Overview

- Last time:
  - thin lens
  - object at infinity
  - image at infinity

$$\begin{pmatrix} \alpha_{\text{right}} \\ x_{\text{right}} \end{pmatrix} = \begin{pmatrix} 1 & -\frac{1}{f} \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \alpha_{\text{left}} \\ x_{\text{left}} \end{pmatrix} \quad \text{where}$$

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_{\text{left}}} - \frac{1}{R_{\text{right}}}\right) \quad \text{Lens maker's Equation}$$

- Today:
  - imaging at finite distances
  - thick lens
  - the human eye

object at infinity

$$x_2 = \alpha_1 f$$

image at infinity

$$\alpha_2 = -\frac{x_1}{f}$$

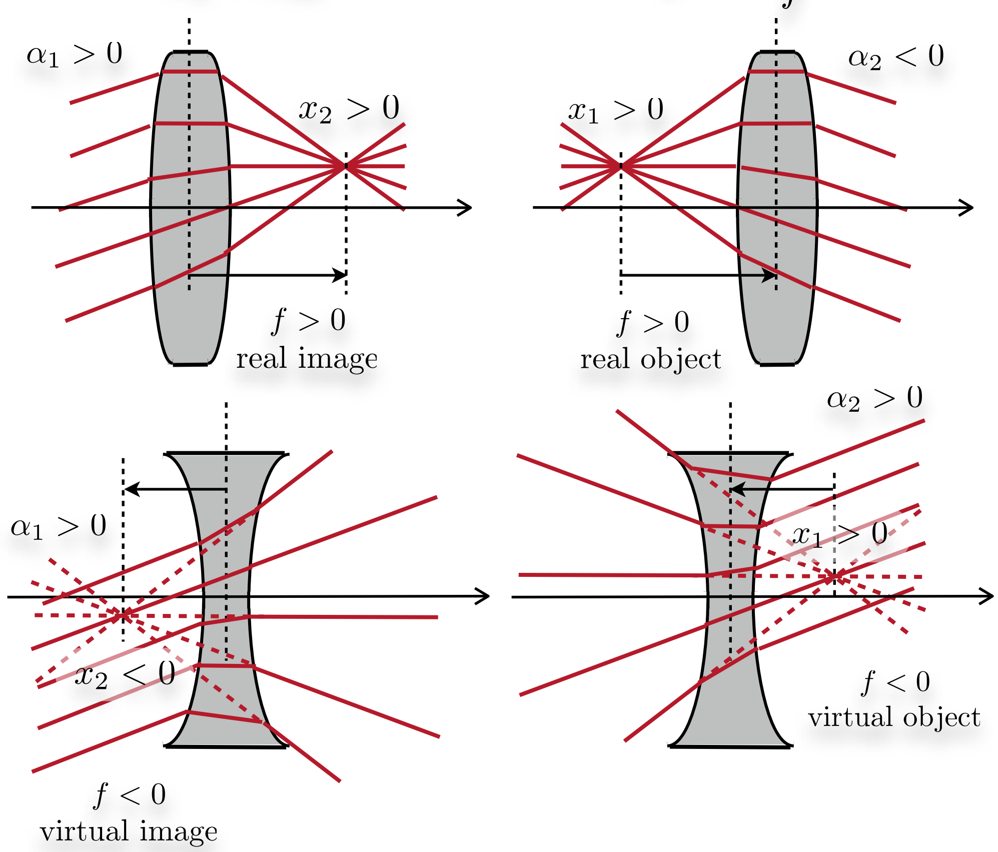
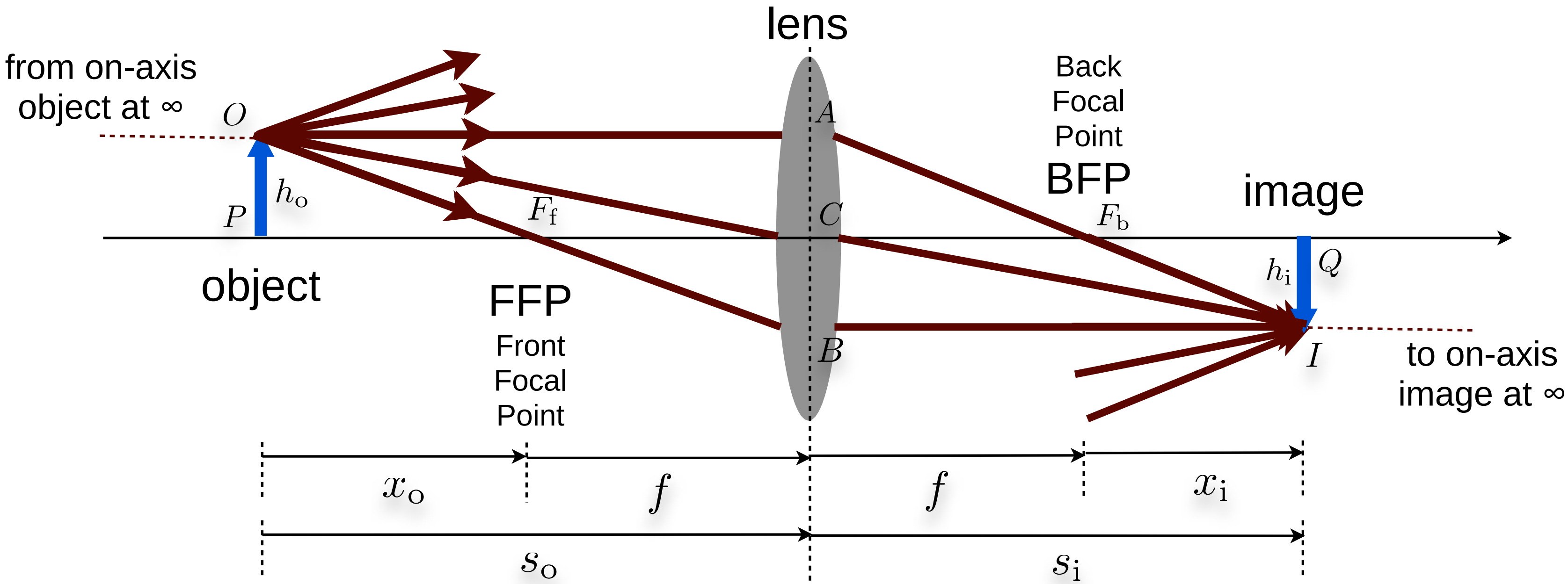


Image formation at finite distances



$$O\hat{P}F_\mathrm{f} \sim B\hat{C}F_\mathrm{f} \Rightarrow \frac{(PO)}{(PF_\mathrm{f})} = \frac{(CB)}{(F_\mathrm{f}C)} \Rightarrow \frac{h_\mathrm{o}}{x_\mathrm{o}} = \frac{-h_\mathrm{i}}{f} \qquad I\hat{Q}F_\mathrm{b} \sim A\hat{C}F_\mathrm{b} \Rightarrow \frac{(QI)}{(F_\mathrm{b}Q)} = \frac{(CA)}{(CF_\mathrm{f})} \Rightarrow \frac{-h_\mathrm{i}}{x_\mathrm{i}} = \frac{h_\mathrm{o}}{f}$$

$$\Rightarrow \boxed{\frac{h_\mathrm{i}}{h_\mathrm{o}} = -\frac{f}{x_\mathrm{o}} = -\frac{x_\mathrm{i}}{f}} \Rightarrow \boxed{x_\mathrm{o}x_\mathrm{i} = f^2}$$

(Newton's form)

$$M_\mathrm{T} \equiv \frac{h_\mathrm{i}}{h_\mathrm{o}}$$

Lateral magnification

Imaging condition

$$O\hat{A}C \sim I\hat{B}C \Rightarrow \frac{(CA)}{(CB)} = \frac{(OA)}{(CQ)} \Rightarrow \frac{h_\mathrm{o}}{-h_\mathrm{i}} = \frac{s_\mathrm{o}}{s_\mathrm{i}} \Rightarrow \boxed{\frac{h_\mathrm{i}}{h_\mathrm{o}} = -\frac{s_\mathrm{i}}{s_\mathrm{o}}} \quad \text{and} \quad \frac{s_\mathrm{i}}{s_\mathrm{o}} = \frac{f}{x_\mathrm{o}} = \frac{f}{s_\mathrm{o} - f} \Rightarrow \boxed{\frac{1}{s_\mathrm{o}} + \frac{1}{s_\mathrm{i}} = \frac{1}{f}}$$

Recall that, since OC is going thorough the optical center of the lens, the emerging ray CI propagates parallel to the incident direction, *i.e.* $OC||CI$.

