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2.830J / 6.780J / ESD.63J Control of Manufacturing Processes (SMA 6303)
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Department of Mechanical Engineering
2.830J/6.780J Control of Manufacturing Processes
Quiz 1
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Open Book and Notes

PROBLEM #1 DESIGN OF A CONTROL CHART BASED ON α AND β ((40%))

You have been asked to design a particular type of xbar chart. It is intended only to detect a mean shift of the amount Δ from the grand mean. Instead of the traditional 3σ limits, you instead are asked to do the following:

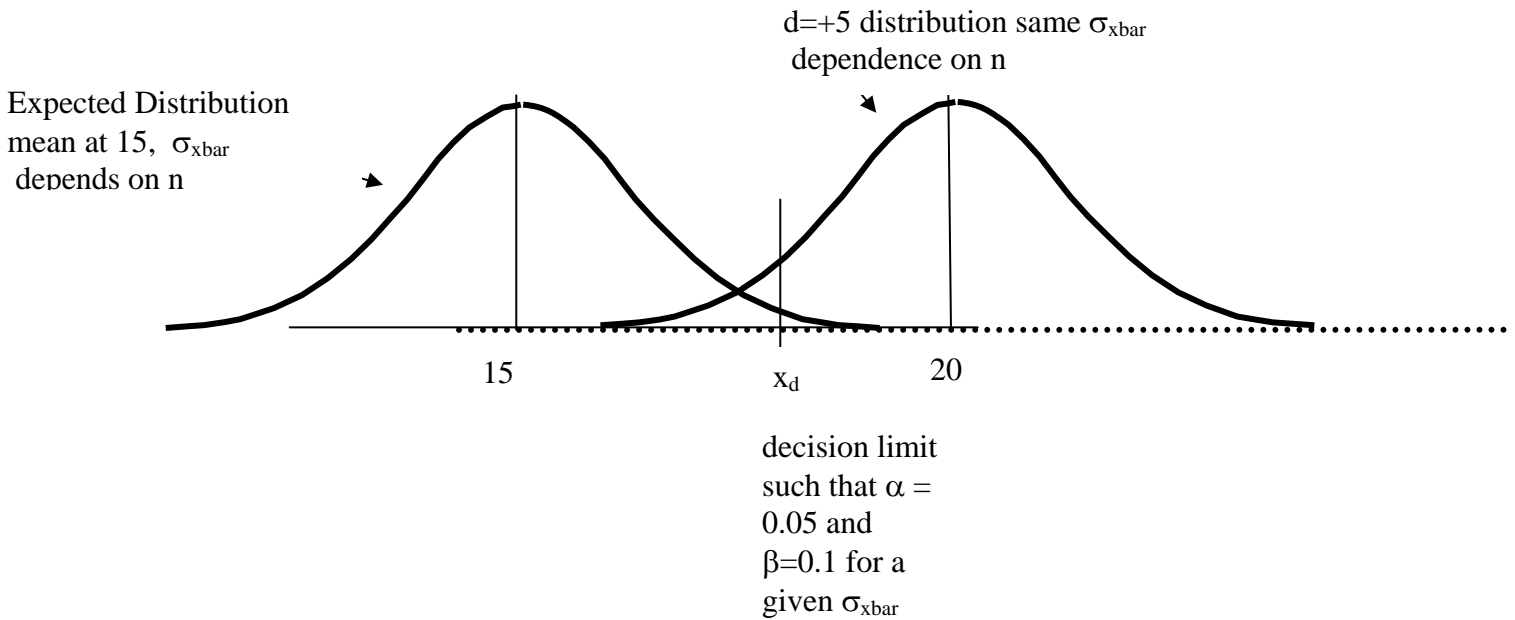
- Assume the expected mean value of the process is 15
- Assume that the parent distribution for the process has a variance of 9
- The desired probability of missing a mean shift of +5 is 0.025 (Type I error)
- The desired probability of signaling a mean shift of +5 when it has not really occurred is 0.1 (Type II error)

You are plotting a conventional Shewhart xbar chart, but the goal is to determine the maximum value of n that will meet these specs.

