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2.830J / 6.780J / ESD.63J Control of Manufacturing Processes (SMA 6303)
Spring 2008

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Massachusetts Institute of Technology
Department of Mechanical Engineering
Department of Electrical Engineering and Computer Science
2.830J/6.780J Control of Manufacturing Processes
Spring 2008

Quiz #2
Thursday – April 24, 2008

In all problems, please show your work and explain your reasoning. Statistical tables for the cumulative standard normal distribution, percentage points of the χ^2 distribution, percentage points of the t distribution, and percentage points of the F distribution (all from Montgomery, 5th Ed.) are provided.

Problem 1 [45%]

An experiment is designed and executed, in which the design or input variable is x and the output variable is y . The input range is normalized to $[-1, +1]$. Experiments are run in the order shown in Table 1 below, with the input setting and output result as given in the table.

Table 1: Full factorial DOE experiment results

Run #	x	y
1	-1	8
2	1	18
3	-1	9
4	1	19
5	-1	10
6	1	20

Part (a) [5%]

Fit a model of the form $y = \beta_0 + \beta_1 x$ to the data, and determine point estimates for β_0 and β_1 .

Part (b) [10%]

Determine the standard error (std. err.) and 95% confidence intervals for the estimates of β_0 and β_1 . Are both parameters significant to 95% confidence or better? Should you include both terms in the model?

Part (c) [5%]

Following good practice, we next examine the residuals (differences between the model prediction values and measured values, for our data). In particular, we consider the residuals as a function of run order. What pattern in the residuals raises a concern? What modifications might you suggest to the experimental design or analysis in light of this?

- (iii) Finally, we also consider an extrapolation of the model beyond the original range of experimentation. Provide a 95% confidence interval prediction for the output value at x equal to 3 (in normalized units), i.e., give the point estimate and 95% confidence interval bounds for $\hat{y}(x = 3)$. Comment on the confidence in model outputs as a function of how far we are extrapolating from our experimental region.

Part (e) [10%]

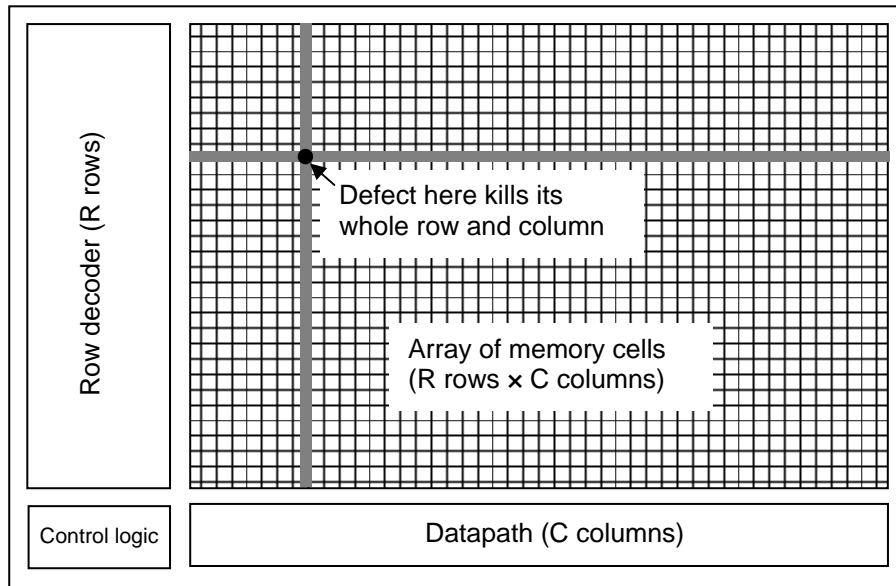
We now perform one additional experimental run to augment Table 1. Setting the input x to 0, the output y is experimentally observed to be equal to 7. Based on this, perform and discuss a lack of fit analysis. To 95% confidence or better, does the model from part (a) show evidence of lack of fit?

Part (f) [5%]

Fit a new model, including the center point data point ($x=0, y=7$). I.e., find α_0 , α_1 , and α_2 for a model of the form $y = \alpha_0 + \alpha_1 \cdot x + \alpha_2 \cdot x^2$. For this part, point estimates are sufficient. Note: it is possible to identify model coefficients by inspection, graphical, or other simplified means; full-scale set up and solution of regression equations is not necessary.