Real and virtual images

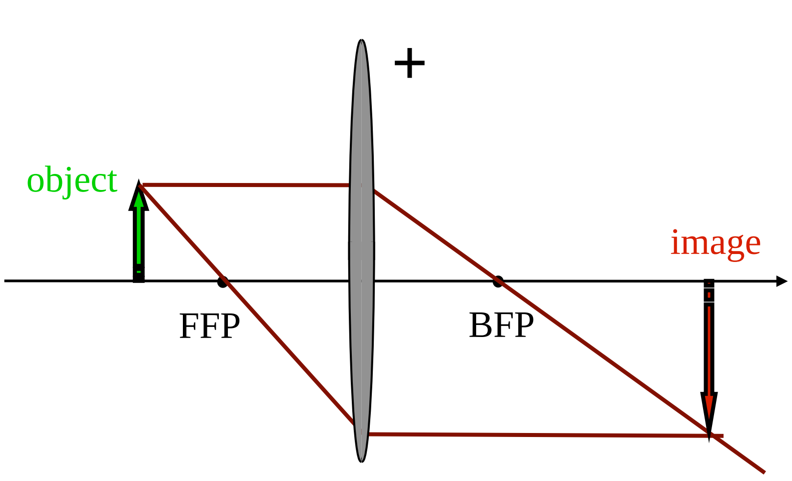


image: real & inverted; $M_T<0$

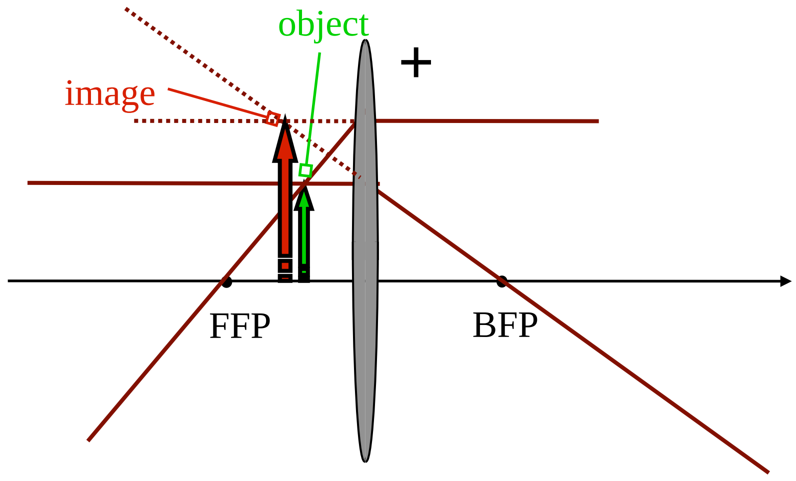


image: virtual & erect; $M_T>1$

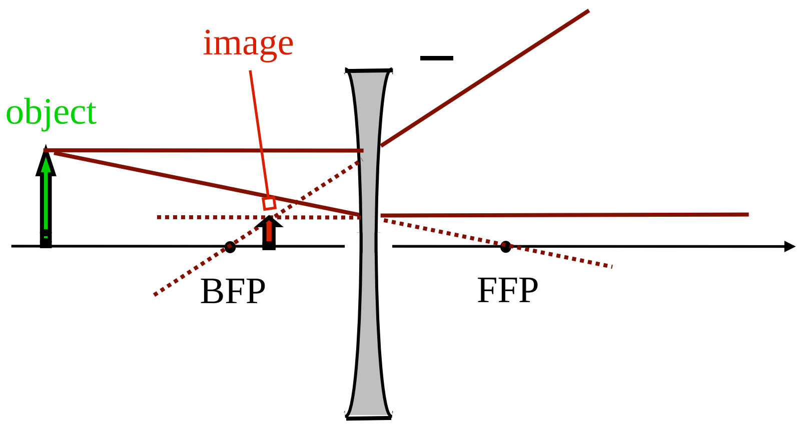


image: virtual & erect; $0<M_T<1$

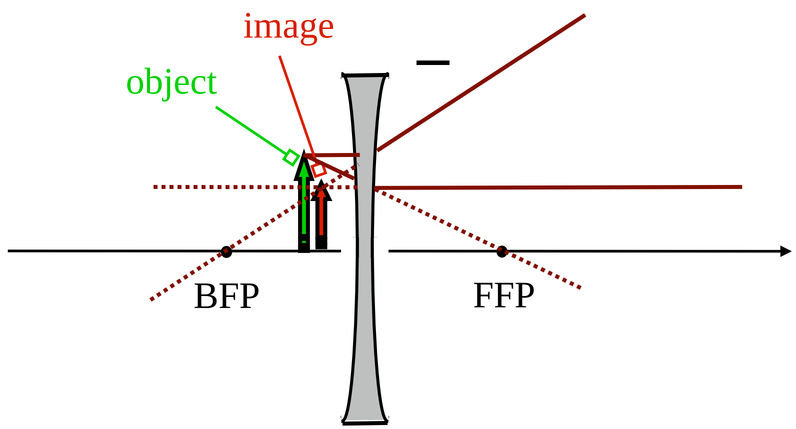
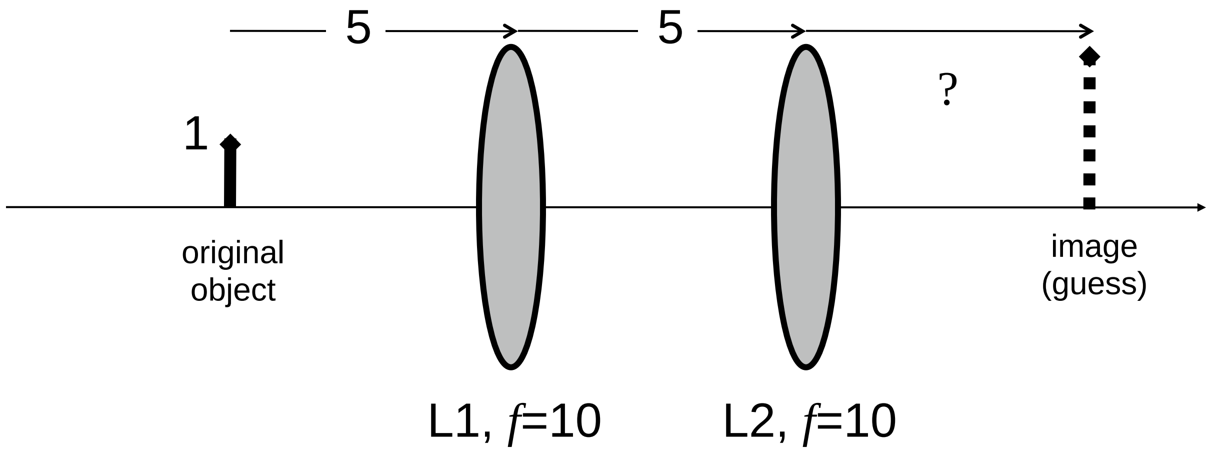


