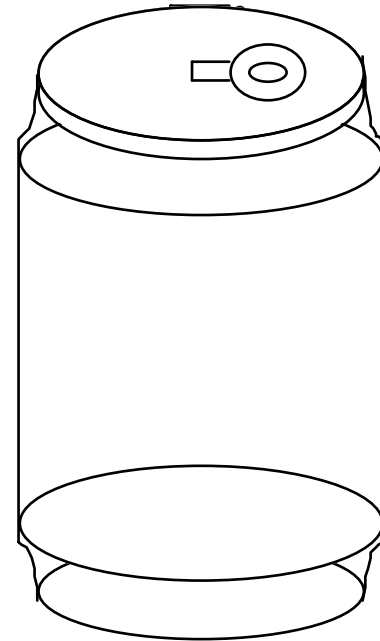
Example 1.3 Beverage Can Design

Consider an aluminum beverage can that contains carbonated drinks.

How many functional requirements must the can satisfy?

How many physical parts does it have?

What are the design parameters (DPs)? How many DPs are there?



Design Matrix

The relationship between {FRs} and {DPs} can be written as

$$\{\text{FRs}\} = [\text{A}] \{\text{DPs}\}$$

When the above equation is written in a differential form as

$$\{\text{dFRs}\} = [\text{A}] \{\text{dDPs}\}$$

[A] is defined as the Design Matrix given by elements :

$$A_{ij} = \partial \text{FR}_i / \partial \text{DP}_j$$

Example

For a matrix A:

$$[A] = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}$$

Equation (1.1) may be written as

$$\begin{aligned} \text{FR1} &= A_{11} \text{DP1} + A_{12} \text{DP2} + A_{13} \text{DP3} \\ \text{FR2} &= A_{21} \text{DP1} + A_{22} \text{DP2} + A_{23} \text{DP3} \\ \text{FR3} &= A_{31} \text{DP1} + A_{32} \text{DP2} + A_{33} \text{DP3} \end{aligned} \tag{1.3}$$

Uncoupled, Decoupled, and Coupled Design

Uncoupled Design

$$[A] = \begin{bmatrix} A_{11} & 0 & 0 \\ 0 & A_{22} & 0 \\ 0 & 0 & A_{33} \end{bmatrix} \quad (1.4)$$

Decoupled Design

$$[A] = \begin{bmatrix} A_{11} & 0 & 0 \\ A_{21} & A_{22} & 0 \\ A_{31} & A_{32} & A_{33} \end{bmatrix} \quad (1.5)$$

Coupled Design

All other design matrices

Design of Processes

$$\{\mathbf{DPs}\} = [\mathbf{B}] \{\mathbf{PVs}\}$$

$[\mathbf{B}]$ is the design matrix that defines the characteristics of the process design and is similar in form to $[\mathbf{A}]$.

Constraints

What are constraints?

Constraints provide the bounds on the acceptable design solutions and differ from the FRs in that they do not have to be independent.

There are two kinds of constraints:

input constraints

system constraints.

New Manufacturing Paradigm – Robust Design

Theorem 4 -- Ideal design

Example: Shaping of Hydraulic Tubes

To design a machine and a process that can achieve the task, the functional requirements can be formally stated as:

FR1= bend a titanium tube to prescribed curvatures

FR2= maintain the circular cross-section of the bent tube

Tube Bending Machine Design (cont's)

Given that we have two FRs,

how many DPs do we need?

Example: Shaping of Hydraulic Tubes

Figure removed for copyright reasons.
See Figure E1.6 in Suh, *Axiomatic Design* (2001).

