2.830J / 6.780J / ESD.63J Control of Manufacturing Processes (SMA 6303)
Spring 2008

MIT 2.830/6.780 Problem Set 2 (2008) — Solutions

Problem 1

Many interesting responses were received. We can discuss on an individual basis when selecting term projects.

Problem 2

Let x be the concentration of particles in particles/ft^3:

$x \sim N(15.08, 0.025)$

Since the acceptable concentration range is 15.00 ± 0.10 particles/ft^3, the probability that a system conforms to specifications is:

$$
\begin{aligned}
P(14.9 \leq x \leq 15.1) \quad &= \quad P(x \leq 15.10) - P(x \leq 14.90) \\
&= \quad \Phi([15.10-15.08]/0.05) - \Phi([14.90-15.08]/0.05) \\
&= \quad \Phi([15.10-15.08]/0.05) - \{1 - \Phi([15.08-14.90]/0.05)\} \\
&= \quad 0.06554 - 1.59 \times 10^{-4} \\
&= \quad 65.5\%
\end{aligned}
$$

Problem 3

Because we know neither the population's mean nor its standard deviation, we need to use a t-test. The sample size is 10 so the number of degrees of freedom is $10 - 1 = 9$. A 99% confidence interval on the mean photoresist thickness is given by:

$$
\overline{x} - t_{(0.01/2),9} \frac{s}{\sqrt{10}} \quad \leq \quad \mu \quad \leq \quad \overline{x} + t_{(0.01/2),9} \frac{s}{\sqrt{10}}
$$

where the sample mean is

$$
\overline{x} = 13.3962 \ \mu m
$$

and the sample variance is

$$
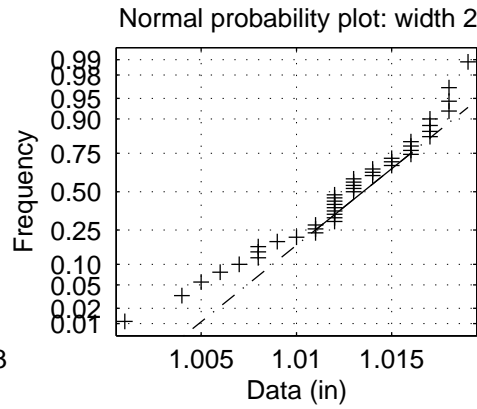s^2 = \frac{1}{9} \sum \left(x - \overline{x} \right)^2 = 1.5277 \times 10^{-5} \, \mu m^2 .
$$
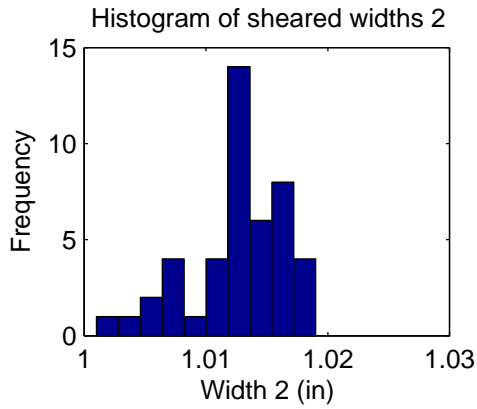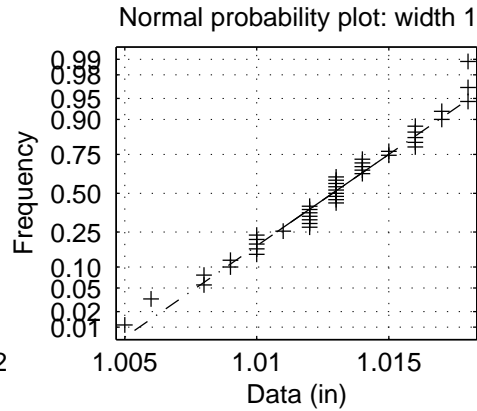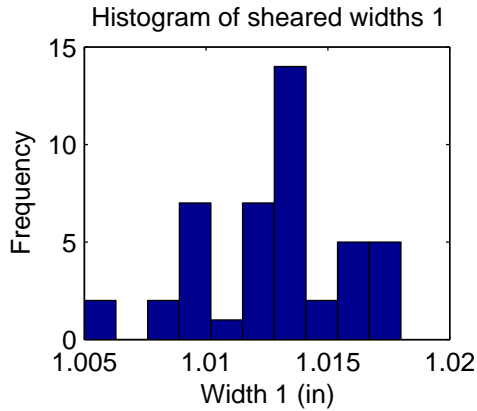
From tables, $t_{0.005,9} = 3.250$. The 99% confidence interval is therefore:

$$
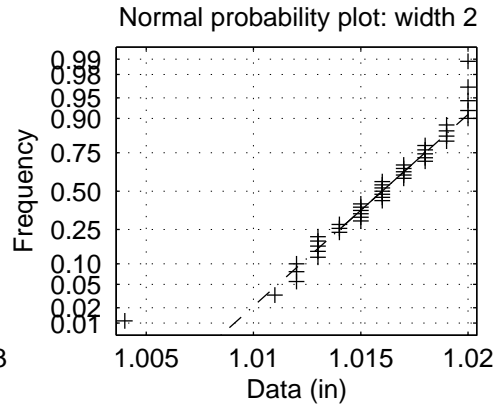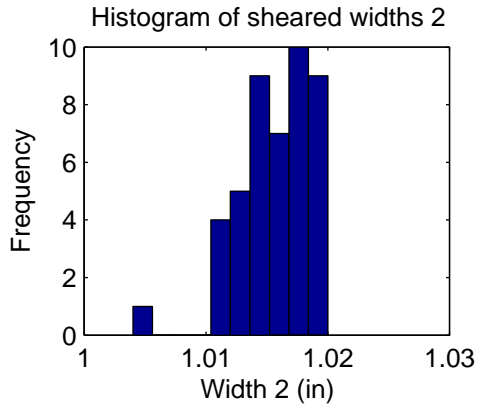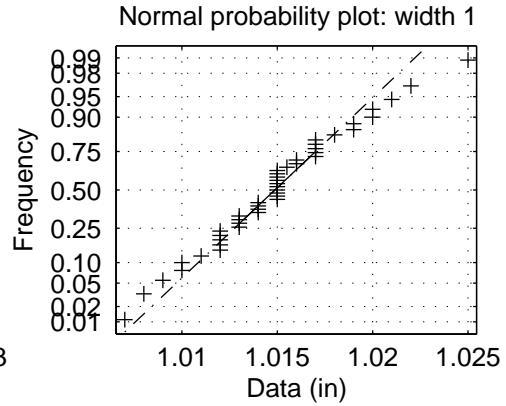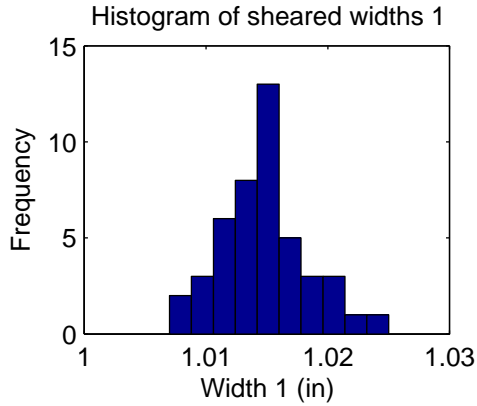13.3922 \ \mu m \leq \mu \leq 13.4002 \ \mu m.
$$

Problem 4

a) Histograms and normal probability plots for **shearing data**, sorted by material and end-of-sample.

Skewness, kurtosis and normality test results for the two materials and two widths:

	Steel		Aluminum	
	Width 1	Width 2	Width 1	Width 2
Skewness	−0.3568	−0.7232	0.3199	−1.1607
Kurtosis*	2.8899	3.3079	3.4622	5.8867
Lillifors test at 5% level	Reject	Reject	Accept	Accept

* Defined as 3 for a normal distribution. Values less than 3 correspond to *leptokurtic* distributions (sharper peak; fatter tails than a normal) and values larger than 3 correspond to *platykurtic* distributions (smaller peak; thinner tails).

The Lilliefors test examines the hypothesis that the distribution is normal. 'Reject' in the table above means that the hypothesis of normality is rejected at the 5% level. For more information see, *e.g.*, Lilliefors (1967) *J. Am. Stat. Assoc.*, vol. 62, pp. 399–402.

