Electrical Overview

Ref: Woud 2.3

N.B. this is a long note and repeats much of what is is the text

$I = \frac{Q}{t}$ $\quad$ $Q = \text{charge}$ $\quad$ $C = 1\,C$ $\quad$ $C = 1\,\text{coul}$ (2.50)

$t = \text{time}$ $\quad$ $\text{min} = 60\,s$ $\quad$ $s = 1\,s$

$I = \text{current}$ $\quad$ $A = 1\,A$

work done per unit charge = potential difference two points $\quad$ $U = \text{volts}$ $\quad$ $V = 1\,V$
aka electromotive force (EMF)

$$\text{Power} = U(t)\cdot I(t) \qquad 1V\cdot 1A = 1\,W \qquad 1V\cdot 1A = 1\,\text{watt} \qquad (2.51)$$

source, resistance, inductance, capacitance

resistance

$\text{resistance} = R$ $\quad$ $\Omega = 1\,\Omega$ $\quad$ $\text{ohm} = 1\,\Omega$ $\quad$ friction in mechanical system

Ohm's law $\quad$ $U(t) = I(t)\cdot R$ (2.52)

power in a resistor ... $\quad$ $\text{Power} = U(t)\cdot I(t) = I(t)^2\cdot R$ $\quad$ $1\Omega\cdot(1A)^2 = 1\,W$ (2.53)

inductance

mass of inertia in mechanical system

$\text{inductance} = L$ $\quad$ $H = 1\,H$ $\quad$ $\text{henry} = 1\,H$

$$U(t) = L\cdot\frac{d}{dt}I(t) \qquad H\cdot\frac{A}{s} = 1\,V \qquad \text{or ...} \qquad I(t) = \int_0^t \frac{U(t)}{L}\,dt \qquad \frac{V\cdot s}{H} = 1\,A \qquad (2.54)$$

$$P = U\cdot I = L\cdot I\cdot\left(\frac{d}{dt}I\right) \qquad H\cdot A\cdot\frac{A}{s} = 1\,W \qquad (2.55)$$

$$\text{inductive\_energy\_stored} = E_{ind} = \int_0^t P(t)\,dt = \int_0^t L\cdot I\cdot\left(\frac{d}{dt}I\right)dt = \int_0^I L\cdot I\,dI \qquad \int_0^I L\cdot I\,dI \rightarrow \frac{1}{2}\cdot I^2\cdot L \qquad (2.56)$$

$$A^2\cdot H = 1\,J$$

capacitance

spring in mechanical system

$\text{capacitance} = C$ $\quad$ $F = 1\,F$ $\quad$ $\text{farad} = 1\,F$

$$I(t) = C\cdot\frac{d}{dt}U(t) \qquad F\cdot\frac{V}{s} = 1\,A \qquad \text{or ...} \qquad U(t) = \int_0^t \frac{I(t)}{C}\,dt \qquad \frac{A\cdot s}{F} = 1\,V \qquad (2.57)$$

$$P = U\cdot I = C\cdot U\cdot\frac{d}{dt}U(t) \qquad F\cdot V\cdot\frac{V}{s} = 1\,W$$

$$\text{capacitive\_energy\_stored} = E_{cap} = \int_0^t P(t)\,dt = \int_0^t C\cdot U\cdot\frac{d}{dt}U(t)\,dt = \int_0^U C\cdot U\,dU \qquad \int_0^U C\cdot U\,dU \rightarrow \frac{1}{2}\cdot U^2\cdot C \qquad (2.58), (2.59)$$

$$V^2\cdot F = 1\,J$$

Kirchhoff's laws

first ...

$$\text{sum\_of\_currents\_towards\_node} = 0 \qquad \sum_{i=1}^{\text{number\_of\_currents}} \left[I_i(t)\right] = 0 \qquad (2.60)$$

second ...

$$\text{sum\_of\_voltages\_around\_closed\_path} = 0$$

direction specified

$$\sum_{i=1}^{\text{number\_of\_voltages}} \left[U_i(t)\right] = 0 \qquad (2.61)$$

series connection of resistance and inductance ...

imposed ... external $U(t) := U_m \cdot \cos(\omega \cdot t)$ $\quad U_m = \text{amplitude\_of\_voltage}$ $\quad V = 1\,V$ (2.62)

$\omega = \text{frequency}$ $\quad Hz = 1\frac{1}{s}$

$t = \text{time}$ $\quad \min = 60\,s$

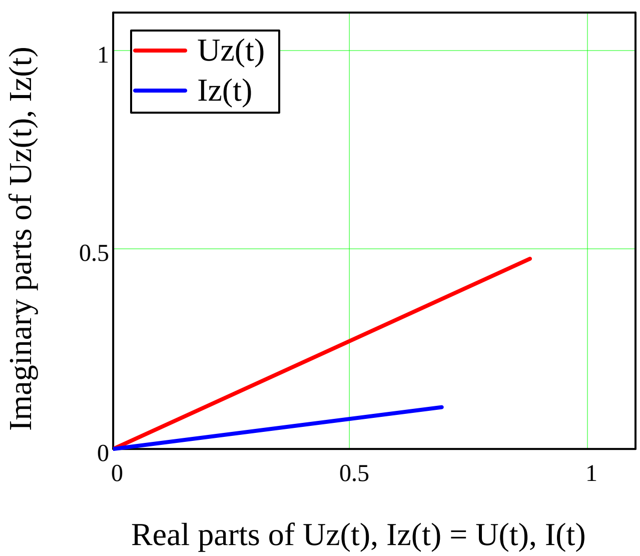
resulting current assumed also harmonic $I(t) := I_m \cdot \cos(\omega \cdot t - \phi)$ $\quad I_m = \text{amplitude\_of\_current}$ $\quad A = 1\,\text{amp}$ (2.63)

