Lecture 7

7/1

Review

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t} + \vec{J}$$

$$\nabla \cdot \vec{D} = \rho_{net}$$

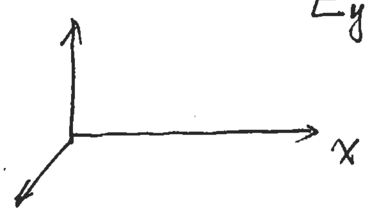
$$\nabla \cdot \vec{B} = 0$$

$$\vec{J} = \sigma \vec{E}$$

If $\sigma = 0$, $\varepsilon =$ constant spatially

$$\Rightarrow \quad \nabla^2 \vec{E} = \mu\varepsilon \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$E_y = E_{y0} \cos(\omega t - kx)$$



$$\Rightarrow \quad k^2 = \mu\varepsilon\omega^2$$

$$k = \frac{\omega}{c}$$

$$c = \frac{1}{\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu_0\varepsilon_0}} \cdot \frac{1}{\sqrt{\mu/\mu_0 \cdot \varepsilon/\varepsilon_0}}$$

$$\frac{c_0}{c} = n = \sqrt{\frac{\mu}{\mu_0}\frac{\varepsilon}{\varepsilon_0}} = \sqrt{\varepsilon_r}.$$

↑ dielectric constant.

$$\vec{E} \perp \vec{H} \perp \vec{k}$$

For most material $\mu = \mu_0 \Rightarrow N = \sqrt{\varepsilon_r}$

dielectric constant.

$$N = n + ik$$

refractive index. extinction coefficient

$$\vec{E}_c = \vec{E}_0 \, e^{-i[\omega t - (n+ik)k_0 x]}$$

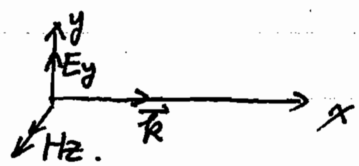
$$= \vec{E}_0 \, e^{-i(\omega t - n k_0 x)} \, e^{-k k_0 x}$$

decay.

if use $E_0 e^{i[\omega t - \vec{k}\cdot\vec{r}]}$

$$N = n - ik$$

From $\vec{E}_c$, $\vec{H}_c$ $\Rightarrow$



$$E_y = E_{y0} \exp[-i(\omega t - N k_0 x)]$$

forward

$$= E_{y0} \exp\left[-i\omega\left(t \mp \frac{N}{c_0} x\right)\right]$$

backward propagation.

$$\nabla \times \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t}$$

$$\begin{pmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{pmatrix} = -\mu \begin{pmatrix} \partial H_x/\partial t \\ \partial H_y/\partial t \\ \partial H_z/\partial t \end{pmatrix}$$

$$H_z = H_{z0} \exp\left[-i\omega\left(t \pm \frac{N x}{c_0}\right)\right] \hat{z}$$

$$H_{z0} = \pm \frac{N}{\mu c_0} E_{y0}$$

forward

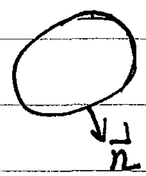
backward

Maxwell Eq. $\vec{H} \cdot \nabla \times \vec{E} = \cdots$

$\vec{E} \cdot \nabla \times \vec{H} = \cdots$

$\Rightarrow$

$$\cancel{\oint dV} \quad -\nabla \cdot (\vec{E} \times \vec{H}) = \int \underbrace{\frac{\partial}{\partial t}\Big(\underset{\substack{\uparrow \\ \text{energy stored} \\ \text{by electric} \\ \text{field}}}{\tfrac{1}{2}\varepsilon \vec{E}\cdot\vec{E}} + \underset{\substack{\uparrow \\ \text{magnetic} \\ \text{field}}}{\tfrac{1}{2}\mu \vec{H}\cdot\vec{H}}\Big)}_{\text{rate of energy change}} + \underbrace{\underset{\substack{\uparrow \\ \text{Joule heating}}}{\vec{E}\cdot\vec{J}}}_{\text{rate of eng deposit}}$$



$$\int dV \qquad = \qquad \int \qquad dV$$

$\Downarrow$

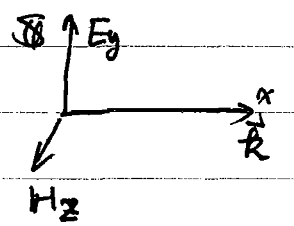
$$-\oiint (\vec{E} \times \vec{H}) d\vec{s} = \int [\qquad] \, dV$$

Poynting vector $\quad \vec{S} = \vec{E} \times \vec{H}$ — surface energy flux

$\quad \hookrightarrow$ changing with 2ω

Time average $\quad \langle \vec{S} \rangle = \frac{1}{T}\int_0^T \vec{S}\, dt = \frac{1}{2} Re\, (\vec{E}_c \times \vec{H}_c^*)$