- a) First sketch out the solution to this problem graphically using normal distributions on a common horizontal axis. Be sure to label all the key variables you will need for the solution.
- b) What is the minimum number of samples (n) that we must include in xbar to meet these specs?
- c) Where will the limits be placed on the xbar chart?

NB: You will need to use the Standard Normal Distribution table (Appendix II) to answer this question.

a) The problem is one of hypothesis testing, where the value of n determines the variance of the distribution for the sample mean (\bar{x}). In this case the shift to be detected (d) is given as +5. This gives the following figure:



b) To find the n- x_d relationship:

$$\alpha/2 = P(x > x_d) \quad \text{for } N(15, \sqrt{\frac{9}{n}})$$

$$\beta = P(x < x_d) \quad \text{for } N(20, \sqrt{\frac{9}{n}})$$

By using standard the standard normal variable Z, we can use the above to find Z_d for each distribution:

$$P(z_1 < (1 - \alpha/2)) = P(z_1 < 0.975) = 1.96 = Z_{d1}$$

$$z_1 = \frac{x - 15}{\sigma_x} \quad \sigma_x = \frac{9}{\sqrt{n}}$$

$$P(z_2 < (\beta)) = P(z_2 < 0.1) = -1.26 = Z_{d2}$$

$$z_1 = \frac{x - 20}{\sigma_x}$$

By converting Z_{d1} and Z_{d2} back to the coordinate x , and equating them we get:

$$x_{d1} = Z_{d1}\sigma_x^- + 15$$

$$x_{d1} = Z_{d2}\sigma_x^- + 20$$

Thus

$$Z_{d1}\sigma_x^- + 15 = Z_{d2}\sigma_x^- + 20$$

or

$$\sigma_x^-(Z_{d1} - Z_{d2}) = 5$$

and substituting

$$\sigma_x^- = \sqrt{\frac{9}{n}} = 5 / (Z_{d1} - Z_{d2}) = 5 / 3.26 = 1.53$$

$$n = 9 / 1.53^2 = 3.85$$

Thus we need at least $n=4$

c) From the above we can get that

$$x_d = Z_{d1}\sigma_x^- + 15$$

$$= 1.96 * \sqrt{9/4} + 15 = 17.94$$

NB: You can actually short cut on this and notice that operating curves for a control chart are a graphical version of this problem.

Problem 2 (35%)

A manufacturing process output is assumed to follow a normal distribution with mean value μ and standard deviation σ . The specifications for the process are in terms of a dimension and given as 300 ± 0.05 mm.

From this the process capability indexes C_p and C_{pk} are computed and found to be 1.2 and 0.9.

- a) What are μ and σ for the process?
- b) What are the fraction out of spec. parts that are expected?
- c) If we had an SPC chart for this process, and the sample size $n=5$, what would be the values for the centerline and UCL, LCL?
- d) Sketch the process distribution and the design specifications showing (approximately) the mean – design target and limit relationships.
- e) If an out of spec part must be reworked at a cost of S\$ 20.00, what is the expected quality loss of this process?

After some time, it is found that although measurement data confirms the estimates of μ and σ , the number of reject parts far exceeds the expected number. You suspect this is because the process is not following a normal distribution, but some other $p(x)$. As a first guess, you assume it is a uniform distribution with the same observed μ and σ as above.

- f) If that is true, how many reject parts should be observed?

a) USL = 300.05 mm, LSL = 299.95 mm

$$C_p = \frac{USL - LSL}{6\sigma} = 1.2 \rightarrow \sigma = \frac{0.1}{6(1.2)} = 0.01389(mm)$$

Assume μ is above the target value

$$C_{pk} = \frac{USL - \mu}{3\sigma} = 0.9 \rightarrow \mu = 300.05 - 3(0.01389)(0.9) = 300.0125(mm)$$

b)