$\phi = \text{phase\_lag\_angle}$

it is useful to represent this parameters as vectors using complex notation, where the values are represented by the real parts

$$Uz(t) := U_m \cdot \cos(\omega \cdot t) + U_m \cdot \sin(\omega \cdot t) \cdot i \qquad Iz(t) := I_m \cdot \cos(\omega \cdot t - \phi) + I_m \cdot \cos(\omega \cdot t - \phi) \cdot i$$

plotting set up



over R voltage drop will be ... $U_R(t) := R \cdot I(t) \rightarrow R \cdot I_m \cdot \cos\left[(-\omega) \cdot t + \phi\right]$ $\quad U_R(t) = R \cdot I_m \cdot \cos(\omega \cdot t - \phi)$ $\quad \cos(\alpha) = \cos(-\alpha)$

over L voltage drop will be ... $U_L(t) := L \cdot \frac{d}{dt} I(t) \rightarrow L \cdot I_m \cdot \sin\left[(-\omega) \cdot t + \phi\right] \cdot \omega$

$$L \cdot \frac{d}{dt} I(t) = -I_m \cdot \omega \cdot L \cdot \sin(\omega \cdot t - \phi) = I_m \cdot \omega \cdot L \cdot \cos\left(\frac{\pi}{2} + \omega \cdot t - \phi\right)$$

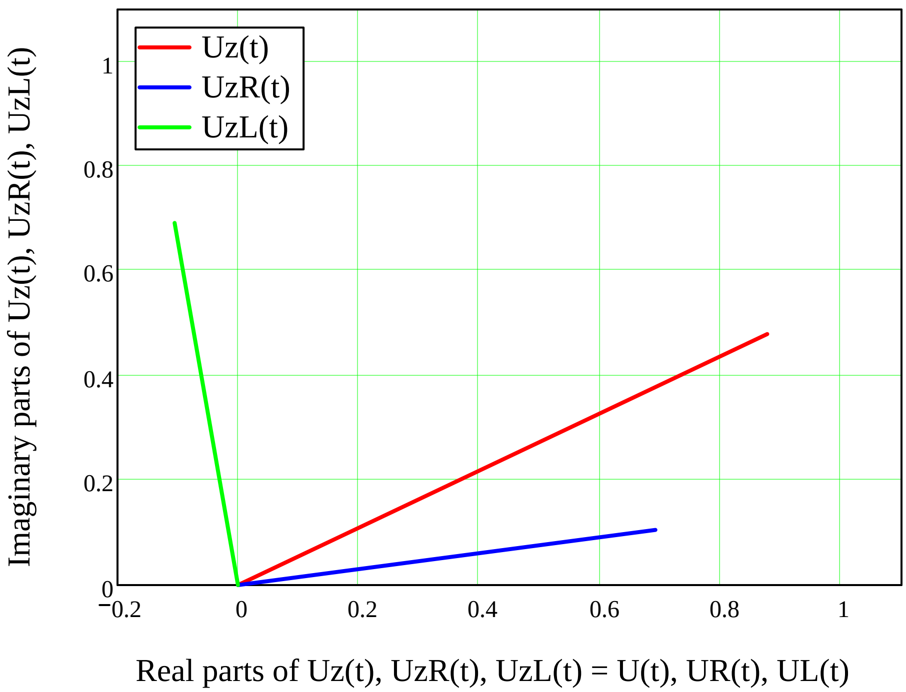
$\cos\left(\frac{\pi}{2}+\alpha\right) \rightarrow -\sin(\alpha)$ or ... $\cos\left(\frac{\pi}{2}+\alpha\right) = \cos\left(\frac{\pi}{2}\right)\cdot\cos(\alpha) - \sin\left(\frac{\pi}{2}\right)\cdot\sin(\alpha) = 0\cdot\cos(\alpha) - 1\cdot\sin(\alpha)$

in complex (vector) notation ...

$$Uz_R(t) := R\cdot I_m\cdot\cos(\omega\cdot t - \phi) + R\cdot I_m\cdot\sin(\omega\cdot t - \phi)\cdot i$$

$$Uz_L(t) := I_m\cdot\omega\cdot L\cdot\cos\left(\frac{\pi}{2} + \omega\cdot t - \phi\right) + I_m\cdot\omega\cdot L\cdot\sin\left(\frac{\pi}{2} + \omega\cdot t - \phi\right)\cdot i$$

plotting set up



at this point these vectors are shown with two unknowns included I_m and ϕ

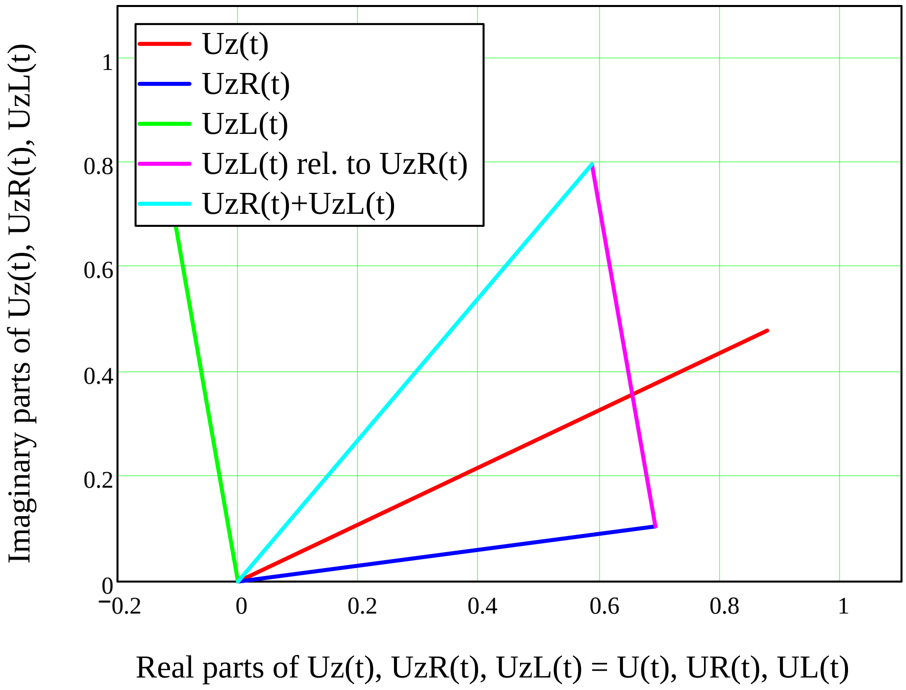
i.e. directions are correct relatively given ϕ and magnitudes arbitrary given I_m

Kirchoff's second law ...