Example: Shaping of Hydraulic Tubes

DP1= Differential rotation of the bending rollers to bend the tube

DP2= The profile of the grooves on the periphery of the bending rollers

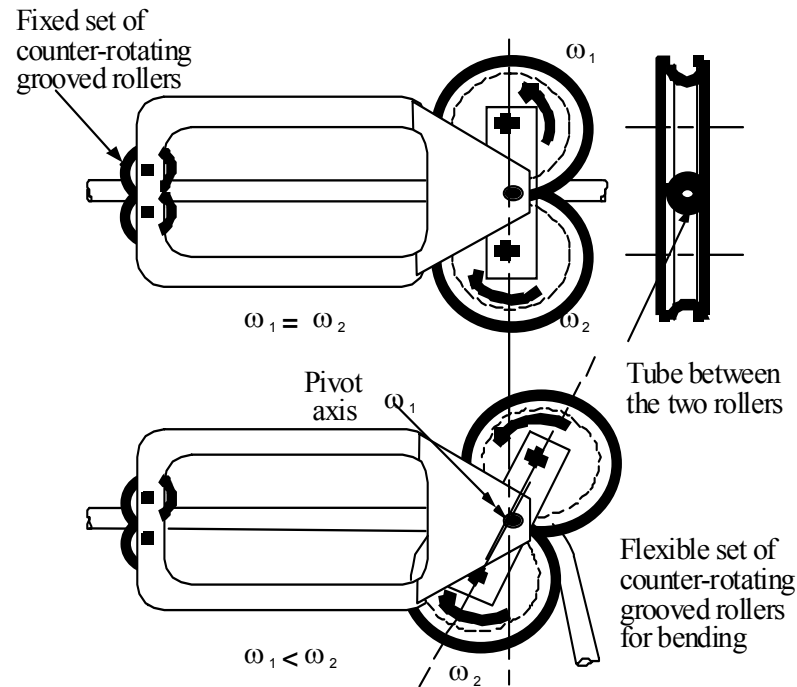


Figure ex.1.4.a

Tube bending apparatus

Example: Van Seat Assembly (Adopted from Oh, 1997)

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See Example 2.6 in Suh, *Axiomatic Design* (2001).

Example: Van Seat Assembly

Traditional SPC Approach to Reliability and Quality

The traditional way of solving this kind of problem has been to do the following:

(a) Analyze the linkage to determine the sensitivity of the error.

Table a Length of linkages and sensitivity analysis

<i>Links</i>	<i>Nominal Length (mm)</i>	<i>Sensitivity (mm/mm)</i>
L12	370.00	3.29
L14	41.43	3.74
L23	134.00	6.32
L24	334.86	1.48
L27	35.75	6.55
L37	162.00	5.94
L45	51.55	11.72
L46	33.50	10.17
L56	83.00	12.06
L67	334.70	3.71

Example: Van Seat Assembly

(b) Assess uncertainty through prototyping and measurement.

The manufacturer of this van measured the distance between the front to rear leg span as shown in Fig. ex.2.5.d. The mean value of FR is determined to be 339.5 mm with a standard deviation of σ_f . Then, we can fit the data to a distribution function. If we assume that the distribution is Gaussian, then the reliability is given by

$$\text{Reliability} = \int_{334}^{346} \frac{1}{\sqrt{2\pi}\sigma_F} e^{-\frac{(FR - \bar{FR})^2}{2\sigma_F^2}} dFR \quad (a)$$

The data plotted in Fig. ex.2.5.d yields a reliability of 95%.

Example: Van Seat Assembly

(c) Develop fixtures and gages to make sure that the critical dimensions are controlled carefully.

(d) Hire inspectors to monitor and control the key characteristics using statistical process control (SPC).

New Manufacturing Paradigm – Robust Design

This design has one FR, i.e., F, the front to rear leg span. This is a function of 10 DPs, i.e., 10 linkages. This may be expressed mathematically as

$$F = f(DP^1, DP^2, \dots, DP^{10})$$
$$\delta F = \frac{\partial f}{\partial DP^x} \delta DP^x + \sum_{i=1, \text{ except } i=x}^{10} \frac{\partial f}{\partial DP^i} \delta DP^i$$

