

Department of Mechanical Engineering
MASSACHUSETTS INSTITUTE OF TECHNOLOGY

2.800 TRIBOLOGY

Fall 2004

Homework

Due: After Thanksgiving

Question 1

The primary mechanisms of friction between dry sliding surfaces are: plowing, deformation and adhesion. Among the three, the dominant mechanism usually is plowing. In this problem we wish to estimate, by the upper-bound method, the friction coefficient μ due to plowing a soft surface by a hard abrasive (or by a wear particle), and in mechanical polishing.

Assume that the particle is spherical of radius R and the depth of indentation is h , where h is less than or equal to R . Assume further that adhesion between the particle and the surface being plowed is negligible. The hardness of the particle is far greater than that of the surface being plowed.

- (a) Express the normal force N on the particle as the product of the projected area of contact (while sliding or plowing) between the particle and the surface **in the direction of loading**. N is a function of R , h and H , where H the hardness of the surface being plowed.
- (b) Similarly, express the tangential force F on the particle as a product of the projected area of contact **in the direction of sliding** and the hardness of the surface being plowed, H , as a function of R , h and H . (Note: You should be able to calculate the required areas in both cases without elaborate integration.)
- (c) The friction coefficient $\mu = F/N$. Express now μ as a function of h/R (i.e., in dimensionless form). (You may note in passing that the friction coefficient is independent of the hardness of the soft surface that is being plowed by the hard particle!)
- (d) Sketch the friction coefficient μ as a function of h/R . Ignore the behavior of μ in the neighborhood of $h/r = 0$.

- (e) What is the greatest (other than singularities, if any) friction coefficient that can be obtained by this model? Don't try differentiation. Just infer from contact geometry. What are the implications for minimizing friction?
- (f) Now, how does adhesion between the particle and the surface affect the friction coefficient? That is, does μ increase or decrease, and approximately by how much?
- (g) The adhesion mechanism dominates when h/R approaches zero. What is the friction coefficient for the adhesion mechanism?
- (h) Suppose now that the particle is a right circular cone of apex angle 2α and adhesion to be zero. How does the friction coefficient depend on the depth of penetration, h ? Don't repeat the above process. You should be able to answer by a glance of the contact geometry and some mental calculation.
- (i) Finally, express the Preston constant as a function of h/R and the hardness H of the material being polished.

Question 2

[Thin, soft metallic films (e.g., Pb, Sn, In, Ag, Au, etc.) are used as solid lubricant films, usually deposited by electroplating, physical vapor deposition, and so on. Such films, often as thin as $0.1 \mu\text{m}$, can be successfully used to lubricate high-vacuum or space tribological systems. The coefficients of sliding friction between a steel slider and a steel counterface plated with a soft metallic film can be very low (0.06 for In and 0.09 for Sn) whereas the sliding friction between uncoated metals is much greater. Thin, soft metallic films have also been used to minimize sliding wear by at least an order of magnitude.]

We wish to develop an expression for the lowest friction coefficient of thin film-coated hard surfaces by combining the Hertz contact stress analysis and the shear yielding of thin, soft films. Consider a flat, hard substrate coated with a thin film being loaded by a hard spherical ball of radius R . The elastic properties (E and ν) of the ball and the flat are the same. We assume that the thin, soft film does not appreciably influence the Hertz analysis of point contacts.

- (a) How is the normal pressure $p(r)$ radially distributed in the circular contact region? Express the pressure distribution in dimensionless form in terms of p_o , (the center pressure) and contact radius, a .
- (b) What is the value of mean pressure p_{mean} (in terms of p_o) in the contact region?
- (c) The maximum pressure that can be sustained without yielding anywhere in the contact is given by $p_o = \tau_y / 0.31$, where τ_y is the shear yield strength of the substrate. Now express the normal contact force, N , in terms of τ_y , a , etc.

- (d) Express the friction force, F , as a product of the contact area and the shear strength of the soft film τ_f .
- (e) What is the friction coefficient, $\mu = F/N$, in terms of the shear yield strengths of the film and the substrate (or in terms of the hardnesses)?
- (f) Estimate the friction coefficient μ if the hardness of the coating is about 20 kg/mm^2 and of the substrate 600 kg/mm^2 . How does it compare with the experimental values quoted above? please state and comment on all the assumptions involved in the approach.

Question 3

Apparently, Lord Rayleigh had used the calculus of variations to show that the film shape in an infinitely wide (normal to the direction of sliding) pad bearing which gives the greatest load-carrying capacity comprises two parallel sections. This design is known as Ryleigh step bearing. We do not contest the result, but wish to employ this bearing to understand the principles of hydrodynamic bearings (see section 7.3.3 of the handout on Hydrodynamic bearings and Fig. 7.5). Please furnish the necessary equations in answering the questions.

- (a) The pressure distribution in the step distribution in the linear regions, either side of the step, is linear. Why?
- (b) The flow rate of the lubricant at each section must be the same. What is it?
- (c) The load carried per unit width of the bearing depends both on the film heights and the step lengths. Show that $h_i/h_o = 1.87$ and $B_i/B_o = 2.59$ for the optimum design.
- (d) Write down a general expression for the traction coefficient of the Rayleigh step bearing.
- (e) When the load carried is the greatest, i. e, for the optimum design, what is the traction coefficient in terms of the relevant quantities.
- (f) How are the design parameters (h_i , h_o , B_i , B_o , etc.) related when the traction coefficient is the least.
- (g) Write down a simple equation for the temperature of the lubricant as it exits the bearing, assuming that the pads of the bearing are thermally insulating.

P.S. Obviously, the homework is too wordy. The intent, however, is to lead you step by step to the possible answer. If watchful, you will see how the solution unfolds gradually.

Happy Thanksgiving!