$$U(t) := U_R(t) + U_L(t) \rightarrow R\cdot I_m\cdot\cos[(-\omega)\cdot t + \phi] + L\cdot I_m\cdot\sin[(-\omega)\cdot t + \phi]\cdot\omega$$

$$U_m\cdot\cos(\omega\cdot t) = R\cdot I_m\cdot\cos(\omega\cdot t - \phi) + L\cdot I_m\cdot\omega\cdot\cos\left(\frac{\pi}{2} + \omega\cdot t - \phi\right)$$

this can be solved for ϕ and I_m after expanding the rhs into sines and cosines and setting cos = cos and sin = sin
easier if think in terms of vectors



for UzR(t) + zL(t) to = Uz(t) magnitude and angle must be =

$Uz(t) \rightarrow U_m \cdot \cos(\omega \cdot t) + i \cdot U_m \cdot \sin(\omega \cdot t)$

$$U_m = \sqrt{\left(R \cdot I_m\right)^2 + \left(L \cdot I_m \cdot \omega\right)^2} = \sqrt{R^2 + (L \cdot \omega)^2} \cdot I_m$$

$Uz_R(t) \rightarrow R \cdot I_m \cdot \cos[(-\omega) \cdot t + \phi] - i \cdot R \cdot I_m \cdot \sin[(-\omega) \cdot t +$

$Uz_L(t) \rightarrow L \cdot I_m \cdot \sin[(-\omega) \cdot t + \phi] \cdot \omega + i \cdot I_m \cdot \omega \cdot L \cdot \cos[(-\omega) \cdot t + \phi$

or ...

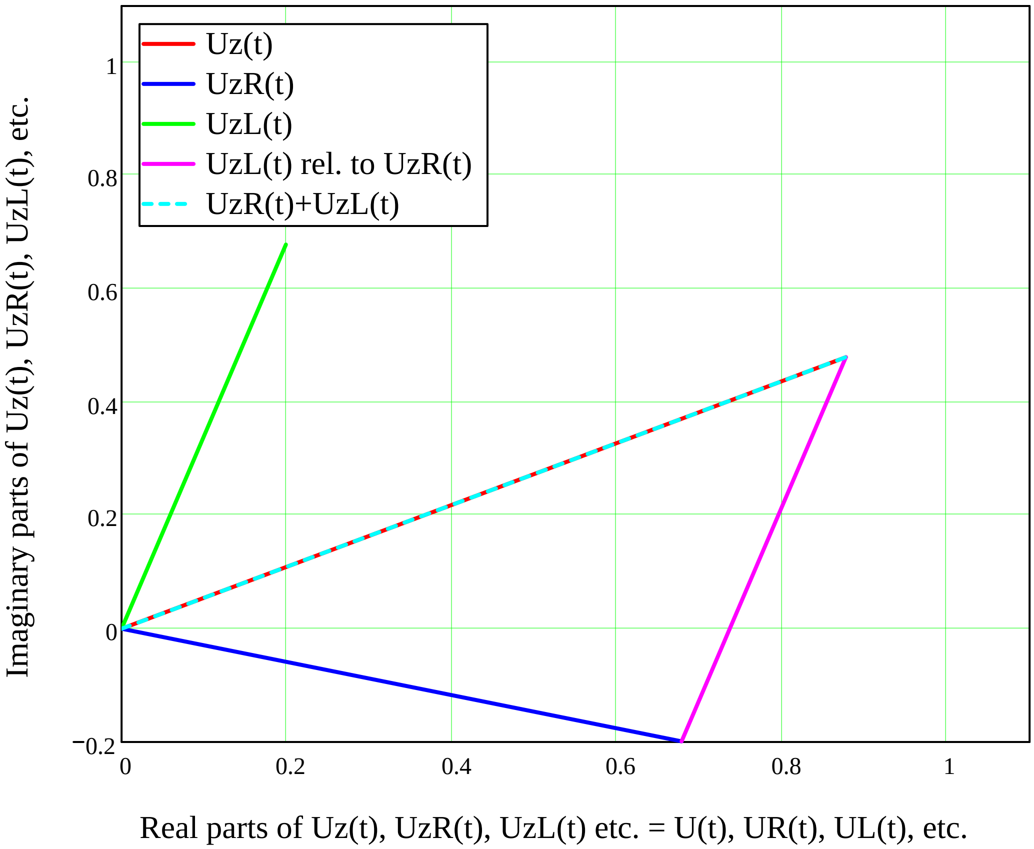
$$I_m = \frac{U_m}{\sqrt{R^2 + (L \cdot \omega)^2}}$$

and ...

$$\phi = \operatorname{atan}\left(\frac{L \cdot \omega}{R}\right)$$

using these relationships in the plot ...

plotting set up



N.B. angle may not appear as right angle due to scales
ϕ shown as lag (positive value with negative sign)

capacitor lead approach (text)

similar for Capacitance

imposed ... external $U(t) := U_m \cdot \cos(\omega \cdot t)$ U_m = amplitude_of_voltage $V = 1\,V$ (2.62)

ω = frequency $Hz = 1\frac{1}{s}$ $min = 60\,s$

t = time

this is different from text: lag phase angle vs. lead angle used

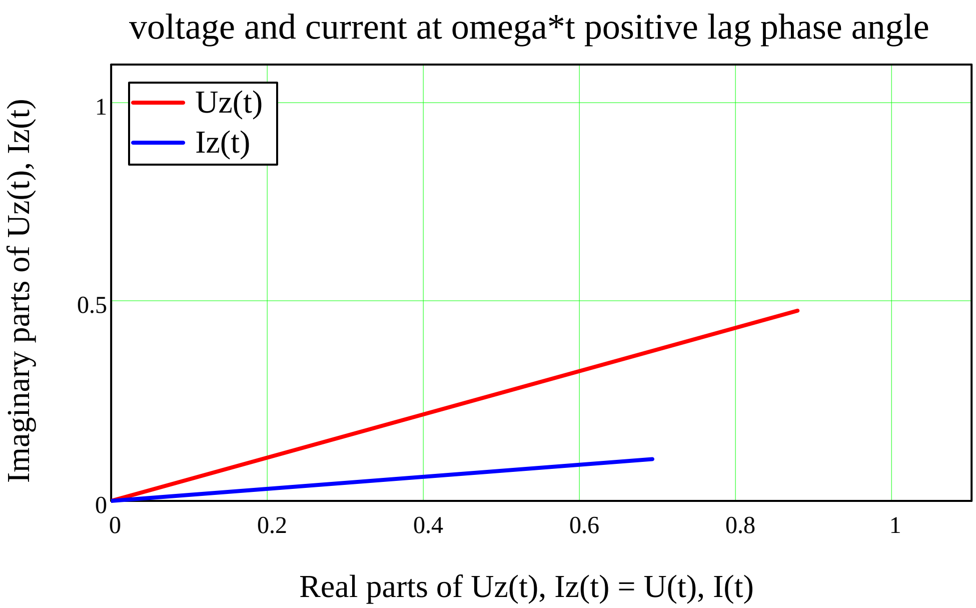
resulting current assumed also harmonic $I(t) := I_m \cdot \cos(\omega \cdot t - \phi)$ I_m = amplitude_of_current $V = 1\,V$

current assumed to have ***lag*** angle. this approach taken to allow common treatment of L and C in circuits ϕ = phase_lag_angle

complex (vector) representation, set up with real part expressed as cos

$Uz(t) = U_m \cdot \cos(\omega \cdot t) + U_m \cdot \sin(\omega \cdot t) \cdot i$ $Iz(t) = I_m \cdot \cos(\omega \cdot t - \phi) + I_m \cdot \sin(\omega \cdot t - \phi) \cdot i$

plotting set up



voltage across capacitor (from above)