What we want to do is to make $\delta F=0$

Decomposition, Zigzagging and Hierarchy

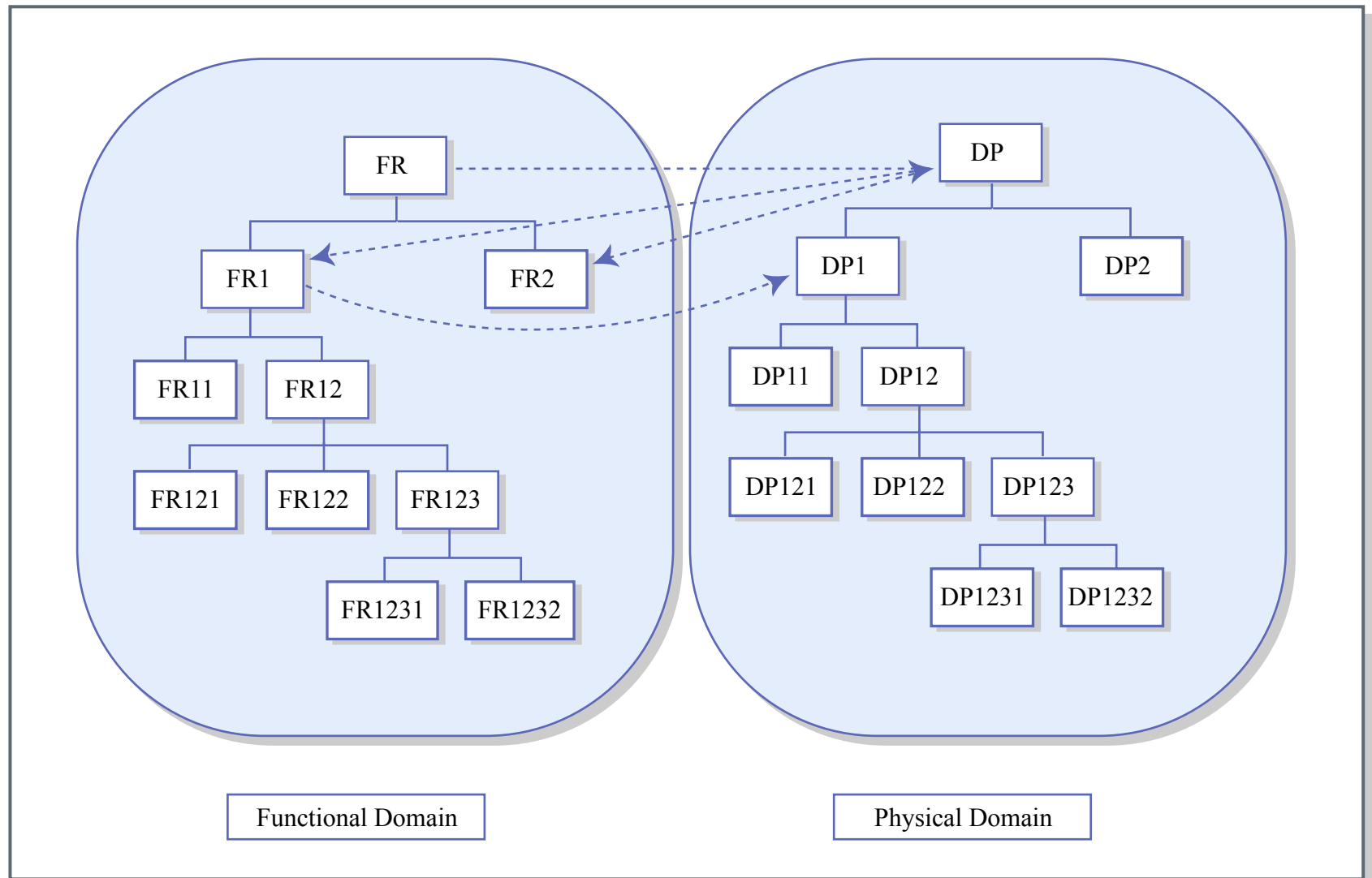


Figure by MIT OCW.

Figure 1.2 Zigzagging to decompose in the functional and the physical domains and create the FR- and DP hierarchies

Identical Design and Equivalent Design

Equivalent Design:

When two different designs satisfy the same set of the highest-level FRs but have different hierarchical architecture, the designs are defined to be equivalent designs.

Identical Design:

When two different designs satisfy the same set of FRs and have the identical design architecture, the designs are defined to be identical designs.

Example: Refrigerator Design

FR1 = Freeze food for long-term preservation

FR2 = Maintain food at cold temperature for short-term preservation

To satisfy these two FRs, a refrigerator with two compartments is designed. Two DPs for this refrigerator may be stated as:

DP1 = The freezer section

DP2 = The chiller (i.e., refrigerator) section.

