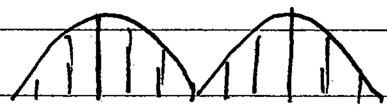
Review of last time

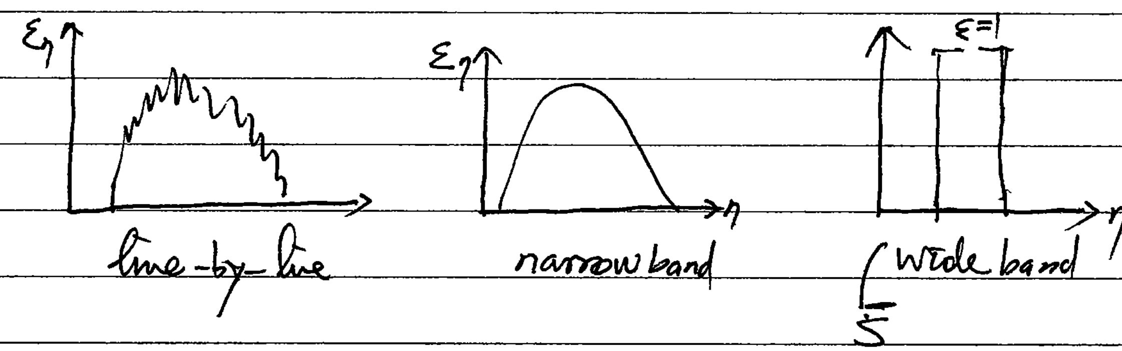


Broading { natural, collision, Doppler   Lorentzian

absorpti coeffi K_η :

emissivity $\quad \varepsilon_\eta = 1 - e^{-K_\eta X}$

narrow band model $\Rightarrow$ { ~~lines~~ Elsasser,
$\bar{S}$, Δ. Goody : random



Total :

Total emissivity & mean absorpti coefficient

$$\varepsilon = \frac{\int_0^\infty I_{b\eta}\varepsilon_\eta d\eta}{\int_0^\infty I_{b\eta} d\eta}$$

$$= \frac{\int_0^\infty I_{b\eta}(1-e^{-k_\eta X})d\eta}{\int_0^\infty I_{b\eta} d\eta}$$

small variati over each band

$$= \sum_{i=1}^{N}\left(\frac{\pi I_{b\eta_0}}{\sigma T^4}\right)_i \int_{\Delta\eta_{band}} (1-e^{-k_\eta X})d\eta$$

$$= \sum_{i=1}^{N}\left(\frac{\pi I_{b\eta_0}}{\sigma T^4}\right)_i A_i$$

└ total band absorptance (effective band width)

overlapping band

$$\overline{\tau}_{a+b} = \frac{1}{\Delta\eta}\int_{\Delta\eta} e^{-k_{\eta a}X} e^{-k_{\eta b}X} d\eta$$

$$\doteqdot \frac{1}{\Delta\eta}\int_{\Delta\eta} e^{-k_{\eta a}X} d\eta \, \frac{1}{\Delta\eta}\int_{\Delta\eta} e^{-k_{\eta b}X} = \overline{\tau}_a \, \overline{\tau}_b$$

$$\varepsilon = 1-\tau \Rightarrow$$

$$\varepsilon_{a+b} = \varepsilon_a + \varepsilon_b - \varepsilon_a\varepsilon_b$$

Planck Mean Asorpti Coefficient

$$k_p \equiv \frac{\int_0^\infty I_{b\eta} k_\eta d\eta}{\int I_{b\eta} d\eta} = \frac{\pi}{\sigma T^4}\int_0^\infty I_{b\eta} k_\eta d\eta$$

* Radiative properties of gases

Donot use n, κ , $n \simeq 1$

EM wave theory

$$\frac{dI_\nu}{dx} = -\kappa_\nu I_\nu$$

absorption coefficient (includes both absorption & stimulated emission).

u upper level n_u

$$: \quad \left(\frac{dn_u}{dt}\right)_{u\to l} = -A_{ul} n_u$$

l lower level n_l

Einstein coefficient for spontaneous emission.

absorption: $\left(\frac{dn_l}{dt}\right)_{l\to u} = n_l B_{lu} \int_{4\pi} I_\nu d\Omega$

At equilibrium rate of absorption = rate of emission

cannot be satisfied by spontaneous.

↳ stimulated emission.

$$\left(\frac{dn_u}{dt}\right)_{u\to l} = -n_u B_{ul} \int_{4\pi} I_\nu d\Omega.$$

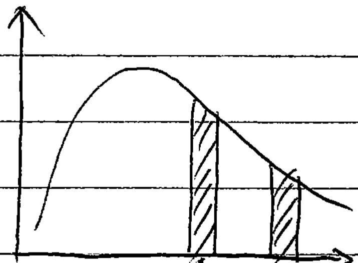
At equilibrium $\frac{n_l}{n_u} = \frac{g_l e^{-E_l/kT}}{g_u e^{-E_u/kT}} = \frac{g_l}{g_u} e^{h\nu/kT}$

$$\Rightarrow \begin{cases} g_u B_{ul} = g_l B_{lu} \\ A_{ul} = \frac{8\pi h \nu^3}{c_0^2} B_{ul} \end{cases}$$

Einstein Relation.

$\overset{I}{\Longrightarrow} \bigcirc$

$$+ \frac{d}{d\Omega}\left(\frac{dn}{dt}\right)_{l\to u} = +(n_l B_{lu} - n_u B_{ul}) I_\nu$$



Total Emissivity

$$\varepsilon \equiv \frac{\int_0^\infty I_{b\eta}\varepsilon_\eta d\eta}{\int_0^\infty I_{b\eta} d\eta}$$

$$= \frac{\int_0^\infty I_{b\eta}(1-e^{-\kappa_\eta X})d\eta}{\int_0^\infty I_{b\eta} d\eta}$$

$$= \sum_{i=1}^{\infty} \left(\frac{\pi I_{b\eta 0}}{\sigma T^4}\right)_i \int_{\Delta\eta_{band}} (1-e^{-\kappa_\eta X})d\eta = \sum_{i=1}^{N} \left(\frac{\pi I_{b\eta 0} A_i}{\sigma T^4}\right)_i$$

↑ If bands do not overlap.

If bands overlap.

species a & b

$$\bar{\tau}_{a+b} = \frac{1}{\Delta\eta}\int_{\Delta\eta} e^{-\kappa_{\eta a}X} e^{-\kappa_{\eta b}X} d\eta \simeq \frac{1}{\Delta\eta}\int_{\Delta\eta} e^{-\kappa_{\eta a}X} d\eta \, \frac{1}{\Delta\eta}\int_{\Delta\eta} e^{-\kappa_{\eta b}X} d\eta$$

↑ overlapping region

$$= \bar{\tau}_a \bar{\tau}_b$$

$$\varepsilon_{a+b} = \varepsilon_a + \varepsilon_b - \varepsilon_a\varepsilon_b$$

↑ if fully overlap.