(2.57) $$U_C(t) = \int_0^t \frac{I(t)}{C}\,dt = \int_0^t \frac{I_m\cdot\cos(\omega\cdot t-\phi)}{C}\,dt = \frac{I_m\cdot\sin(\omega\cdot t-\phi)}{C\cdot\omega} = \frac{I_m}{C\cdot\omega}\cdot\cos\left(\omega\cdot t-\phi-\frac{\pi}{2}\right)$$

using complex (vector) notation

$$Uz(t) := U_m\cdot\cos(\omega\cdot t) + U_m\cdot\sin(\omega\cdot t)\cdot i$$

$$Iz(t) := I_m\cdot\cos(\omega\cdot t-\phi) + I_m\cdot\cos(\omega\cdot t-\phi)\cdot i$$

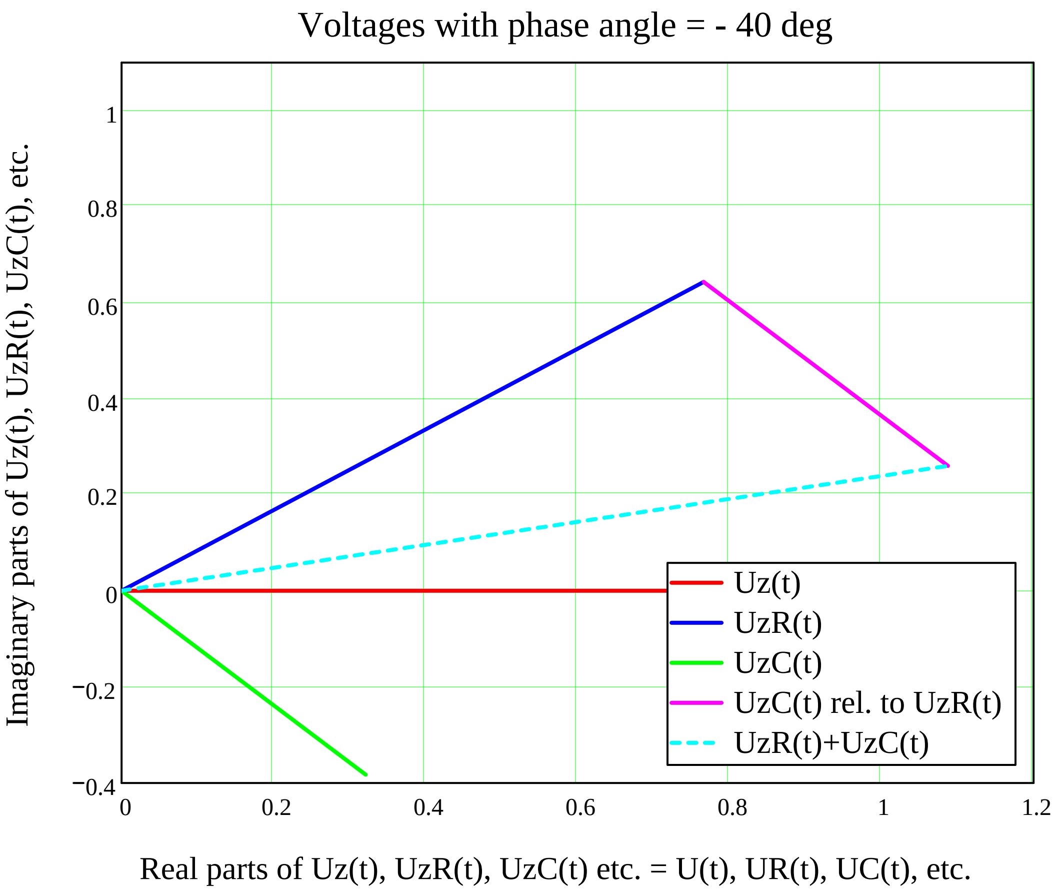
$$Uz_C(t) := \frac{I_m}{C\cdot\omega}\cdot\cos\left(\omega\cdot t-\phi-\frac{\pi}{2}\right) + \frac{I_m}{C\cdot\omega}\cdot\sin\left(\omega\cdot t-\phi-\frac{\pi}{2}\right)\cdot i$$

$$Uz_R(t) := R\cdot I_m\cdot\cos(\omega\cdot t-\phi) + R\cdot I_m\cdot\sin(\omega\cdot t-\phi)\cdot i$$

Kirchoff's second law for resistor with capacitor...

$$Uz(t) := Uz_R(t) + Uz_C(t) \rightarrow \Omega\cdot I_m\cdot\cos[(-\omega)\cdot t+\phi] - i\cdot\Omega\cdot I_m\cdot\sin[(-\omega)\cdot t+\phi] - \frac{I_m}{C\cdot\omega}\cdot\sin[(-\omega)\cdot t+\phi] - i\cdot\frac{I_m}{C\cdot\omega}\cdot\cos[(-\omega)\cdot t+\phi]$$

