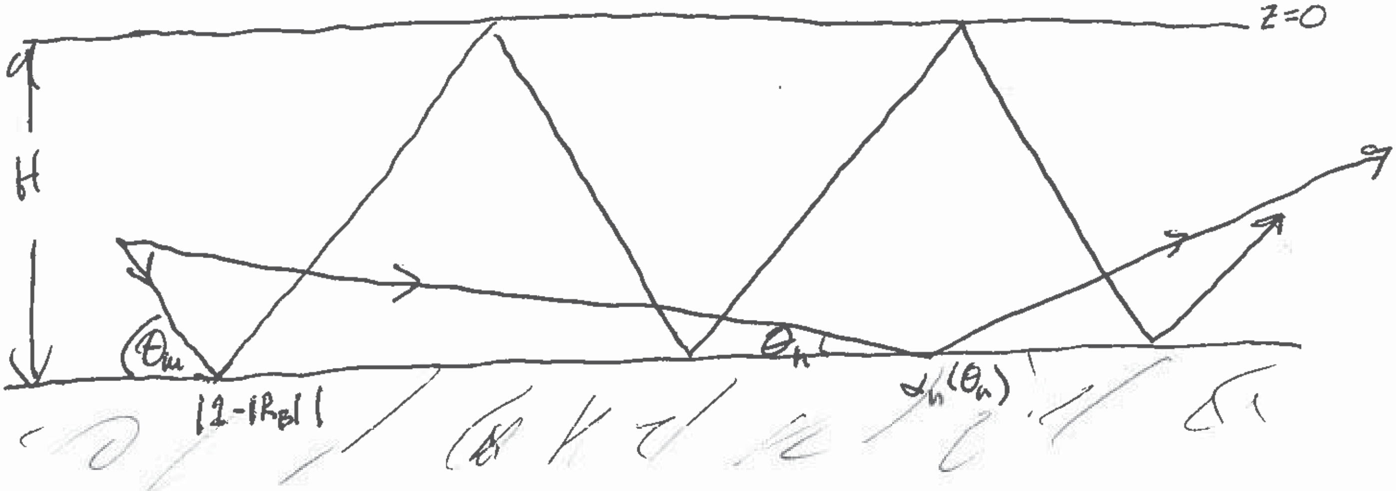
# Problem Set #3. "Mode Madness"

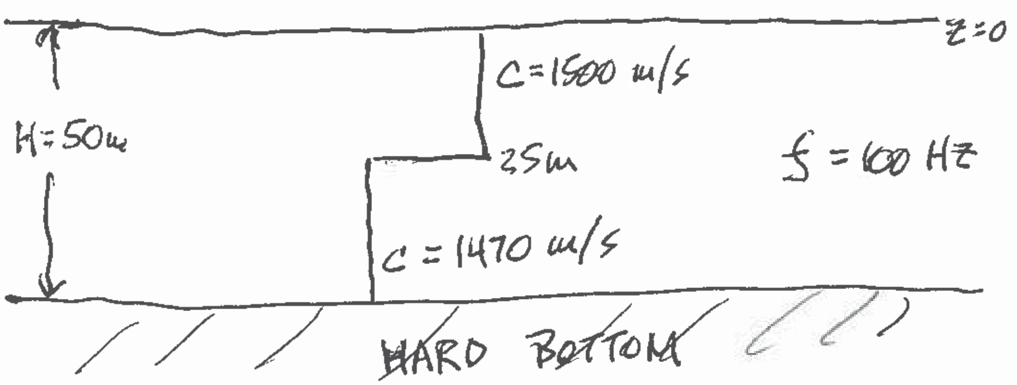
1) Hard Bottom Waveguide. (Easy) Consider a 50m deep, 1500 m/s water column overlying a hard ocean bottom. Also, consider a source at 25m depth, transmitting a 100 Hz CW tone.
    a. Generate the waveguide eigenvalues and mode functions.
    b. What are the phase velocities and group velocities of the hard bottom modes?
    c. What is the cutoff frequency of each of the modes?
    d. What are the mode cycle distances of all of the mode pairs?
    e. What are the eigenangles (equibalent ray angles) for each of the modes?
2) Pekeris waveguide. For the same setup as in problem #1, now consider a Pekeris waveguide, i.e. the bottom becomes an isovelocity halfspace with $c_B = 1700$ m/s, $\rho_B = 1.7$ g/cc (typical of sand). Also, in doing this problem, please compare your results to the hard bottom case (Problem 31) where appropriate.
    a. Generate the trapped and continuum (virtual) waveguide eigenvalues and mode functions.
    b. What are the mode cutoff frequencies?
    c. What are the mode cycle distances for all mode pairs?
    d. What are the trapped and virtual mode eigenangles?
    e. Using Eq. 5.165 of Frisk, what is the frequency of the Airy phase for the trapped modes? (Numerical calculations might be easier here!)
    f. For receivers at 15 and 35m depth, calculate $|p(r; z, z_0|$ from 1m out to 10 km. Use dB units. Can you easily see the cycle distance effects in the interference pattern?