Problem 2 [20%]

Consider the random-access memory chip shown below, having R rows and C columns:



We are interested in investigating the defect tolerance of the memory array. Small particles (which may be regarded as causing point defects) land on the memory chip during fabrication, with a per-area density of D_0 . Particles landing on the datapath, row decoder or control logic do not cause faults (these circuits are very robustly designed), but when a defect lands on one of the memory cells, each cell having an area A , the entire **row** and the entire **column** of cells in which the afflicted cell sits are rendered inoperative.

Assume that the spatial density of defects is very tightly distributed, so that we can write the proportion of memory cells *not* hit by a particle during fabrication as $Y = \exp(-AD_0)$.

Part (a) [3%]

Write down an expression in terms of Y , R , and C for the number, F , of individual memory cells hit by particles during fabrication.

Part (b) [10%]

Write down expressions in terms of Y , R and C for (i) the proportion of columns that are operative after fabrication, and (ii) the proportion of rows operative.

Part (c) [7%]

Write down an expression in terms of Y , R and C for the overall proportion, P , of memory cells that is available for use after fabrication.

Part (d) [optional; for up to 5 bonus percentage points]

Show that the proportion of usable memory cells is maximized when the memory array is square.

Problem 3 [35%]

Part (a) [20%]

Measurements are made of the threshold voltages of MOSFET devices at three randomly chosen locations on each of three wafers themselves chosen randomly from a lot. The deviations of the measured threshold voltages from their target value are shown below. Complete the nested variance analysis. Cells requiring a value to be inserted have a thick border. Some calculations have already been done for you.

Wafer #	Site #	Threshold voltage deviation (mV)	Wafer average (mV)	Squared deviations of point from grand ave. (mV ²)	Squared deviations of wafer ave. from grand ave. (mV ²)	Squared deviations of point from wafer ave. (mV ²)
1	1	1		1		1
1	2	2		0		0
1	3	3	2	1	0	1
2	1	2		0		0
2	2	2		0		0
2	3	2	2	0	0	0
3	1	4		4		4
3	2	0		4		4
3	3	2	2	0	0	0
		Grand average:	2			
			SS_D	10		
			SS_W		0	
			SS_E			10

ANOVA (in mV ²)					
Source	Degrees of Freedom	SS	MS	F ₀	F _{crit} (5% level)
WAFER					
ERROR				N/A	N/A
C TOTAL				N/A	N/A

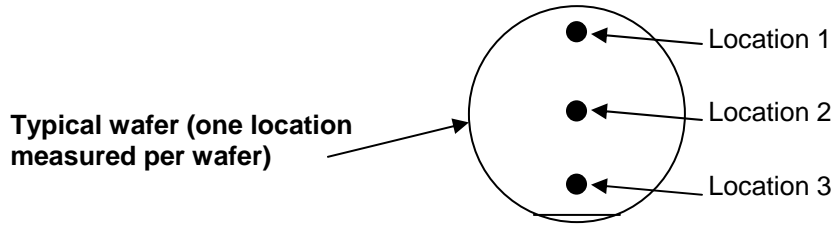
VARIANCE COMPONENTS (in mV ²)				
Variation source	MS	# data in SS	Observed variance	Estimated variance
ERROR (site-to-site)				
WAFER (wafer-to-wafer)				
TOTAL				

Part (b) [5%]

What do you conclude from the nested variance analysis above?

Part (c) [10%]

Now you are given threshold voltage deviation data from the same process, but this time taken from nine separate wafers. From each wafer one measurement is taken at one of three specified (*not* randomly chosen) locations. The objective is to check for evidence of any systematic (fixed) relationship between threshold voltage deviation and position on the wafer. Complete the ANOVA below.



Wafer #	Site (location on wafer)	Threshold voltage deviation (mV)	Per-location average (mV)
1	1	1	7/3
2	1	2	
3	1	4	
4	2	2	4/3
5	2	2	
6	2	0	
7	3	3	7/3
8	3	2	
9	3	2	
		grand average:	2

Source of variation	Sum of squares	Degrees of freedom	Mean square	F ₀	F _{crit} (5% level)
Between groups					
Within groups				N/A	N/A
Total			N/A	N/A	N/A

Part (c), cont'd.

Is there evidence at the 95% confidence level of a significant systematic dependence of threshold voltage on site location?