Example: Refrigerator Design

FR1 = Freeze food for long-term preservation

FR2 = Maintain food at cold temperature for short-term preservation

DP1 = The freezer section

DP2 = The chiller (i.e., refrigerator) section.

$$\begin{Bmatrix} FR1 \\ FR2 \end{Bmatrix} = \begin{bmatrix} X0 \\ 0X \end{bmatrix} \begin{Bmatrix} DP1 \\ DP2 \end{Bmatrix}$$

Example: Refrigerator Design

Having chosen the DP1, we can now decompose FR1 as:

FR11 = Control temperature of the freezer section in the range of -18 C +/- 2 C

FR12 = Maintain the uniform temperature throughout the freezer section at the preset temperature

FR13 = Control humidity of the freezer section to relative humidity of 50%

Example: Refrigerator Design

FR11 = Control temperature of the freezer section in the range of $-18\text{ C} \pm 2\text{ C}$

FR12 = Maintain the uniform temperature throughout the freezer section at the preset temperature

FR13 = Control humidity of the freezer section to relative humidity of 50%

DP11 = Sensor/compressor system that turn on and off the compressor when the air temperature is higher and lower than the set temperature in the freezer section, respectively.

DP12 = Air circulation system that blows air into the freezer section and circulate it uniformly throughout the freezer section at all times

DP13 = Condenser that condenses the moisture in the returned air when its dew point is exceeded

Example: Refrigerator Design

Similarly, based on the choice of DP2 made, FR2 may be decomposed as:

FR21 = Control the temperature of the chilled section in the range of 2 to 3 C

FR22 = Maintain a uniform temperature throughout the chilled section within 1 C of a preset temperature

Example: Refrigerator Design

FR21 = Control the temperature of the chilled section in the range of 2 to 3 C

FR22 = Maintain a uniform temperature throughout the chilled section within 1 C of a preset temperature

DP21 = Sensor/compressor system that turn on and off the compressor when the air temperature is higher and lower than the set temperature in the chiller section, respectively.

DP22 = Air circulation system that blows air into the freezer section and circulate it uniformly throughout the freezer section at all times

Example: Refrigerator Design

Several slides removed for copyright reasons.
See Example 1.7 in Suh, *Axiomatic Design* (2001).

Example: Refrigerator Design

The design equation may be written as:

$$\begin{Bmatrix} FR12 \\ FR11 \\ FR13 \end{Bmatrix} = \begin{bmatrix} X & O & O \\ X & X & O \\ X & O & X \end{bmatrix} \begin{Bmatrix} DP12 \\ DP11 \\ DP13 \end{Bmatrix}$$

Equation (a) indicates that the design is a decoupled design.

	DP22	DP21
FR22	X	0
FR21	X	X

Full DM of Uncoupled Refrigerator Design

		DP1			DP2	
		DP12	DP11	DP13	DP22	DP21
FR1	FR12	X	0	0	0	0
	FR11	X	X	0	0	0
	FR13	X	0	X	0	0
FR2	FR22	0	0	0	X	0
	FR21	0	0	0	X	X

Full DM of Uncoupled Refrigerator Design

		DP1			DP2		
		DP12	DP11	DP13	DP22	DP21	
FR1	FR12	X	0	0	0	0	
	FR11	X	X	0	0	0	
	FR13	X	0	X	0	0	
FR2	FR22	X	0	0	0	0	
	FR21	0	0	0	X	0/X	