Examining the normal probability plots of the data, it would seem fairly reasonable to model the distributions of widths as normal, although there are some reasons to suspect that they may *not* be normal – *e.g.* the steel, width 1, data are 'missing' an upper tail; the steel, width 2, normal probability plot has a marked curve; the aluminum, width 2, data are 'missing' an upper tail. Could the data be skewed towards the lower values for some physical reason? A possible operator error involves not pressing the material hard against the stop on the shear, giving a shorter part, whereas one cannot push the material beyond the stop. The rather large kurtosis of aluminum width 2 data provides a further suggestion that the underlying distribution might not be normal. Meanwhile, the steel data fail the Lilliefors test of normality at the 5% level.

b) To test whether there is a significant taper in the parts, consider that we have two measurements, w_1 and w_2, per part. A paired test is needed. If we could assume that the amount of taper, w_1-w_2, was normally distributed, we could use a paired t-test; however, since the normal probability plots shown above raise some questions over whether any of the dimensions are normally distributed, we will not make this assumption and will use instead a Wilcoxon signed rank test, which does not assume any particular distribution. The test is implemented in Matlab as p = signrank(x), where x is a vector of differences between pairs, w_1-w_2 in this case, and p is the probability of observing the given x, or one more 'extreme', by chance if the median of x was in fact zero.

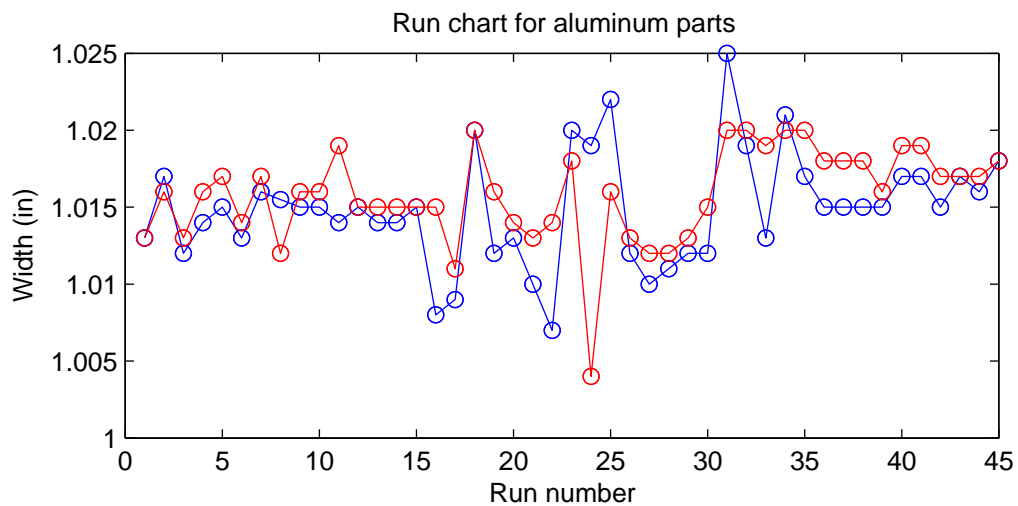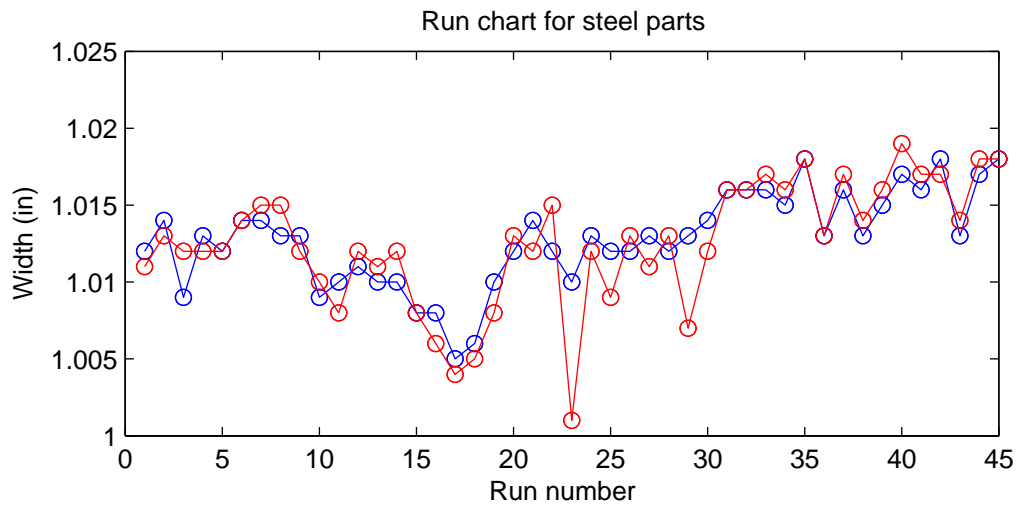For the steel parts: $p = 0.738$
For the aluminum parts: $p = 0.002$
(There are 45 samples for each material.)