plotting set up

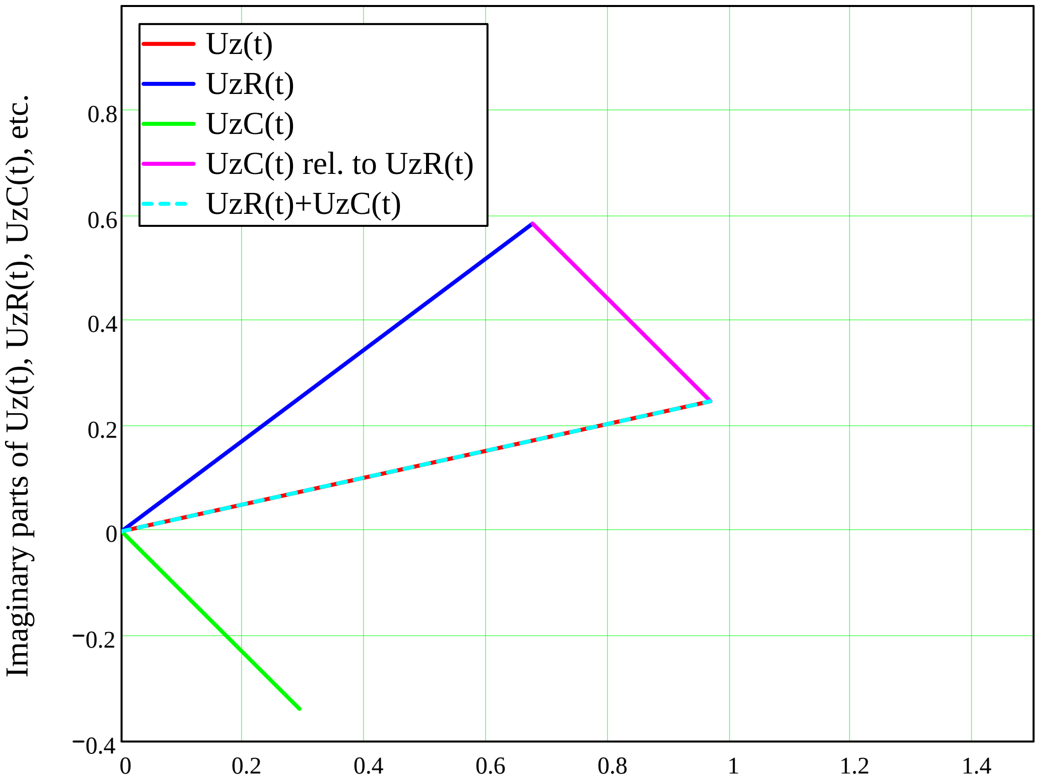


N.B. angle is distorted due to scales; angle between UzR(t) and UzC(t) is $\pi/2$)

$$Uz(t) = U_m \cdot \cos(\omega \cdot t) + U_m \cdot \sin(\omega \cdot t) = R \cdot I_m \cdot \cos(\omega \cdot t - \phi) + i \cdot R \cdot I_m \cdot \sin(\omega \cdot t - \phi) + \frac{I_m}{C \cdot \omega} \cdot \sin(\omega \cdot t - \phi) - i \cdot \frac{I_m}{C \cdot \omega} \cdot \cos(\omega \cdot t - \phi)$$

look at solution plotted

plotting set up



Real parts of Uz(t), UzR(t), UzC(t) etc. = U(t), UR(t), UC(t), etc.

$$Uz(t) = U_m \cdot \cos(\omega \cdot t) + U_m \cdot \sin(\omega \cdot t) = R \cdot I_m \cdot \cos(\omega \cdot t - \phi) + i \cdot R \cdot I_m \cdot \sin(\omega \cdot t - \phi) + \frac{I_m}{C \cdot \omega} \cdot \sin(\omega \cdot t + \phi) - i \cdot \frac{I_m}{C \cdot \omega} \cdot \cos(\omega \cdot t + \phi)$$

magnitude similar to above ...

$$I_m = \frac{U_m}{\sqrt{R^2 + \frac{1}{(\omega \cdot C)^2}}}$$

phase angle is negative; hence referred to as ***lead*** angle

$$\phi = -\text{atan}\left(\frac{1}{\omega \cdot C \cdot R}\right)$$

N.B. angle may not appear as right angle due to scales

ϕ shown as ***lead*** (negative value with negative sign) $\phi 1 = -26.565\ \text{deg}$ in this numerical example

so with both L and C

$$\phi = \text{atan}\left(\frac{\omega \cdot L}{R} - \frac{1}{\omega \cdot C \cdot R}\right)$$

Section 2.3.4

Direct current (DC) $P = U\cdot\ = I^2\cdot R$

Single phase alternating current (AC) $P(t) = U(t)\cdot I(t)$

typically sinusoidal ... $U(t) := U_m\cdot\cos(\omega\cdot t)$ $I(t) := I_m\cdot\cos(\omega\cdot t - \phi)$ lag phase angle used

average power ... $P_a := \lim_{T\to\infty}\left(\frac{1}{T}\cdot\int_0^T U(t)\cdot I(t)\,dt\right) \to \frac{1}{2}\cdot U_m\cdot I_m\cdot\cos(\phi)$

in practice effective values are used

$$U_e := \lim_{T\to\infty}\sqrt{\frac{1}{T}\cdot\int_0^T \left(U_m\cdot\cos(\omega\cdot t)\right)^2 dt} \to \frac{1}{2}\cdot 2^{\frac{1}{2}}\cdot\left(U_m^{\,2}\right)^{\frac{1}{2}} \qquad U_e := \frac{U_m}{\sqrt{2}} \qquad U_e = effective\_voltage$$

$$I_e := \lim_{T\to\infty}\sqrt{\frac{1}{T}\cdot\int_0^T \left(I_m\cdot\cos(\omega\cdot t - \phi)\right)^2 dt} \to \frac{1}{2}\cdot 2^{\frac{1}{2}}\cdot\left(I_m^{\,2}\right)^{\frac{1}{2}} \qquad I_e := \frac{I_m}{\sqrt{2}} \qquad I_e = effective\_current$$

so average power becomes ... $P_a := U_e\cdot I_e\cdot\cos(\phi)$ $\cos(\phi) = power\_factor$

what is current required in two systems with same effective voltage but larger phase lag?

here forward e subscript dropped and U == U_e, I == I_e

some power and current definitions

$apparent\_power = V\cdot A = U\cdot I$ $I = current$ $A = 1\,amp$

$real\_power = U\cdot\ \cdot\cos(\phi)$ $W = 1\,W$ same for current $load\_current = I\cdot\cos(\phi)$ $A = 1\,amp$

$reactive\_power = U\cdot I\cdot\sin(\phi)$ $V\cdot A$ $reactive\_current = I\cdot\sin(\phi)$ $A = 1\,amp$

three phase alternating current

$ORIGIN := 1$

$i := 1 .. 3$

$$\alpha := \begin{pmatrix} 0 \\ 2\cdot\frac{\pi}{3} \\ 4\cdot\frac{\pi}{3} \end{pmatrix}$$ phase angle for respective phases and ...