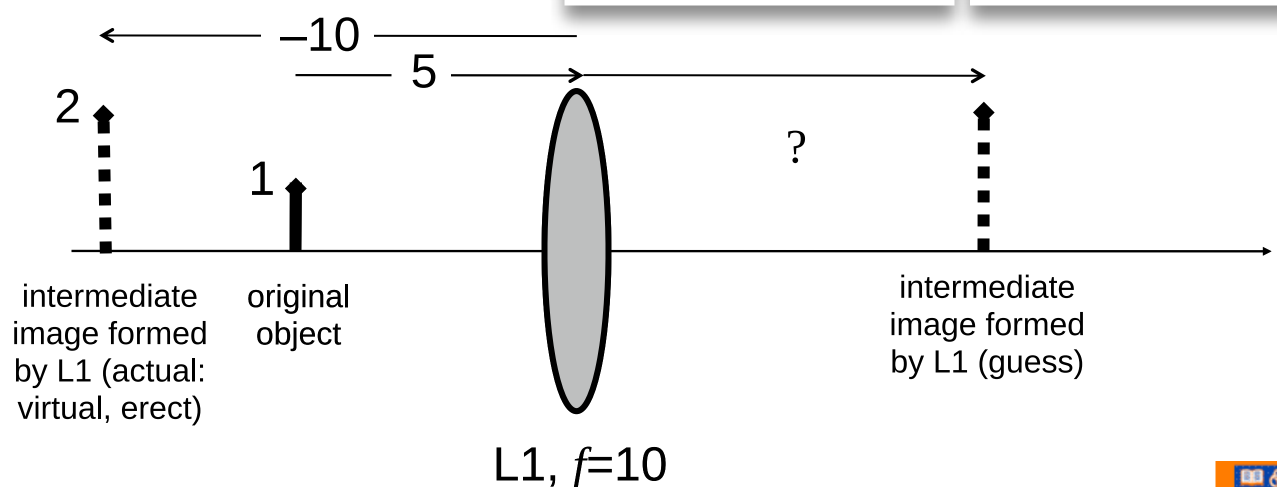
image: virtual & erect; $0<M_T<1$

Example: composite lens \1

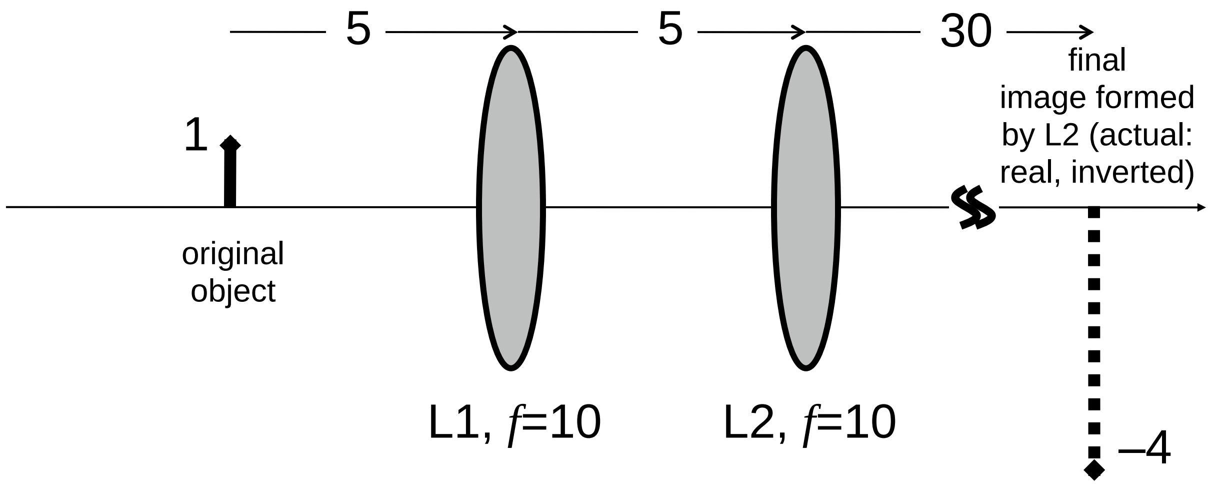


We seek the image location and lateral magnification for the composite lens imaging system shown above. We will solve the problem by repeated application of the imaging condition and lateral magnification relationships that we derived in Slide #2.

Begin by considering the first lens in isolation: $\frac{1}{5}+\frac{1}{s_1{}'}=\frac{1}{10} \Rightarrow s'=-10;$ $m_{1,l}=-\frac{s_1{}'}{5}=-\frac{-10}{5}=+2.$



Example: composite lens \2



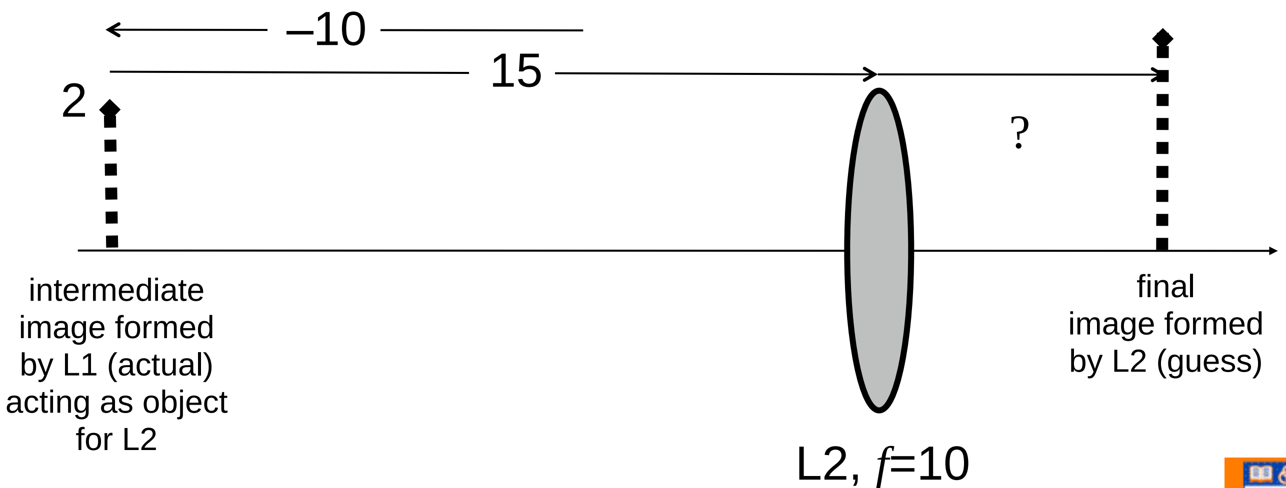
Next we consider L2 in isolation, with object identical to the image formed by L1 in isolation.

$$\frac{1}{15}+\frac{1}{s'}=\frac{1}{10}\Rightarrow s'=+30;$$

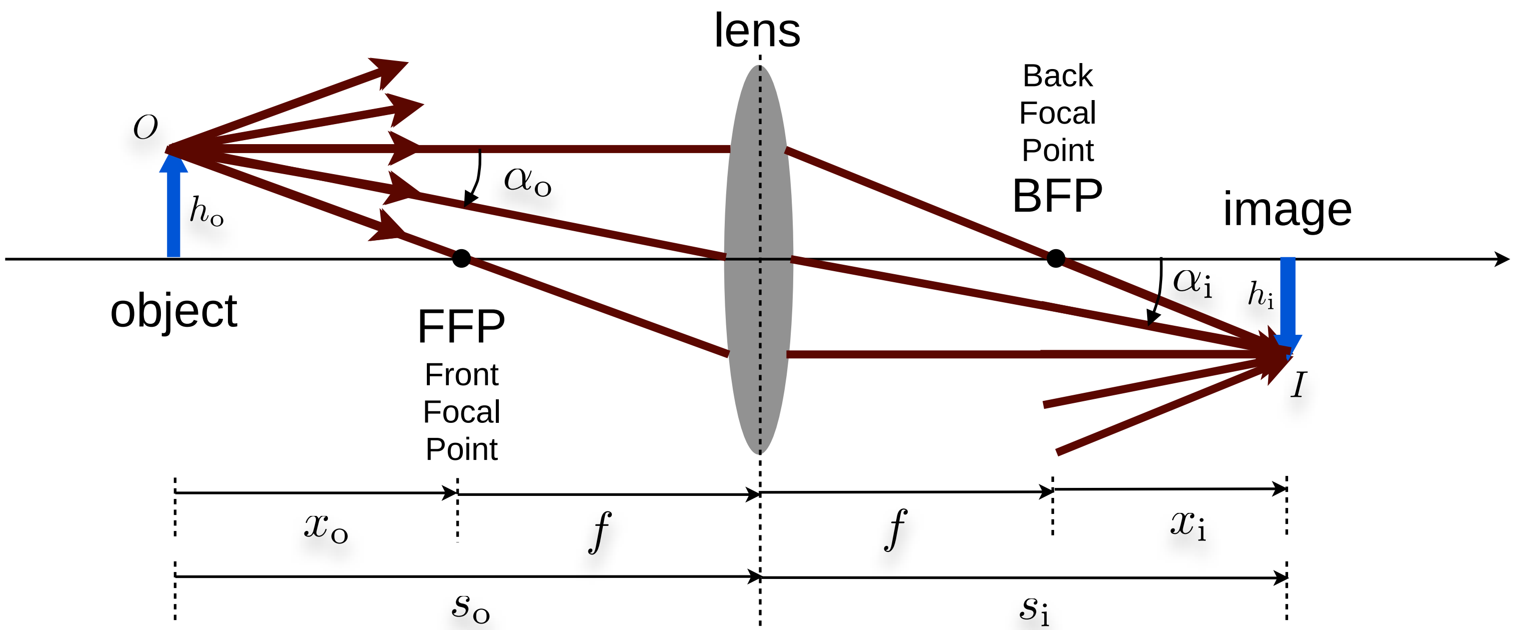
$$m_{2,l}=-\frac{s'}{15}=-\frac{+30}{15}=-2.$$