$\boxed{P3}$



3) Attenuation in range. For the mode amplitudes $A_n$ (trapped and virtual), we can estimate their attenuation in range due to the bottom by considering the loss per bottom reflection per unit range. Consider the diagram above, where the mode n loses a small amount $\alpha_n(\theta_n)$ at each reflection in the trapped mode region and virtual mode m loses a (assumed) small amount $1 - |R_B|$ at each reflection past critical angle. Using the simple geometry in the Figure P3, formulate and solve the simple differential equation for $A_n(r)$ for each case. Is this a useful result?

P4



4) Numerical mode codes. Consider the hard bottom waveguide in Figure P4. Using either shooting or finite difference methods, find the mode eigenvalues and eigenvectors. (10 points extra credit if you do both!)
5) Wavenumber integral. For the hard bottom waveguide and source in Problem #1, generate the Green's function (magnitude and phase) using Eq. 6.189 of Frisk (page 173 in the notes copy). A noteworthy feature of the Green function is the phase shift of π at a zero or a resonance. What causes this???

6) Adiabatic modes. A classic ocean acoustics experiment was the so-called "Scrimger experiment" in which a motorboat created a small wake wave that propagated across an acoustic transmission range (see Figure P6).

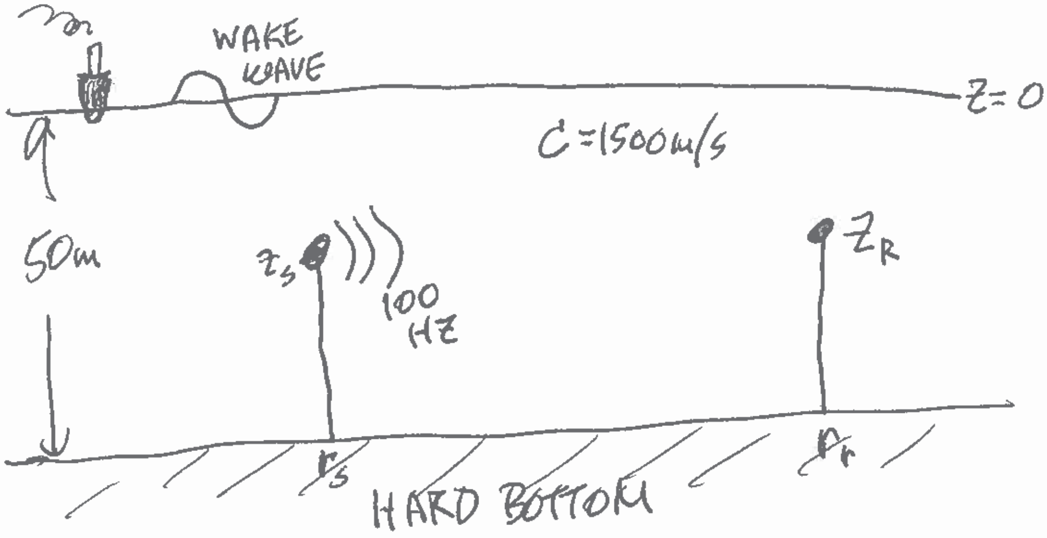
   Consider a single full cycle sine wave wake of A=1m, $\lambda$ =10m, a source-receiver distance of 1 km, and (yet again) the simple hard bottom waveguide from Problem #1. The wake crosses over the source position, transits the middle ergion, and then crosses the receiver position. Using the adiabatic approximation to the normal mode solution, describe:

   a. The variation in the acoustic normal mode phases , travel times, and amplitudes as the wave propagates through the three regimes described above.
7) Coupled modes. Consider the same waveguide and boat wake as in Problem 6.
   a. Using the class notes on "sound field in a range dependent waveguide", particularly Eqs. 3.19 and 3.23, comment on whether the waveguide is adiabatic or coupled when the wake is in it.
   b. Show how the mode travel times (including coupled) vary as the wake travels across the acoustic range. (Since the wake is small in extent compared to the S/R distance, take it as a point scatterer for this question.)

EXTRA CREDIT (5 points for 1.2, and 1 point for each right prediction in 3)

1) Show the steps and geometry used in going from the 2D Fourier transfor to the Hankel transfor,
2) Show all the steps (in detail) in deriving the fully coupled mode equations.
3) Before 3/19 noon, predict the Men's and Women's NCAA Basketball tournament Final four and National Champion.

P6



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