Inside a material



$$S_x = \tfrac{1}{2}Re\, E_y H_z^*$$

$$= \frac{1}{2} Re\left[E_{y0} \exp\left[-i\omega\left(t - \frac{N}{C_0}x\right)\right] \cdot \frac{N^*}{\mu C_0} E_{y0}^* \exp\left[i\omega\left(t - \frac{N^*}{C_0}x\right)\right]\right]$$

$$= \frac{1}{2\mu C_0} |E_{y0}|^2 \; Re\left\{ N^* \exp\left[i\omega \frac{N - N^*}{C_0} x\right]\right\}$$

$$= \frac{1}{2\mu C_0}\, n\, |E_{y0}|^2 \cdot \exp\left(-\frac{4\pi k}{\lambda_0} x\right)$$

$$= \frac{1}{2\mu c_0} n |E_{y0}|^2 e^{-\alpha x}$$

$$\alpha = \frac{4\pi K}{\lambda_0} \quad \text{— absorption coefficient}$$

$$d = \frac{1}{\alpha} = \frac{\lambda_0}{4\pi K} \quad \text{— skin depth}$$

Heat generation $\quad \dot{q} = -\nabla\cdot\vec{S}$

$$= \frac{1}{2\mu c} n|E_{y0}|^2 \cdot \alpha\, e^{-\alpha x}$$

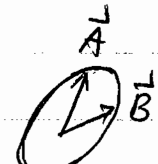
* polarization

$$\vec{E} = Re[\vec{E}_c]$$
$$= Re\{(\vec{A} \mp \vec{B} i) e^{-i\omega(t - \frac{N}{c}x)}\}$$

$$= \vec{A} \cos[\omega(t - \tfrac{N}{c}x)] \mp \vec{B} \sin[\omega(t - \tfrac{N}{c}x)].$$

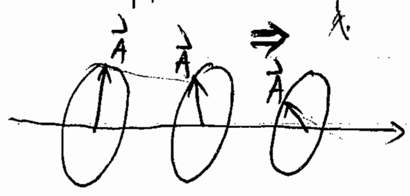
physically, source ~~to~~ emit 2 waves, one lags the other by 90° in phase and is in a different direct.

at $x=0$ $\quad \vec{E}(x=0,t) = \vec{A}\cos\omega t + \vec{B}\sin\omega t$



elliptical

at different x, same time.



helical

Either $\vec{A}$ or $\vec{B} = 0 \rightarrow$ linearly polarized

$\vec{A} = \vec{B}$ $\quad$ circularly polarized.

Decompose into $E_{\parallel}$ & $E_{\perp}$ perpendicular components.

$(E_{\parallel} = a_{\parallel} e^{-i\delta_{\parallel}}, \; E_{\perp} = a_{\perp} e^{-i\delta_{\perp}}$

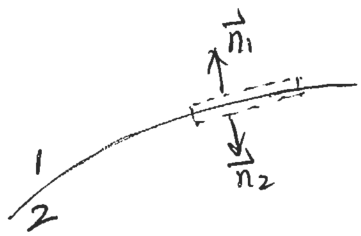
$\Rightarrow$ stokes parameter

$$I^2 = Q^2 + U^2 + V^2$$

$$I = a_{\parallel}^2 + a_{\perp}^2$$
$$Q = a_{\parallel}^2 - a_{\perp}^2$$
$$U = 2a_{\parallel}a_{\perp}\cos(\delta_{\parallel} - \delta_{\perp})$$
$$V = 2a_{\parallel}a_{\perp}\sin(\delta_{\parallel} - \delta_{\perp})$$

thermal radiation : unpolarized

* Interface conditions



$$\nabla \cdot \vec{D} = \rho_{free}$$

$$\int_V \nabla \cdot \vec{D}\, dV = \int \rho_{free}\, dV$$

$$\oiint \vec{D} \cdot d\vec{S} = \int \rho_{free}\, dV$$

$$\vec{n}_1 \cdot \vec{D}_1 S + \vec{n}_2 \cdot \vec{D}_2 \cdot S = \rho_s \cdot S$$

$$\Rightarrow \quad \vec{n} \cdot (\vec{D}_1 - \vec{D}_2) = \rho_s.$$

└ surface charge

$$\vec{n} \cdot (\vec{H}_1 - \vec{H}_2) = 0$$

$$(\vec{E}_1 - \vec{E}_2) \times \vec{n} = 0$$

$$(\vec{H}_1 - \vec{H}_2) \times \vec{n} = \vec{J}_s$$

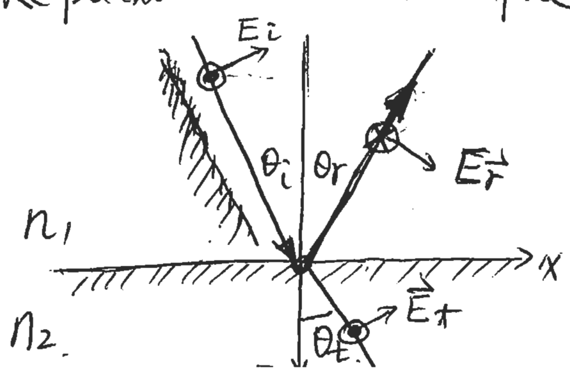
└ surface current density

8/2

* Reflection at one Interface : plane wave

Plane of incidence

linearly polarized.



$\boxed{\ }_i$ in TM wave = ∥ wave = p wave

TE wave = ⊥ wave = s wave