The overall (composite) magnification is

$$m_l=m_{1,l}m_{2,l}=(+2)\times(-2)=-4.$$



Imaging condition using ray transfer matrices



$$\begin{pmatrix}\alpha_i\\ h_i\end{pmatrix} = \begin{pmatrix}1 & 0\\ s_i & 1\end{pmatrix}\begin{pmatrix}1 & -\frac{1}{f}\\ 0 & 1\end{pmatrix}\begin{pmatrix}1 & 0\\ s_o & 1\end{pmatrix}\begin{pmatrix}\alpha_o\\ h_o\end{pmatrix}$$

$$= \begin{pmatrix}1-\frac{s_o}{f} & -\frac{1}{f}\\ s_i+s_o-\frac{s_i s_o}{f} & 1-\frac{s_i}{f}\end{pmatrix}\begin{pmatrix}\alpha_o\\ h_o\end{pmatrix}$$

$$\begin{pmatrix}\alpha_i\\ h_i\end{pmatrix} = \begin{pmatrix}1-\frac{s_o}{f} & -\frac{1}{f}\\ 0 & 1-\frac{s_i}{f}\end{pmatrix}\begin{pmatrix}\alpha_o\\ h_o\end{pmatrix}$$

$$= \begin{pmatrix}-\frac{x_o}{f} & -\frac{1}{f}\\ 0 & -\frac{x_i}{f}\end{pmatrix}\begin{pmatrix}\alpha_o\\ h_o\end{pmatrix}$$

Imaging object point O to image point I requires that *all* rays departing from O meet again at I, *independently* of ray departure angle α_o. Equivalently, we require that a divergent spherical wave originating from O must *focus*, *i.e.* converge at I.

Mathematically, this is expressed as

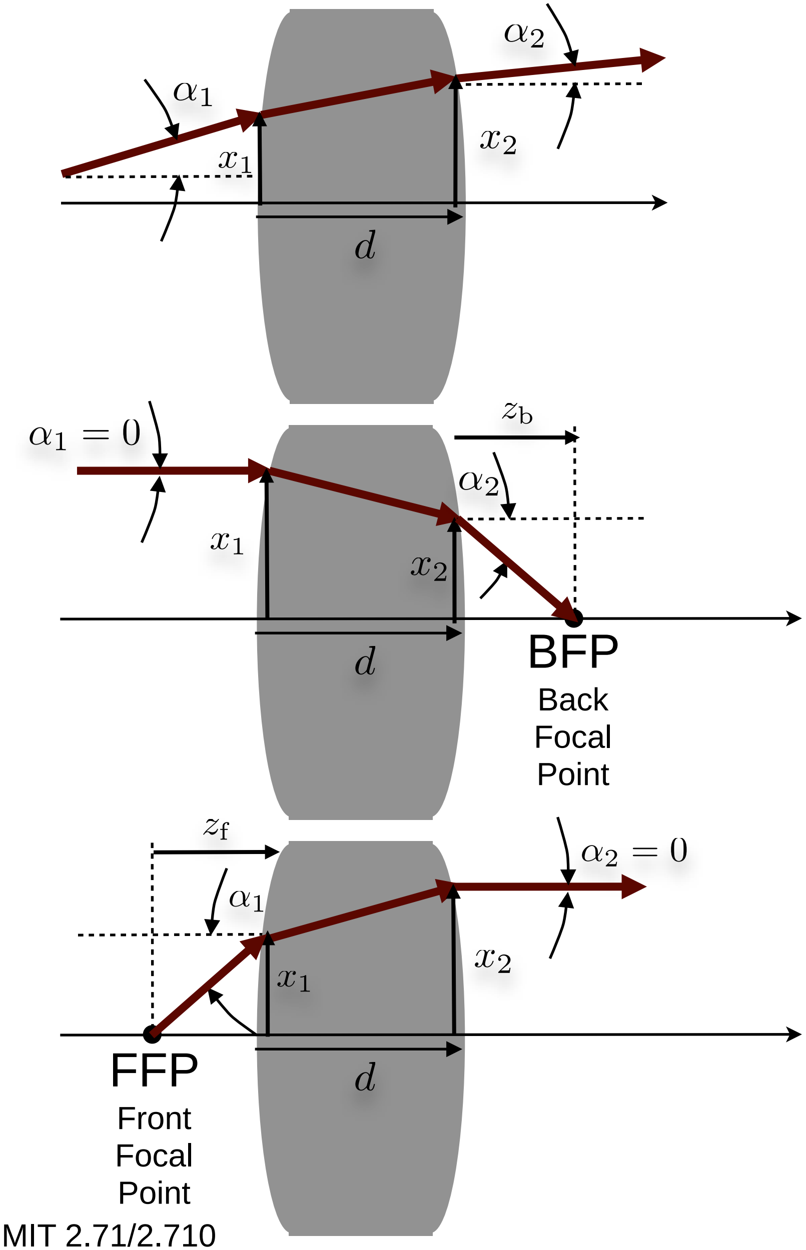
$$\frac{\partial h_i}{\partial \alpha_o} = 0 \Rightarrow s_i + s_o - \frac{s_i s_o}{f} = 0 \Rightarrow \boxed{\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f}}.$$

Imaging condition

$$M_T \equiv \frac{h_i}{h_o} = 1 - \frac{s_i}{f} = -\frac{x_i}{f}$$ consistent with Slide #2.

Angular magnification $$M_A \equiv \frac{\partial \alpha_i}{\partial \alpha_o} = 1 - \frac{s_o}{f} = -\frac{x_o}{f}.$$ Note $M_A = \frac{1}{M_T}$

Thick lens



$$\begin{pmatrix}\alpha_2\\ x_2\end{pmatrix} = \begin{pmatrix}1 & -\frac{1-n}{R_2}\\ 0 & 1\end{pmatrix}\begin{pmatrix}1 & 0\\ \frac{d}{n} & 1\end{pmatrix}\begin{pmatrix}1 & -\frac{n-1}{R_1}\\ 0 & 1\end{pmatrix}\begin{pmatrix}\alpha_1\\ x_1\end{pmatrix}$$

$$= \begin{pmatrix}1+\frac{n-1}{n}\frac{d}{R_2} & -\left[(n-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)+\frac{(n-1)^2 d}{nR_1R_2}\right]\\ \frac{d}{n} & 1-\frac{n-1}{n}\frac{d}{R_1}\end{pmatrix}\begin{pmatrix}\alpha_1\\ x_1\end{pmatrix}$$

We define $\quad \frac{1}{\mathrm{EFL}} \equiv (n-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)+\frac{(n-1)^2 d}{nR_1R_2}$

On–axis object at infinity $\Rightarrow \alpha_1 = 0$.
Where do the rays focus? *i.e.*, $z_b = ?\ : x_2 = 0$

$$\begin{pmatrix}\alpha_2\\ 0\end{pmatrix} = \begin{pmatrix}1 & 0\\ z_b & 1\end{pmatrix}\begin{pmatrix}1+\frac{n-1}{n}\frac{d}{R_2} & -\frac{1}{\mathrm{EFL}}\\ \frac{d}{n} & 1-\frac{n-1}{n}\frac{d}{R_1}\end{pmatrix}\begin{pmatrix}0\\ x_1\end{pmatrix} \Rightarrow$$

$$\begin{pmatrix}\alpha_2\\ 0\end{pmatrix} = \begin{pmatrix}-\frac{x_1}{\mathrm{EFL}}\\ x_1\left(-\frac{z_b}{\mathrm{EFL}}+1-\frac{n-1}{n}\frac{d}{R_1}\right)\end{pmatrix} \Rightarrow \begin{cases}\alpha_2 = -\frac{x_1}{\mathrm{EFL}}\\ z_b = (\mathrm{EFL})\left(1-\frac{n-1}{n}\frac{d}{R_1}\right)\end{cases}$$