Crew survivability system for the Orbital Space Plane

Design of Crew Survivability System for OSP

The highest-levels of FRs were decomposed

to develop

**the detailed design of TPS, Landing System,
and Sensing System for Meteorite Damage.**

High-level Decomposition

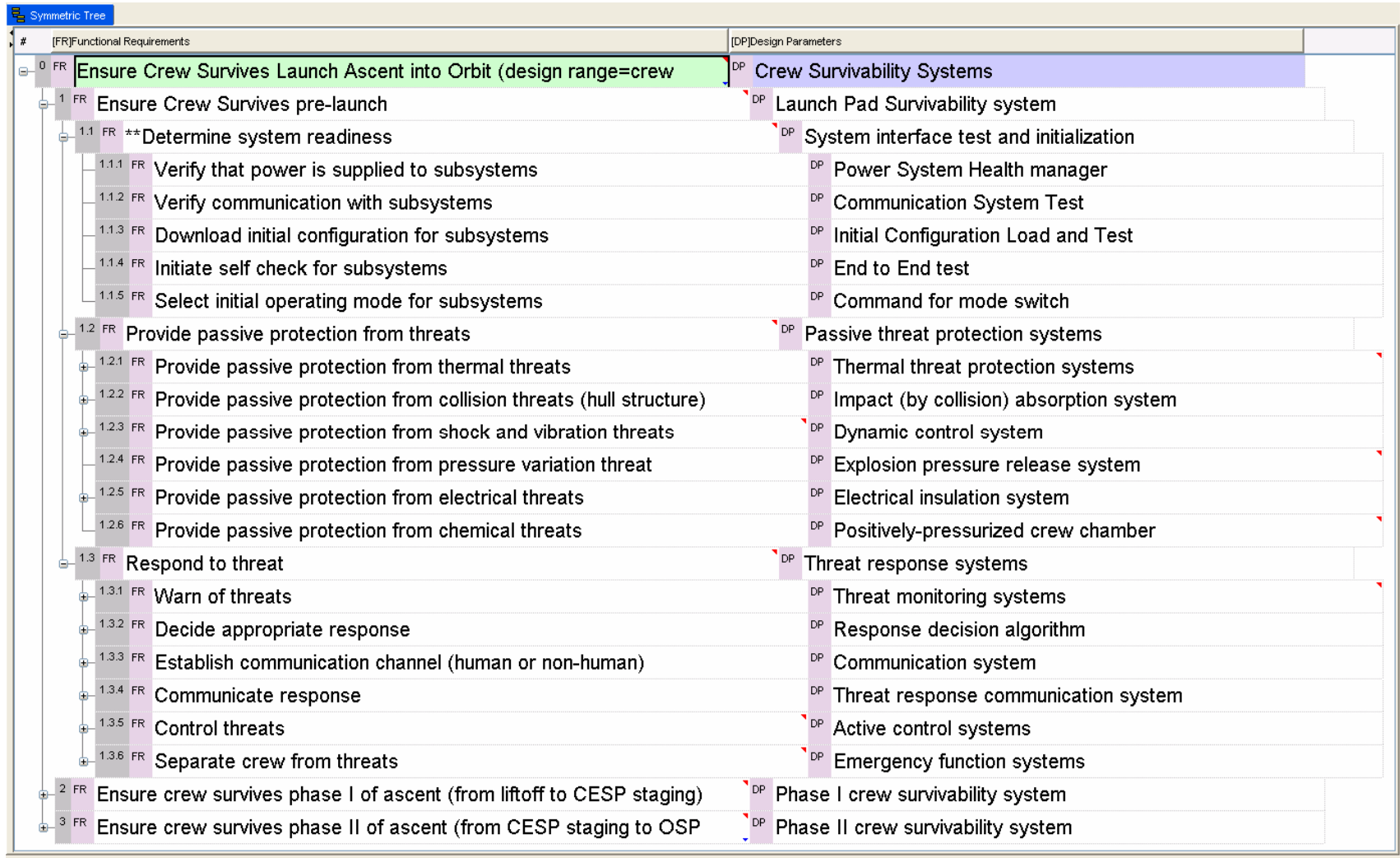
Functional Requirements (FR)

Ensure crews survive launch ascent into Orbit

Design Parameters (DP)

Crew survivability systems

High-level Decomposition (Acclaro, Courtesy of ADSI)



Courtesy of Axiomatic Design Solutions, Inc. Used with permission.

Design Matrix

(Software - Acclaro, Courtesy of ADSI)

Acclaro Designer - [C:\Documents and Settings\Taesik\My Documents\LM_surv\Report_CD\CrewSurv.ad3]