At the 5% level, therefore, there is no statistically significant taper in the steel parts but there is a significant taper in the aluminum parts. Even removing the outlier at sample 24 of the aluminum dataset, the *p*-value increases only to 0.005; there is still a significant taper. A histogram plot of the differences shows that the median of w_1-w_2 is slightly negative for aluminum.

[The use of a paired t-test is acceptable as a solution if, in the previous part, the data were interpreted as coming from a normal distribution.]

c) Run charts for the parts produced (blue symbols: w_1; red symbols: w_2):


Looking at these run charts, it is difficult to say conclusively whether there is a mean shift for either material as the run progresses. Perhaps the width of the steel parts drifts up over time, but to make a quantitative evaluation of whether or not the process is stationary, we can use a control chart. That is what we will practice in Assignment 3.

(Incidentally, 'eyeballing' the run chart for aluminum parts suggests to me a consistent taper, with w_2 being slightly larger)

d) For whole **injection molding** dataset:

Histogram and normal probability plot:
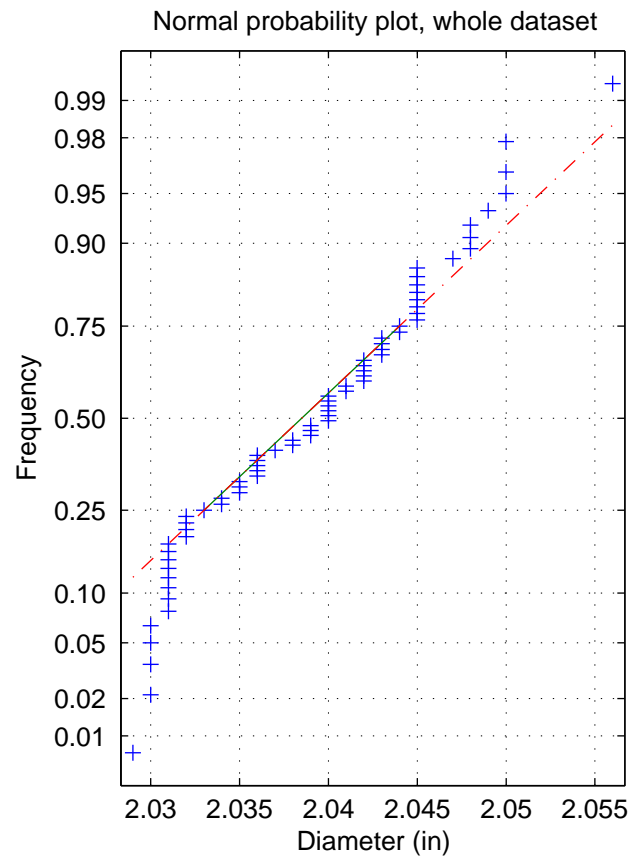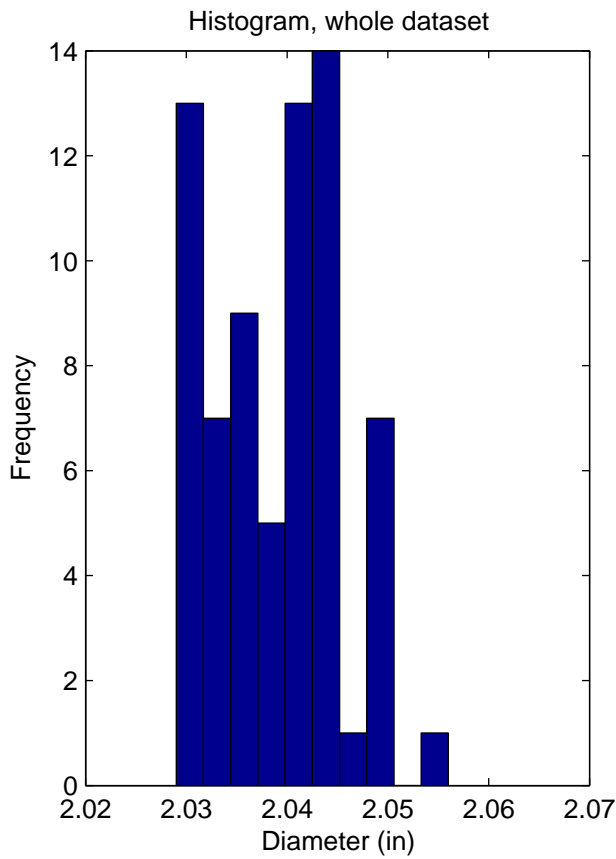
The kurtosis is 2.2; the form of the normal probability plot suggests that in the center of the distribution an assumption of normality is reasonable, while the tails seem to be 'missing' when the experimental data are compared to a normal distribution. Skewness = 0.21; there is not a particularly marked skew in the data. At the 5% level the assumption of normality is rejected by the Lilliefors test.