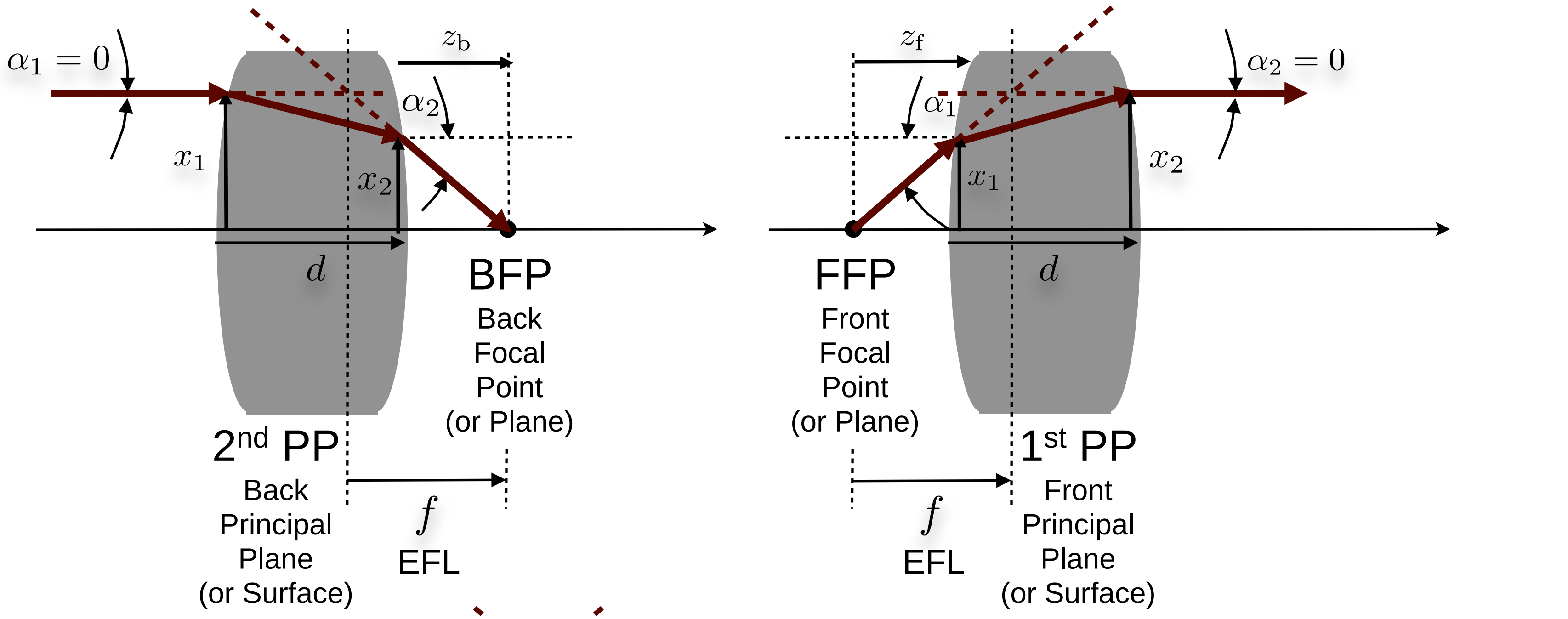
$$\Rightarrow \begin{cases}P \equiv \frac{1}{\mathrm{EFL}} & \text{is the } \textbf{power} \text{ of the thick lens;}\\ f \equiv (\mathrm{EFL}) & \text{is the } \textbf{effective focal length};\\ z_b \equiv (\mathrm{BFL}) & \text{is the } \textbf{back focal length}.\end{cases}$$

Similarly, by requiring an on–axis point object at finite distance z_f to produce an on–axis image at infinity, *i.e.* $\alpha_2 = 0$, we find

$$z_f \equiv (\mathrm{FFL}) = (\mathrm{EFL})\left(1+\frac{n-1}{n}\frac{d}{R_2}\right) \qquad x_2 = \alpha_1\,(\mathrm{EFL})$$

$z_f \equiv$ (BFL) is the **front focal length.**

Focal Lengths and Principal Planes



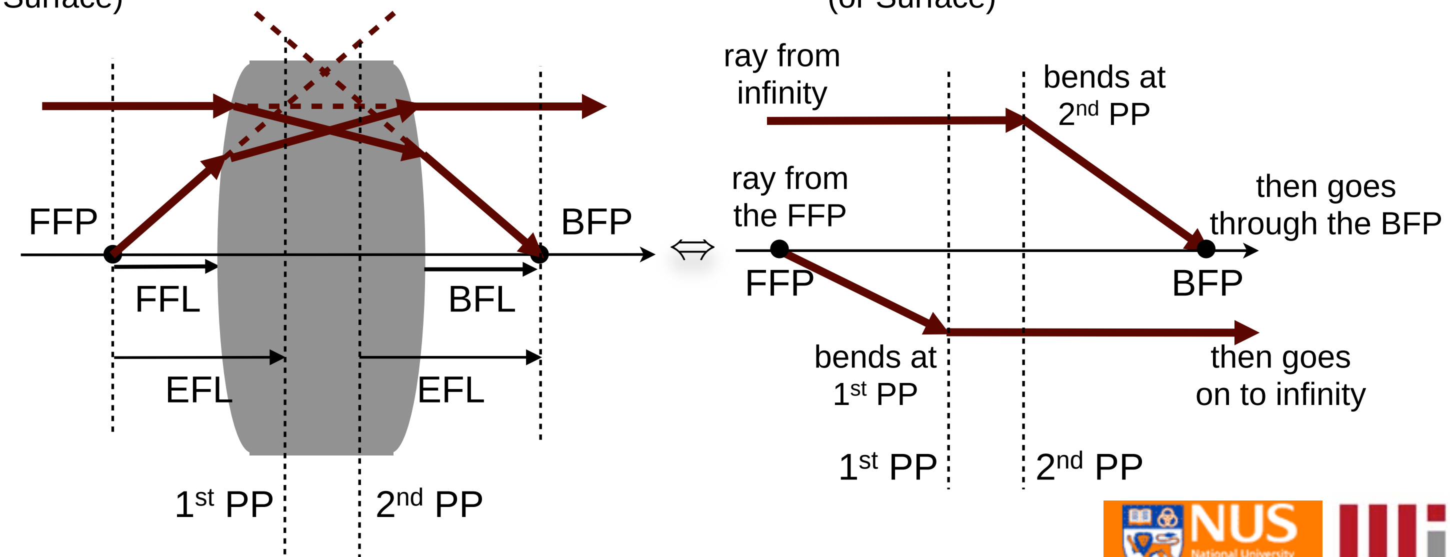
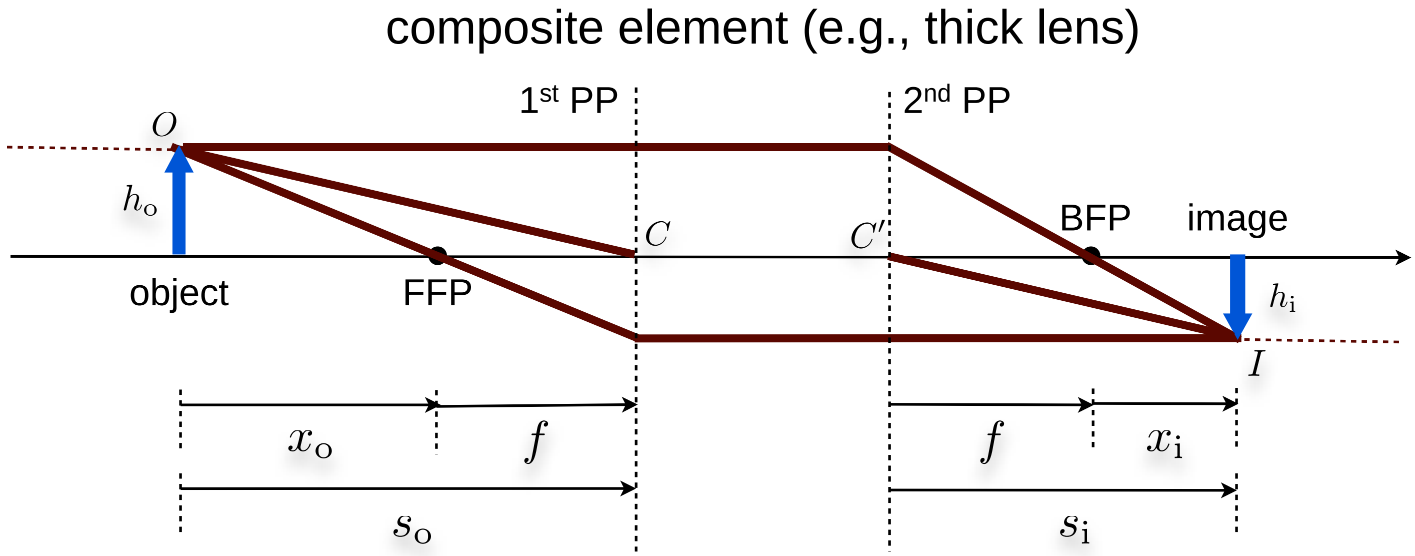