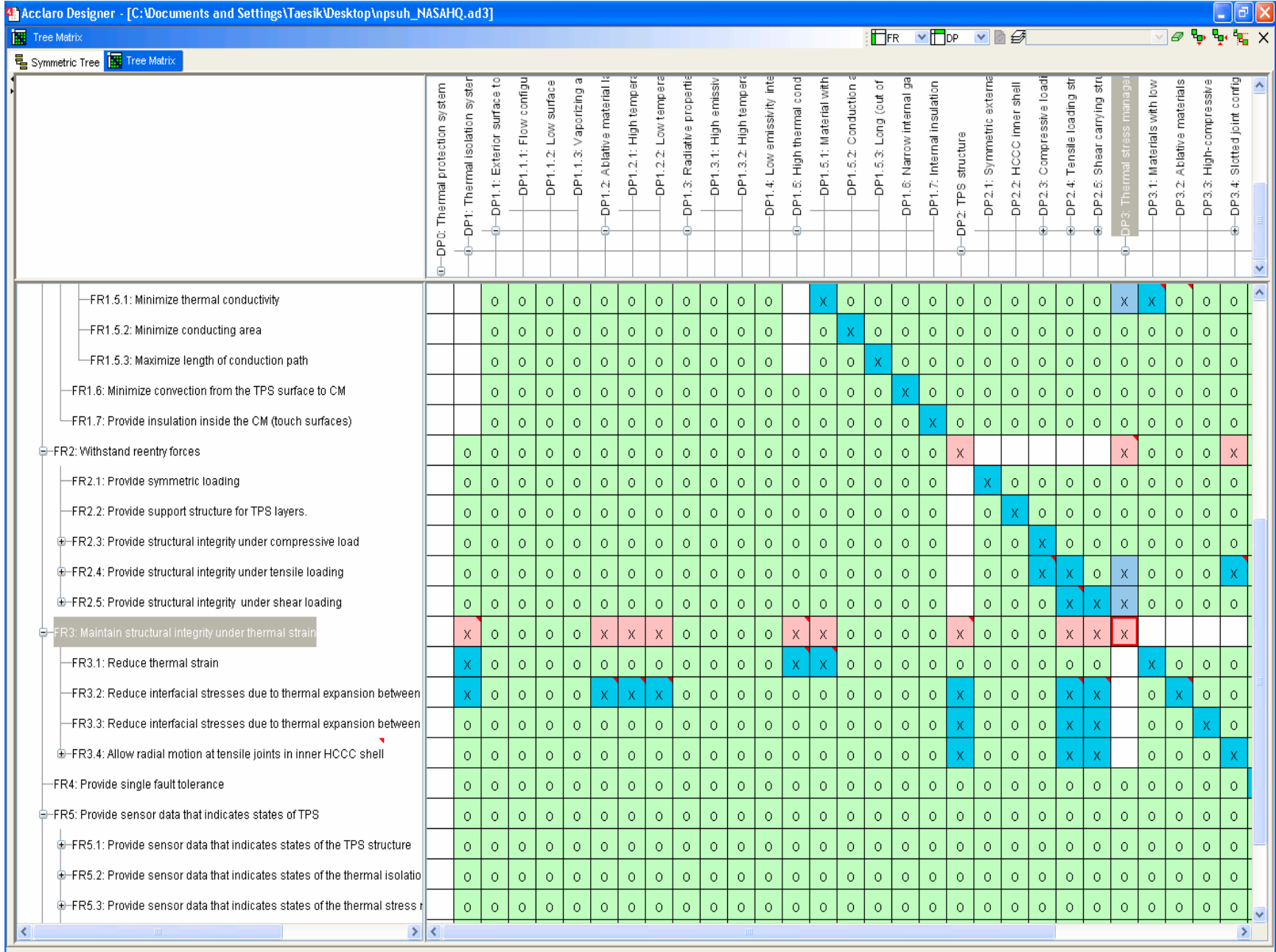
Tree Matrix

Tree Matrix

FR DP

	DP0: Crew Survivability Systems	DP1: Launch Pad Survivability	DP1.1: System interface t	DP1.1.1: Power System	DP1.1.2: Communicat	DP1.1.3: Initial Config	DP1.1.4: End to End t	DP1.1.5: Command fo	DP1.2: Passive threat pro	DP1.2.1: Thermal thre	DP1.2.2: Impact (by co	DP1.2.3: Dynamic cor	DP1.2.4: Explosion pr	DP1.2.5: Electrical ins	DP1.2.6: Positively-pr	DP1.3: Threat response s	DP1.3.1: Threat monit	DP1.3.2: Response de	DP1.3.3: Communicat	DP1.3.4: Threat respo	DP1.3.5: Active contr	DP1.3.6: Emergency f	DP2: Phase I crew survivabil	DP3: Phase II crew survivabil	
FR0: Ensure Crew Survives Launch Ascent into Orbit (design range=crew s	X																								
FR1: Ensure Crew Survives pre-launch		X																					O	O	
FR1.1: **Determine system readiness			X						O	O	O	O	O	O	O	O	O	O	O	O	O	O	O	O	
FR1.1.1: Verify that power is supplied to subsystems				X	O	O	O	O	O	O	O	O	O	O	O	O	O	O	O	O	O	O	O	O	
FR1.1.2: Verify communication with subsystems				O	X	O	O	O	O	O	O	O	O	O	O	O	O	O	O	O	O	O	O	O	
FR1.1.3: Download initial configuration for subsystems				O	O	X	O	O	O	O	O	O	O	O	O	O	O	O	O	O	O	O	O	O	
FR1.1.4: Initiate self check for subsystems				O	O	O	X	O	O	O	O	O	O	O	O	O	O	O	O	O	O	O	O	O	
FR1.1.5: Select initial operating mode for subsystems				O	O	O	O	X	O	O	O	O	O	O	O	O	O	O	O	O	O	O	O	O	
FR1.2: Provide passive protection from threats				O	O	O	O	O	X								O	O	O	O	O	O	O	O	
FR1.2.1: Provide passive protection from thermal threats				O	O	O	O	O		X							O	O	O	O	O	O	O	O	
FR1.2.2: Provide passive protection from collision threats (hull s				O	O	O	O	O		O	X						O	O	O	O	O	O	O	O	
FR1.2.3: Provide passive protection from shock and vibration thr				O	O	O	O	O		O	X	X					O	O	O	O	O	O	O	O	
FR1.2.4: Provide passive protection from pressure variation thr				O	O	O	O	O		O	O	O	X				O	O	O	O	O	O	O	O	
FR1.2.5: Provide passive protection from electrical threats				O	O	O	O	O		O	O	O	O	X			O	O	O	O	O	O	O	O	
FR1.2.6: Provide passive protection from chemical threats				O	O	O	O	O		O	O	O	O	O	X		O	O	O	O	O	O	O	O	