$U_{p_i} := U_m\cdot\cos(\omega\cdot t - \alpha_i)$

$I_{p_i} := I_m\cdot\cos(\omega\cdot t - \alpha_i - \phi)$

$$\sum_{i=1}^{3} U_{p_i}\ \text{expand} \to 0 \qquad \sum_{i=1}^{3} I_{p_i}\ \text{expand} \to 0$$

star connection ... $I_{L_1} = I_{p_1}$ $I_{L_2} = I_{p_2}$ $I_{L_3} = I_{p_3}$

$i := 1..2$ $U_{L_i} := U_{p_i} - U_{p_{i+1}}$ $U_{L_3} := U_{p_3} - U_{p_{31}}$

$i := 1..3$ $Uz_{p_i} := U_m \cdot \cos(\omega \cdot t - \alpha_i) + U_m \cdot \sin(\omega \cdot t - \alpha_i) \cdot i$

$i := 1..2$ $Uz_{L_i} := Uz_{p_i} - Uz_{p_{i+1}}$ $Uz_{L_3} := Uz_{p_3} - Uz_{p_1}$

$i := 1..3$ e.g. Uz_{L_1} simplify $\rightarrow U_m \cdot \cos(\omega \cdot t) + i \cdot U_m \cdot \sin(\omega \cdot t) + U_m \cdot \cos\left(\omega \cdot t + \frac{1}{3} \cdot \pi\right) + i \cdot U_m \cdot \sin\left(\omega \cdot t + \frac{1}{3} \cdot \pi\right)$

magnitude is ... from trigonometry...

$$U_m * \sqrt{\left(\cos(\omega \cdot t) + \cos\left(\omega \cdot t + \frac{\pi}{3}\right)\right)^2 + \left(\sin(\omega \cdot t) + \sin\left(\omega \cdot t + \frac{\pi}{3}\right)\right)^2} \text{ expand } \rightarrow \left(3 \cdot \cos(\omega \cdot t)^2 + 3 \cdot \sin(\omega \cdot t)^2\right)^{\frac{1}{2}}$$

$\text{magnitude} := U_m \cdot \sqrt{3}$ (2.85)

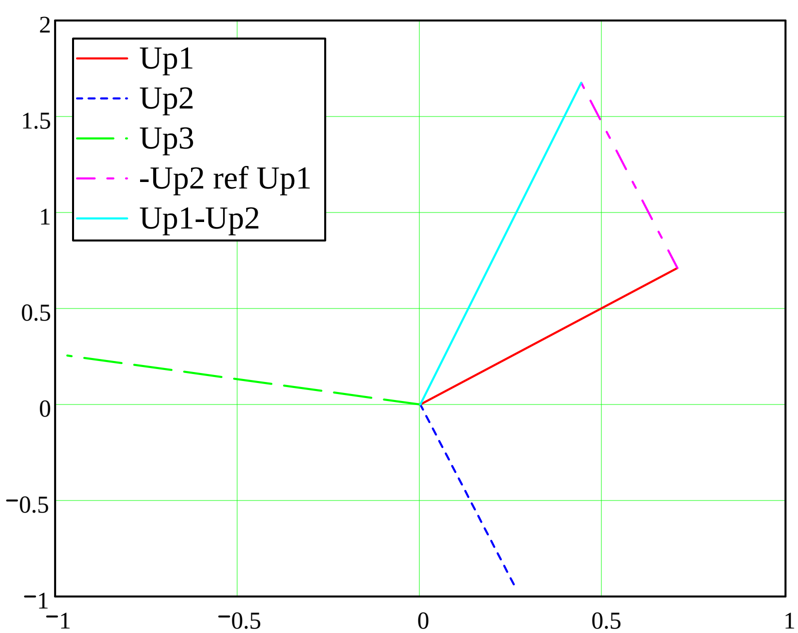
angle relative to ω*t (set ω*t = 0)

$$Uz_{L_1} \left| \begin{array}{l} \text{simplify} \\ \text{substitute}, t = 0 \end{array} \right. \rightarrow U_m \cdot \cos(0) + i \cdot U_m \cdot \sin(0) + U_m \cdot \cos\left(\frac{1}{3} \cdot \pi\right) + i \cdot U_m \cdot \sin\left(\frac{1}{3} \cdot \pi\right)$$

$$\text{angle}_{\omega t} = \text{atan}\left(\frac{\sin\left(\frac{\pi}{3}\right)}{1 + \cos\left(\frac{\pi}{3}\right)}\right) \qquad \text{atan}\left(\frac{\sin\left(\frac{\pi}{3}\right)}{1 + \cos\left(\frac{\pi}{3}\right)}\right) = 30\,\text{deg}$$

for plotting ... $i := 1..3$ $\omega 1 := 1$ $t1 := 0.79$

$\phi 1 := 1$ $U1_m := 1$ $U1_{p_i} := U1_m \cdot \cos(\omega 1 \cdot t1 - \alpha_i)$ $U1z_{p_i} := U1_m \cdot \cos(\omega 1 \cdot t1 - \alpha_i) + U1_m \cdot \sin(\omega 1 \cdot t1 - \alpha_i) \cdot i$



similarly in a delta connection ... current has same geometry $U_L = U_p$ (2.86) $I_L = I_p \cdot \sqrt{3}$ (2.87)

2.3.5 Magnetic induction

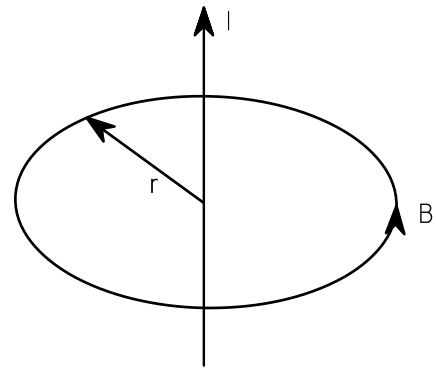
$B = \mu \cdot \frac{I}{2 \cdot \pi \cdot r}$ $B = \text{flux\_density}$ $T = 1\,\text{tesla}$ $T = 1\,\frac{Wb}{m^2}$ $T = 1\,\frac{kg}{amp \cdot s^2}$

(2.90)