e) For the four process settings in turn:
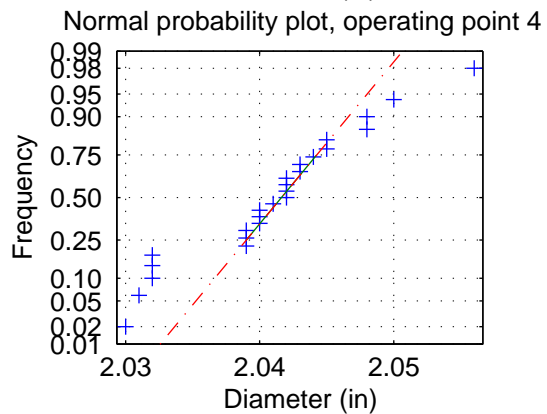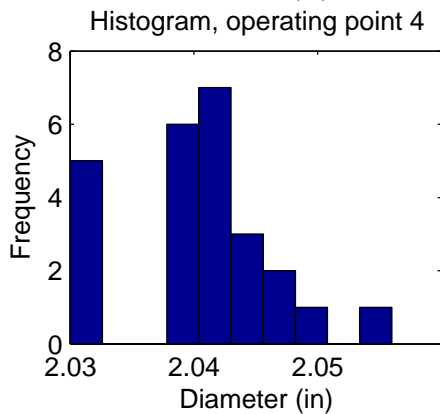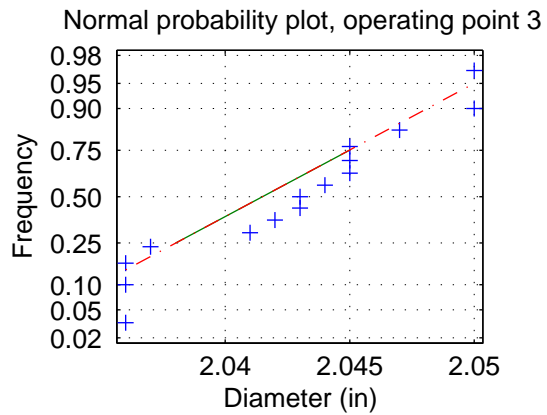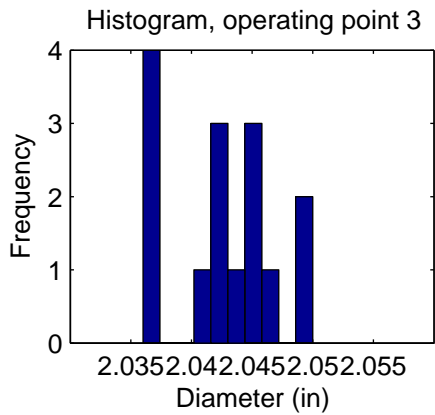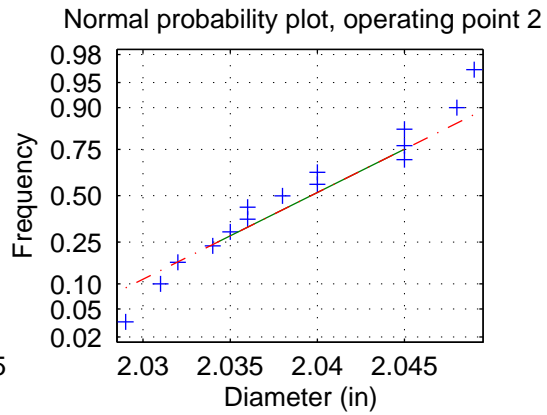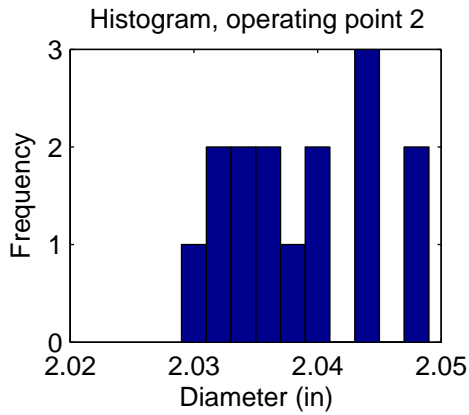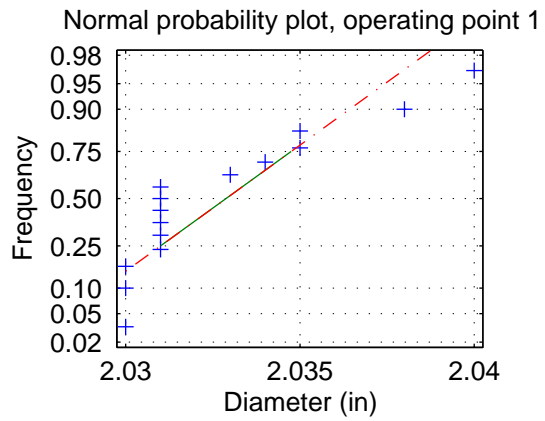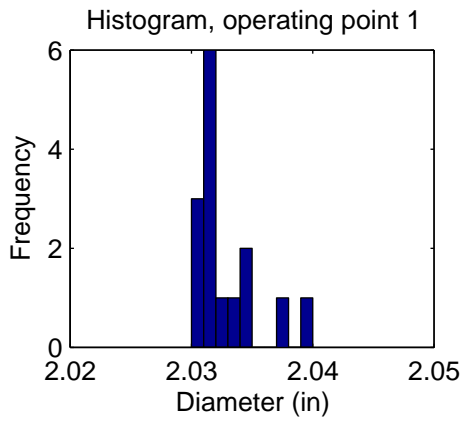
The skewness, kurtosis and Lilliefors test results are shown below:

Operating point	1	2	3	4
Sample mean (in)	2.0327	2.0389	2.0427	2.0410
Sample s.d. (in)	0.0031	0.0063	0.0047	0.0062
Skewness	1.1690	0.1463	−0.1153	0.0733
Kurtosis	3.2550	1.8031	2.0158	3.0531
Lilliefors test (5%)	Reject	Accept	Accept	Reject
Sample size	15	15	15	25

After separating the data associated with each operating point, the Lilliefors test accepts the hypothesis of normality at the 5% level in two of the four cases.

f) We wish to compare the mean diameter produced at operating points 1 (low hold, low velocity) and 4 (high hold, high velocity). Since the tests of normality above suggested that the underlying distributions were not normal, we cannot use a t-test. Instead we use a Wilcoxon rank sum test, which is distribution-independent. It is implemented in Matlab as `[p,h] = ranksum(x,y)` and tests the hypothesis that two given vectors `x` and `y` come from distributions with equal medians. `p` is the probability of observing the given `x` and `y`, or samples more 'extreme', by chance, if the medians of `x` and `y` are in fact equal. `h` is the result of testing at the 5% level the hypothesis that `x` and `y` come from distributions with equal medians, and is equal to 1 if the null hypothesis is to be rejected.

For operating points 1 and 4, the rank sum test gives a p-value of 7×10^{-5}, and the hypothesis that the underlying means are equal is rejected at the 5% level. Therefore there is significant evidence of a difference in median output between these operating points: the 'low hold, low velocity' condition gives smaller-diameter parts.

[The use of a t-test is acceptable as a solution if, in the previous part, the data were interpreted as coming from a normal distribution.]