FR1.3: Respond to threat				O	O	O	O	O	X	X		X				X							O	O	
FR1.3.1: Warn of threats				O	O	O	O	O	X	X							X	O	O	O	O	O	O	O	
FR1.3.2: Decide appropriate response				O	O	O	O	O	O	O	O	O	O	O	O	O		X	O	O	X	X	O	O	
FR1.3.3: Establish communication channel (human or non-hum				O	O	O	O	O	O	O	O	O	O	O	O	O	O	X	X	O	O	O	O	O	
FR1.3.4: Communicate response				O	O	O	O	O	O	O	O	O	O	O	O	O	O	O	O	X	O	O	O	O	
FR1.3.5: Control threats				O	O	O	O	O	O	O	O	O	O	O	O	O	O	O	O	O	X	O	O	O	
FR1.3.6: Separate crew from threats				O	O	O	O	O	X	X		X					O	O	O	O	O	X	O	O	
FR2: Ensure crew survives phase I of ascent (from liftoff to CESP stagir		O	O	O	O	O	O	O	O	O	O	O	O	O	O	O	O	O	O	O	O	O	O	X	O
FR3: Ensure crew survives phase II of ascent (from CESP staging to OS		O	O	O	O	O	O	O	O	O	O	O	O	O	O	O	O	O	O	O	O	O	O	O	X

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Design Outcome (selected examples)

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Design of Low Friction Sliding Surfaces without Lubricants

What are the FRs?

What are the constraints?

Design of Low Friction Sliding Surfaces without Lubricants

FR_1 = Support the normal load

FR_2 = Prevent particle generation

FR_3 = Prevent particle agglomeration

FR_4 = Remove wear particles from the
interface

Constraint: No lubricant

Friction at Dry Sliding Interface

Undulated Surface for Elimination of Particles

Figures removed for copyright reasons.
See Figures 7.11 & 7.13 in Suh, *Complexity* (2005).

Design of Low Friction Sliding Surfaces without Lubricants

The design equation:

$$\left\{ \begin{array}{l} FR_1 \\ FR_2 \\ FR_3 \\ FR_4 \end{array} \right\} = \left[\begin{array}{cccc} X & 0 & 0 & 0 \\ 0 & X & x & 0 \\ 0 & 0 & X & 0 \\ 0 & 0 & 0 & X \end{array} \right] \left\{ \begin{array}{l} DP_1 \\ DP_2 \\ DP_3 \\ DP_4 \end{array} \right\} = \left[\begin{array}{cccc} X & 0 & 0 & 0 \\ 0 & X & x & 0 \\ 0 & 0 & X & 0 \\ 0 & 0 & 0 & X \end{array} \right] \left\{ \begin{array}{l} A \\ R \\ \lambda \\ V \end{array} \right\}$$

Suggested Solution

**Transform the system with time-dependent
combinatorial complexity**

to

a system with time-dependent periodic complexity.