Image formation with composite elements



To find the imaging condition for the composite element we can use the principal planes as follows:

- trace an on-axis ray from infinity through O to the 2nd PP then bend so that it goes through the BFP;
- trace a ray from O through the FFP then bend at the 1st PP so that it goes to infinity on-axis;
- the intersection of the traced rays is the image point I;
- the ray from O through the intersection of the 1st PP with the optical axis should emerge at the intersection C of the 2nd PP with the optical axis and also go through the image point I; moreover, if the indices of refraction to the left and right of the composite are the same, then $OC \| C'I$.
- It is easy to see that the similar triangle arguments that we used in the case of the single thin lens apply here as well; therefore, the imaging condition and magnification relations remain the same with the notation as shown above.

$$x_o x_i = f^2; \qquad \frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f}; \qquad M_T = -\frac{x_i}{f} = -\frac{f}{x_o} = -\frac{s_i}{s_o}; \qquad M_A = \frac{1}{M_T}.$$

Imaging systems in nature: chambered eyes

Image removed due to copyright restrictions.
Please see Fig. 1 a,c,d,g in Fernald, Russell D. "Casting a Genetic Light on the Evolution of Eyes." *Science* 313 (2006): 1914-1918.

Imaging systems in nature: compound eyes

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Please see Fig. 1 b, e, f, h in Fernald, Russell D.
"Casting a Genetic Light on the Evolution of Eyes." *Science*
313 (2006): 1914-1918.

The human eye

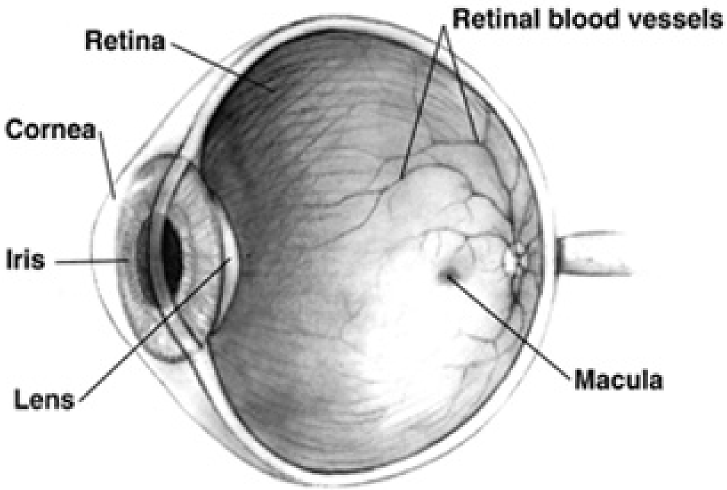
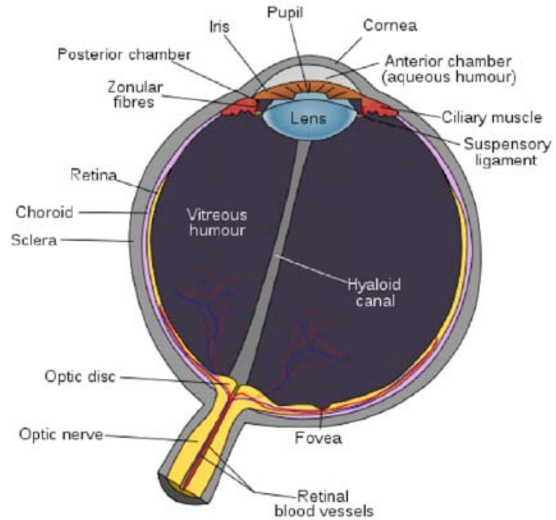


Image courtesy of NIH National Eye Institute.



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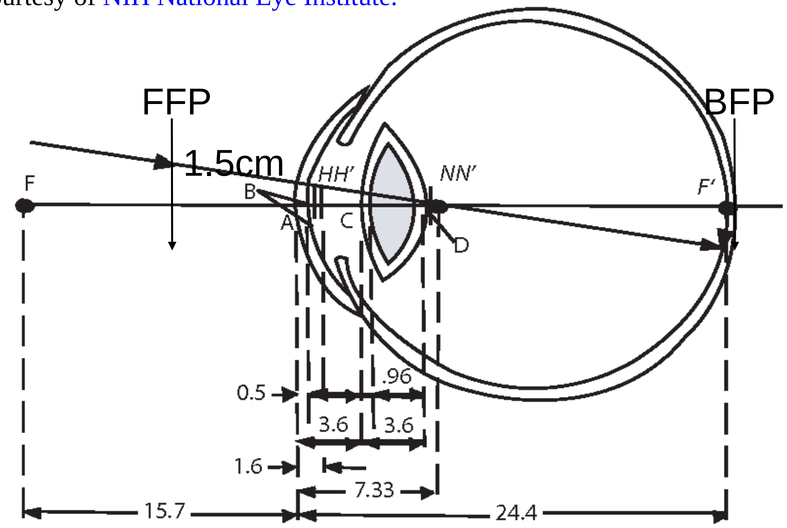
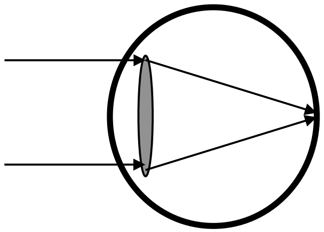
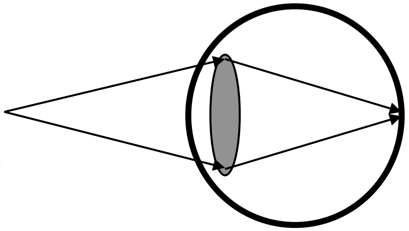


FIGURE 10B
Schematic eye as developed by Gullstrand, showingthe real andinverted image on the retina (dimensions are inmillimeters).

Remote objects:
unaccommodated eye
(lens muscles relaxed)



Nearby objects:
accommodated eye
(lens muscles contracted)



from *Fundamentals of Optics* by F. Jenkins & H. White

Eye defects and their correction

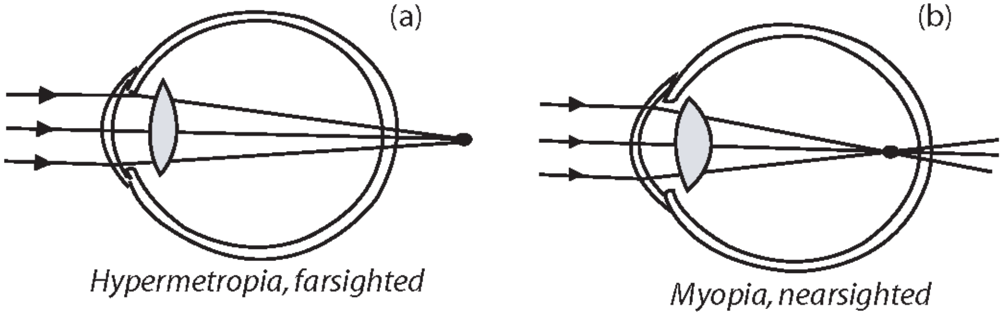


FIGURE 10K
Typical eye defects largely present in the adult population.

from *Fundamentals of Optics*
by F. Jenkins & H. White

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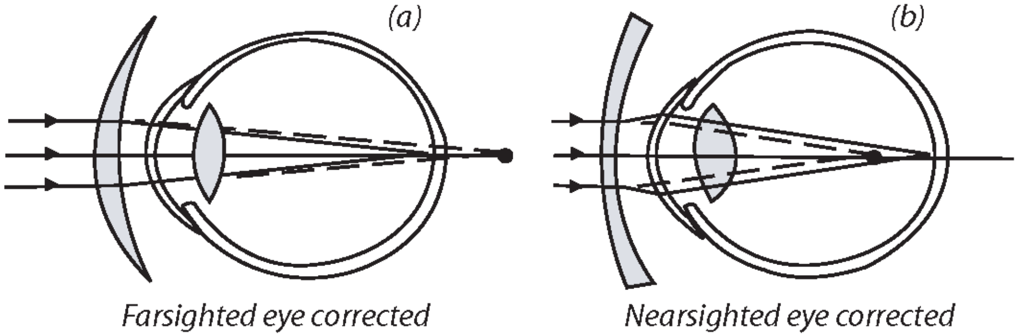
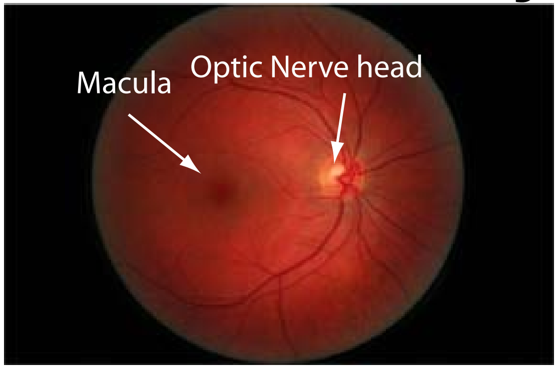


FIGURE 10L
Typical eye defects can be corrected by spectacle lenses.