$\mu = \text{permeability\_of\_medium}$ $\frac{H}{m} = 1\,\frac{m \cdot kg}{A^2 \cdot s^2}$ $\frac{H}{m} = 1\,\frac{henry}{m}$

$\mu = \mu_0 \cdot \mu_R$ $\mu_0 = \text{permeability\_of\_vacuum}$ $\mu_0 := 4 \cdot \pi \cdot 10^{7}\,\frac{H}{m}$

$\mu_R = \text{permeability\_of\_medium\_relative\_to\_vacuum}$ unitless



magnetic field around wire carrying current

derived from Biot-Savart law

e.g. magnetic field at point P results from motion of charged particle at velocity V in vacuum

$B = \frac{\mu_0}{4 \cdot \pi} \cdot q \cdot \frac{\vec{V} \times \vec{a_r}}{r^2}$ T

<u>parameters</u>   <u>units and equivalents</u>

$B = \text{flux\_density}$ $T = 1\,\text{tesla}$ $T = 1 \times 10^4\,\text{gauss}$ $T = 1\,\frac{1}{m^2}\,Wb$

$\mu_0 = \text{permeability\_in\_vacuum}$ $\mu_0 := 4 \cdot \pi \cdot 10^{-7}\,\frac{H}{m}$ $\frac{H}{m} = 1\,\frac{N}{A^2}$ $\frac{H}{m} = 1\,\frac{newton}{amp^2}$

$q = \text{charge}$ $C = 1\,\text{coul}$ $C = 1\,A \cdot s$

$\vec{V} = \text{velocity\_vector\_of\_charge}$ $\frac{m}{s}$

$\vec{a_r} = \text{unit\_vector\_from\_charge\_q\_to\_point\_P}$

$r = \text{distance\_from\_P\_to\_charge}$ m

units check

$\frac{H}{m} \cdot C \cdot \frac{m}{s} \cdot \frac{1}{m^2} = 1\,T$

differential form $dB = \frac{\mu_0}{4 \cdot \pi} \cdot dq \cdot \frac{\vec{V} \times \vec{a_r}}{r^2}$

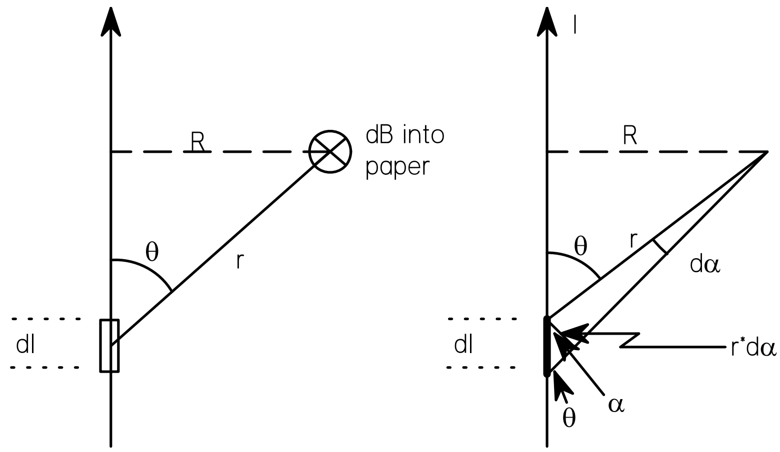
line currents ... $q \cdot \vec{V} = I \cdot \vec{dl}$

so .. $dB = \frac{\mu_0}{4 \cdot \pi} \cdot I \cdot \frac{\vec{dl} \times \vec{a_r}}{r^2}$ $B = \int \frac{\mu_0}{4 \cdot \pi} \cdot \frac{I}{r^2}\,\vec{dl} \times \vec{a_r}$

e.g. long straight wire with current I

$$dB = \frac{\mu_0}{4\cdot\pi}\cdot I\cdot\frac{\vec{dl}\times\vec{a_r}}{r^2} = \frac{\mu_0}{4\cdot\pi}\cdot I\cdot\frac{dl\cdot\sin(\theta)}{r^2}$$

see figure at right



geometry for solution set up

$$dl\cdot\sin(\theta) = r\cdot d\alpha$$

$$\frac{dl\cdot\sin(\theta)}{r^2} = \frac{r\cdot\frac{d\alpha}{\sin(\theta)}\cdot\sin(\theta)}{r^2} = \frac{d\alpha}{r}$$

$d\alpha$ $\quad$ $\sin(\theta) = \frac{r\cdot d\alpha}{dl}$ $\quad$ $\cos(\alpha) = \frac{R}{r}$

$$r = \frac{R}{\cos(\alpha)} \qquad \frac{d\alpha}{r} = \frac{\cos(\alpha)\cdot d\alpha}{R}$$

$$dB = \frac{\mu_0}{4\cdot\pi}\cdot I\cdot\frac{\vec{dl}\times\vec{a_r}}{r^2} = \frac{\mu_0}{4\cdot\pi}\cdot I\cdot\frac{dl\cdot\sin(\theta)}{r^2} = \frac{\mu_0}{4\cdot\pi}\cdot I\cdot\frac{\cos(\alpha)\cdot d\alpha}{R}$$

$$B = \int 1\, dB = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\mu_0}{4\cdot\pi}\cdot I\cdot\frac{\cos(\alpha)}{R}\, d\alpha \qquad \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\mu_0}{4\cdot\pi}\cdot I\cdot\frac{\cos(\alpha)}{R}\, d\alpha \to \frac{1}{2}\cdot\frac{\mu_0}{\pi}\cdot\frac{I}{\Omega} \qquad B = \frac{\mu_0\cdot I}{2\cdot\pi\cdot R}$$

Q.E.D.