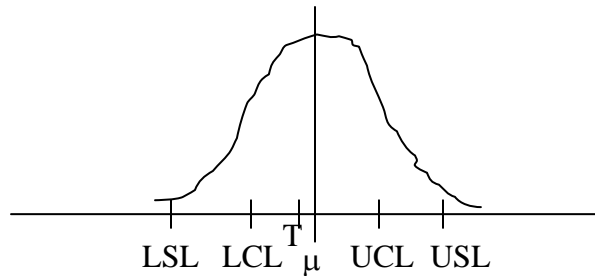
$$[1 - \Phi(\frac{USL - \mu}{\sigma})] + [1 - \Phi(\frac{\mu - LSL}{\sigma})] = 2 - \Phi(2.7) - \Phi(4.5) = 2 - 0.9965 - (\sim 1) = 0.0035$$

$$UCL = \mu + \frac{3\sigma}{\sqrt{n}} = 300.0125 + \frac{3(0.01389)}{\sqrt{5}} = 300.0311$$

c)

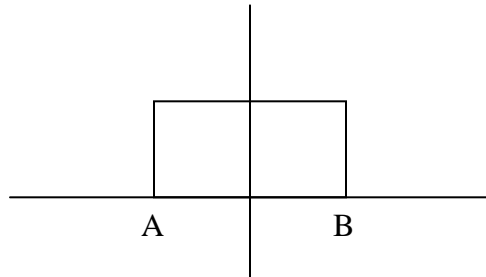
$$LCL = \mu - \frac{3\sigma}{\sqrt{n}} = 300.0125 - \frac{3(0.01389)}{\sqrt{5}} = 299.9939$$

d) LSL=299.95,LCL=299.9939,T = 300, μ = 300.0125,UCL=300.0311,USL=300.05.



e) $20(0.0035) = 0.07$ (\$/piece)

f)



$$A + B = 2(300.0125) = 600.025$$

$$A - B = (0.01389)\sqrt{12} = 0.048 \Rightarrow A = 300.0365, B = 299.9885 \Rightarrow zero$$

Therefore the “unknown” distribution cannot be a uniform one, but must be something other than uniform or normal.

PROBLEM #3 (25%) SPC FOR SHORT PRODUCTION

SPC is best applied to repetitive production, where we can develop an accurate set of statistics for the process and then look for how stationary the statistics remain. However, there are many situations where the process produces very few identical parts. One example is a machining job shop where every part made could be different. Over all these parts, the dimensions and material types can vary widely

It has been proposed to measure each part and plot the error based on the target dimension instead of plotting the dimension itself. In this way a run chart can be created plotting error instead of the absolute dimension or its sequential average (\bar{x})

- a) How would you create an \bar{x} (in this case an (“ \bar{e} ”) chart for this data? What would be the centerline and UCL LCL?

The target value for the error is 0, but we would still use the grand mean from initial production runs. The UCL and LCL would be calculated using deviations of the error from the grand mean. But the data used for these statistics would be coming from many different parts instead of many similar ones.

- b) Why would this approach violate the basic assumptions of Shewhart?

The assumption is that all the data is coming from an identical probability distribution with the same mean and variance. However, we know that as we change machining conditions, as we change part dimensions, tool types and material types, the mean of the process (or in this case the error of the process) will vary. Thus we will not have a stationary process, and the chart will not be valid.

- c) How will the chart vary from a “proper” Shewhart chart in both the limits and the appearance of the data on the chart?

Since we have taken a grand mean for a non-stationary process, we will not only have an incorrect mean for any given case, the variance will be greater than expected for any given run. This is because we are modeling a variable mean process as one that is stationary. In doing so this moving mean will be reflected in an inflated estimate of the process variance. The data will tend to not fall into the expected confidence intervals correctly and will appear more uniform than normal, since the mean is actually shifting with each new part.

- d) What would a C_{pk} value for this process mean?

C_{pk} here would be difficult to assess since we would most likely have different tolerances for different parts, and as stated above a overall process variance that

ignores the variance on that a particular part. If we used a fixed tolerance it could be a means of globally assessing performance of the equipment. But it could not be used to predict the number of defective parts to be expected.