The eye's "digital camera": retina



Retina

Image by Danny Hope at Wikimedia Commons.

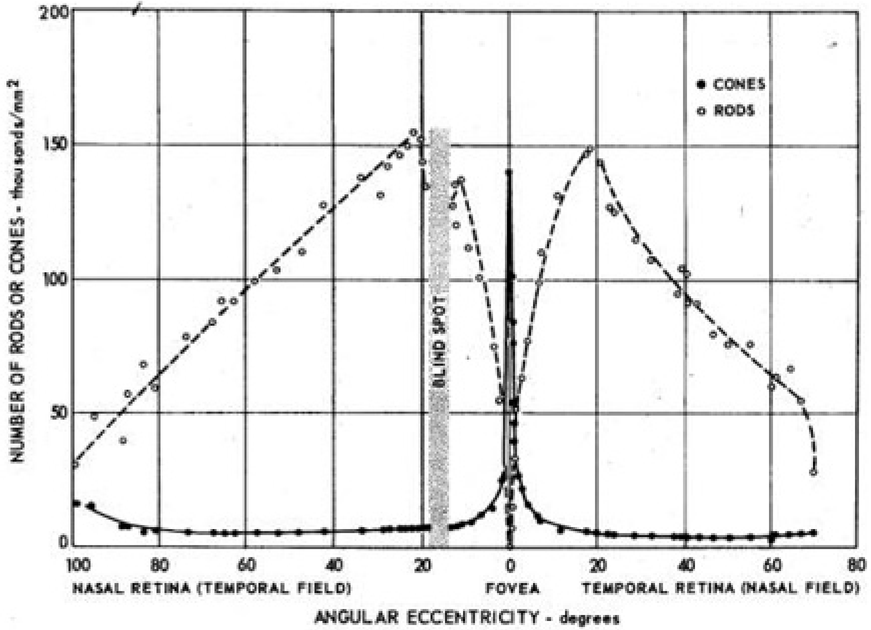
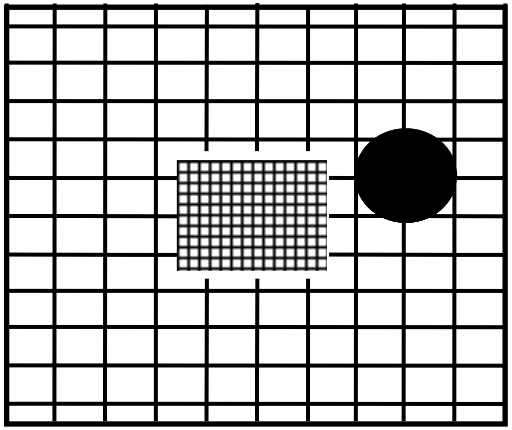
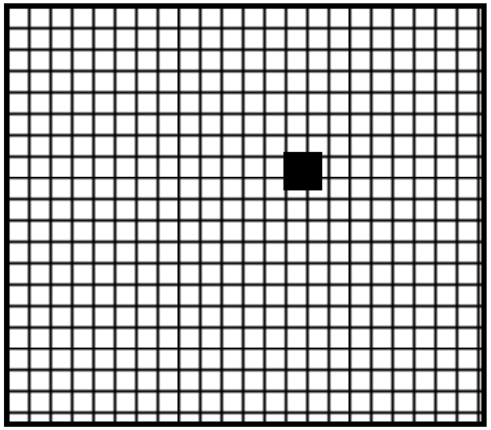


Image by the NRC Committee on Undersea Warfare.

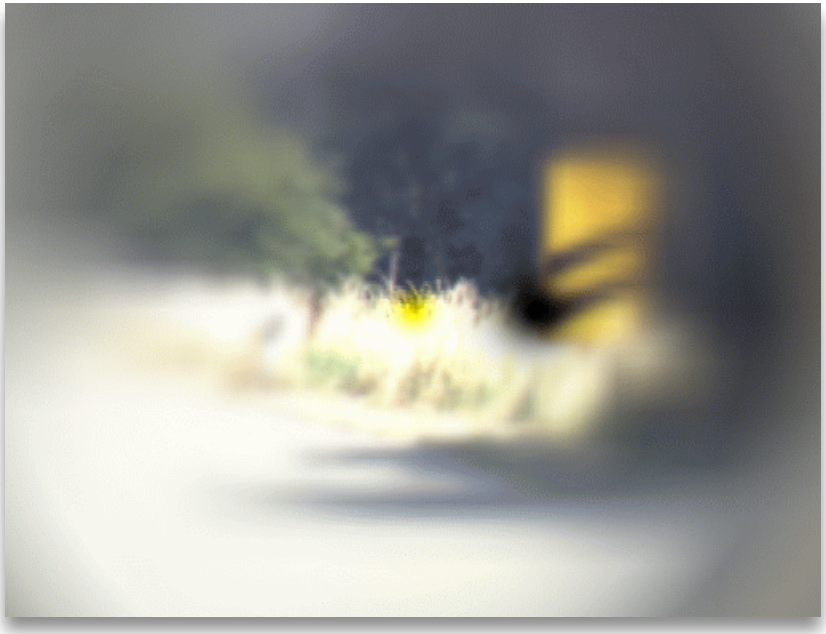
Retina:
- ➡ variable sampling rate
- ➡ "dead pixels" (fovea) are compensated for