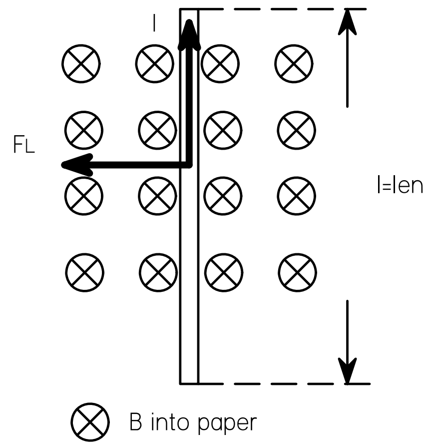
if area not vacuum, substitute $\mu = \mu_r * \mu_0$ ...

magnetic flux density over an area AA

$\Phi := \int B\, dAA$ $\quad$ $AA = \text{enclosed\_area}$ $\quad$ to distinguish from A (ampere) $\quad$ $Wb = 1\,\text{weber}$ $\quad$ $Wb = 1\frac{kg\cdot m^2}{A\cdot s^2}$ $\quad$ $A = 1\,\text{amp}$

Lorentz force

a force will act on a current carrying conductor when it is placed in a magnetic field



$F_L = B \cdot I \cdot len$ (2.92)

$F_L = Lorentz\_force$ $\quad N = 1\,newton$

$B = flux\_density$ $\quad T = 1\,tesla$

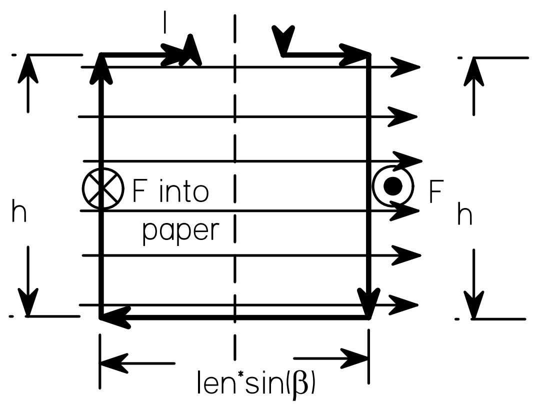
$I = current\_thru\_conductor$ $\quad A = 1\,amp$

$len = length$ $\quad m$

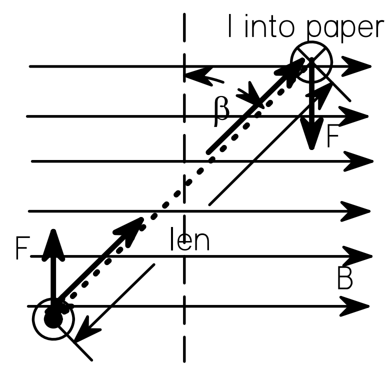
$T \cdot A \cdot m = 1\,N$

$F_L = I \cdot \overrightarrow{len} \times B$ where x is vector cross product and magnitude is $B \cdot I \cdot \sin(angle)$ right hand rule applies

view of single coil in magnetic field (B) with current (I) (slightly revised from text; len*sin(β)



side view



top view

force on one segment (h) of coil $\quad F = I \times B \cdot h$ $\quad$ N.B. I is perpendicular to B => $F = I \cdot B \cdot h$

torque on coil depends on β $\quad M = F \cdot len \cdot \sin(\beta)$ $\quad len \cdot \sin(\beta) = distance\_between\_couple\_of\_force\_F$

$M = F \cdot len \cdot \sin(\beta) = I \cdot B \cdot h \cdot len \cdot \sin(\beta) = I \cdot B \cdot AA \cdot \sin(\beta) = I \cdot \Phi \cdot \sin(\beta)$ $\quad AA = area\_of\_coil = enclosed\_area$ $\quad$ AA to distinguish from A (ampere)

account for multiple windings (turns) (N) $\quad M = N \cdot I \cdot \Phi \cdot \sin(\beta)$

general relationship recognizing proportionality to I*Φ $\quad M = K_m \cdot \Phi \cdot I$ $\quad K_m = constant\_for\_given\_motor$ $\quad$ (2.93)

Faraday's Law

Voltage is generated in conductor when moving in magnetic field

$E = -B \cdot len \cdot v$

$E = induction\_potential = electromotive\_force$ $\quad V = 1\,volt$

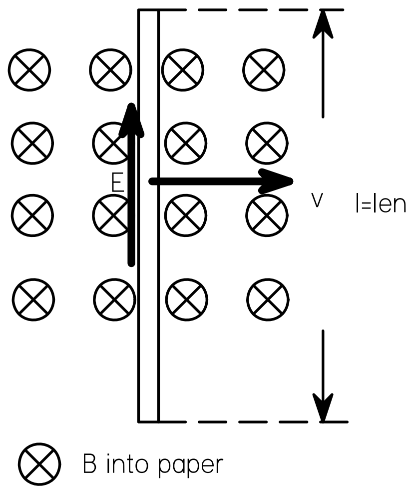
$B = flux\_density$ $\quad T = 1\,tesla$

$v = velocity$ $\quad \frac{m}{s}$

$len = length\_of\_conductor$ $\quad m$

units check

$T \cdot \frac{m}{s} \cdot m = 1\,V$



E as shown is positive value and direction minus sign is consistent with observation that E as shown would produce a current in the same direction which in turn would produce a force opposite to velocity.

vector form ... $E = -(B \times v) \cdot len$

consistent with text ... multiply by sin(α) where α is angle between B and v

may also be expressed as ... $E = -\frac{d}{dt}\Phi$ for single turn and .. for N turns $E = -N\frac{d}{dt}\Phi$ $\quad \frac{Wb}{s} = 1\,V$ (2.95)

since ... $\Phi = B \cdot Area$ $\quad E = -\frac{d}{dt}\Phi = -\frac{d}{dt}(B \cdot Area) = -B\frac{d}{dt}Area = -B \cdot len\frac{d}{dt}x$ as ... $Area = len \cdot x$

in coil rotating in constant magnetic field B $\quad \Phi = B \cdot Area \cdot \cos(\beta) = B \cdot Area \cdot \cos(\omega \cdot t)$ where ...

$Area = area\_enclosed\_in\_coil$

and with N turns ... $E = -N\frac{d}{dt}(B \cdot Area \cdot \cos(\omega \cdot t)) \rightarrow E = N \cdot B \cdot Area \cdot \sin(\omega \cdot t) \cdot \omega$

substituting ... $\omega = 2 \cdot \pi \cdot n$ $\quad n = rpm$ $\quad rpm = 6.283\,\frac{rad}{min}$ $\quad E = N \cdot 2 \cdot \pi \cdot n \cdot B \cdot Area \cdot \sin(\omega \cdot t)$

as abve for motor constant ... express ... $E = K_E \cdot \Phi \cdot n$ $\quad K_E = constant\_for\_given\_motor$

$E = induced\_electromotive\_force$ $\quad V = 1\,volt$

$\Phi = magnetic\_flux$ $\quad Wb = 1\,weber$

$n = rotation\_speed$ $\quad rpm = 0.105\,\frac{1}{sec}$ $\quad 1Wb \cdot 60rpm = 6.283\,V$