Digital camera:
- ➡ fixed sampling rate
- ➡ "dead pixels" (fovea) are noticeable

Retina vs your digital camera

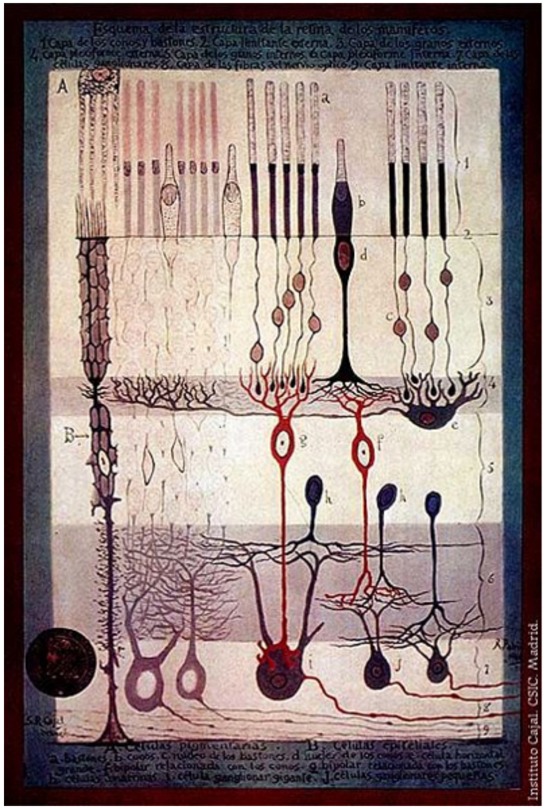


Retinal image

CCD image

http://www.klab.caltech.edu/~itti/

Spatial response of the retina – lateral connections



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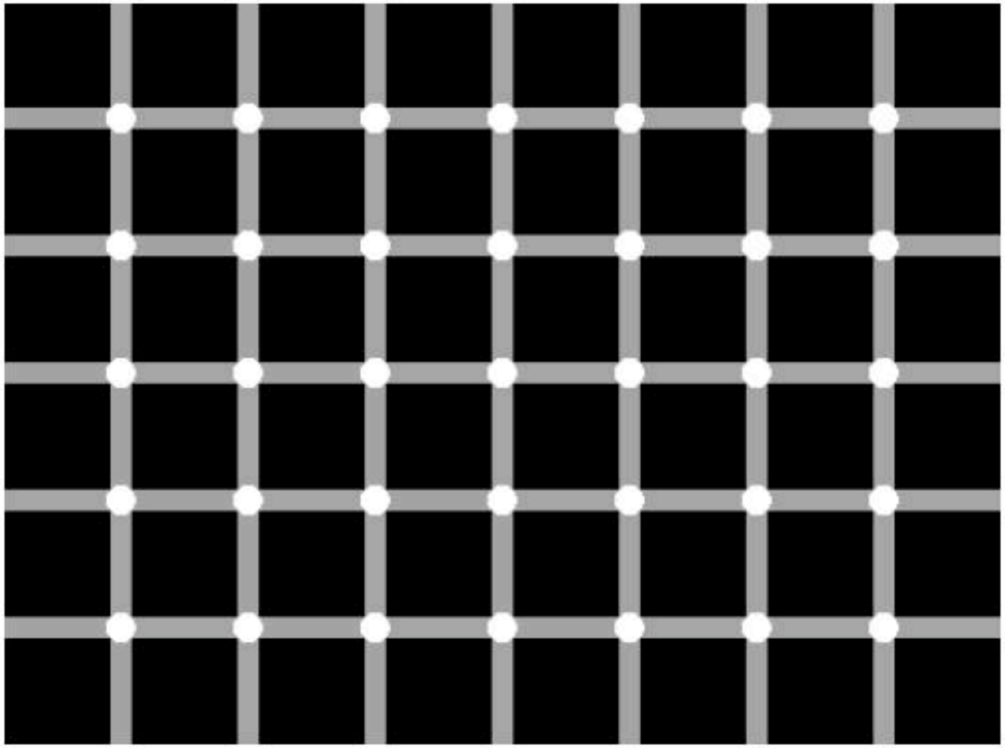
Please see http://williamcalvin.com/bk4/bk4.htm

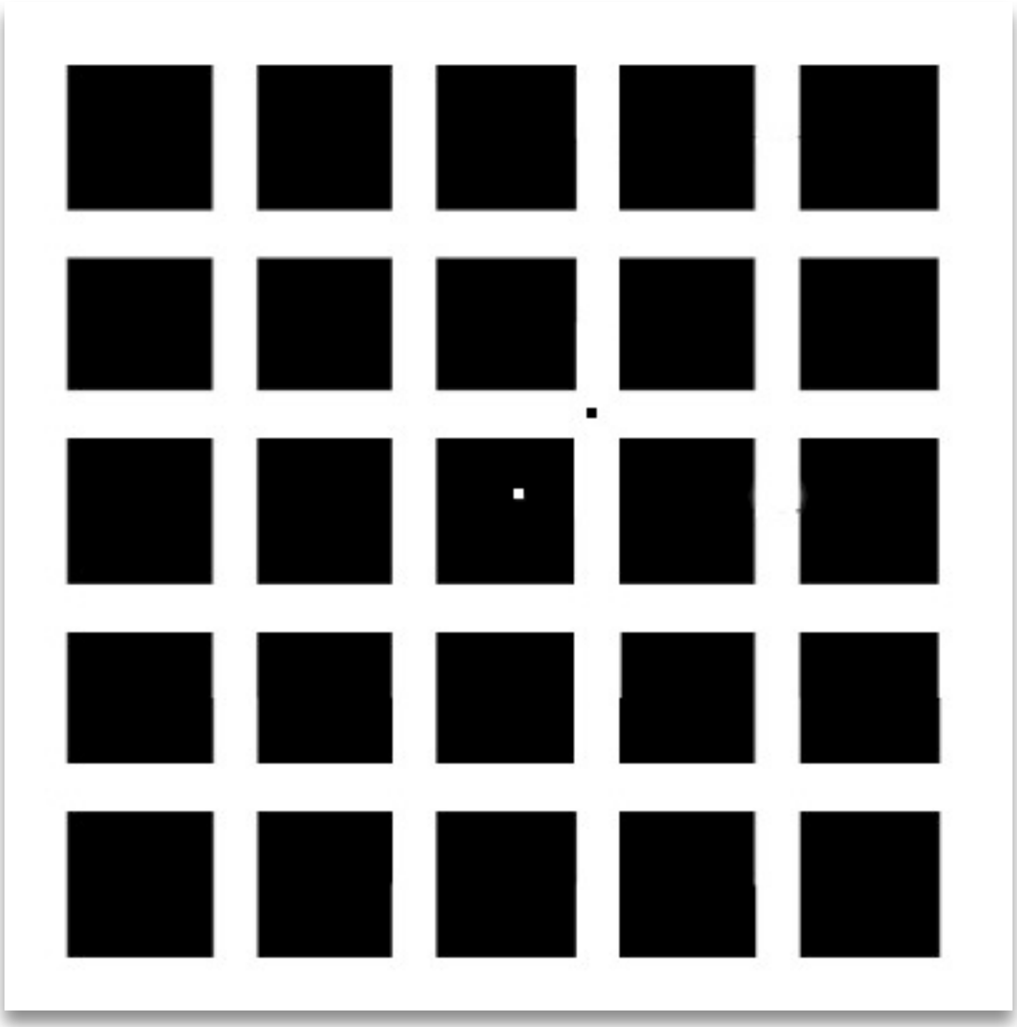
Image from Ramón y Cajal, Santiago.
"Structure of the Mammalian Retina." Madrid, 1900.

What do you see?



http://www.phys.ufl.edu/~avery/

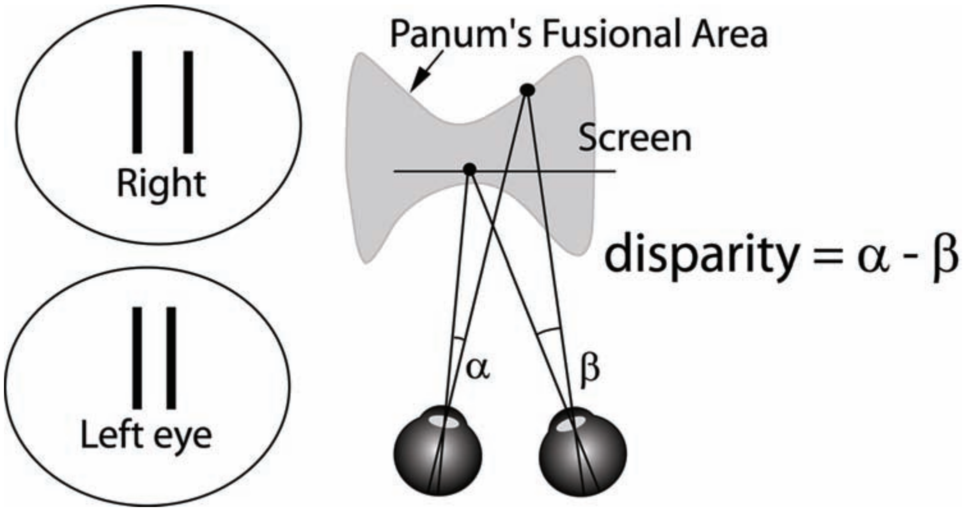
Temporal response: after-images



Courtesy of David T. Landrigan. Used with permission.

http://dragon.uml.edu/psych/

Seeing 3D: binocular vision



http://www.ccom.unh.edu/vislab/VisCourse

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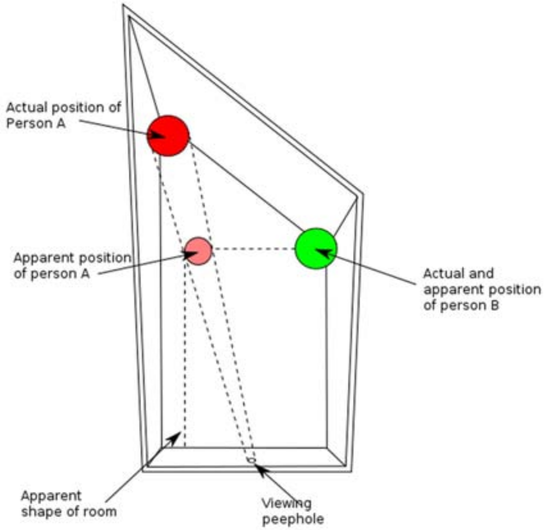

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