Review:

What we learnt so far:

How to calculate radiation exchange
view factor,
energy balance at surfaces
radiative properties of surfaces.
EM wave basis . input refractive index

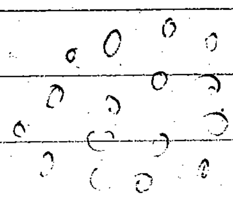
Type of problems you can deal with

- radiation furnace . semiconductor equipment
- lighting, solar radiation, ~~to~~
- Coatings

↳ physics : ballistic transport.

Where we are going:

Clouds, particles, combustion, global warming.



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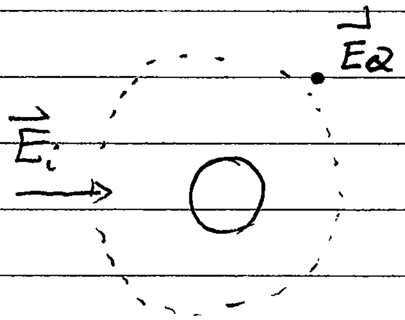
radiative properties of particles
radiative properties of gases
master equation.

Radiative properties of particles.

* ~~§~~ Scattering ~~from particles~~



2 — a point outside particle

$$\vec{E_2} = \vec{E_i} + \vec{E_s}$$
$$\vec{H_2} = \vec{H_i} + \vec{H_s}$$

$$\langle \vec{S} \rangle = \frac{1}{2} Re(\vec{E_2} \times \vec{H_2}^*)$$

$$= \frac{1}{2} Re(\vec{E_i} \times \vec{H_i}^*) + \frac{1}{2} Re(\vec{E_s} \times \vec{H_s}^*) + \frac{1}{2} Re(\vec{E_i} \times \vec{H_s}^* + \vec{E_s} \times \vec{H_i}^*)$$

$$= \langle \vec{S_i} \rangle + \langle \vec{S_s} \rangle + \langle \vec{S_e} \rangle$$

incident — scattering — interaction of incident & scattered fields

Consider an imaginary sphere surrounding the particle

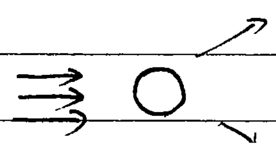
$\oint \langle \vec{S} \rangle \cdot d\vec{A}$ is net energy flow in normal direction

< 0 — absorbed inside

$$-W_a = \cancelto{0}{\int \langle \vec{S_i} \rangle \cdot d\vec{A}} + \underbrace{\int \langle \vec{S_s} \rangle \cdot d\vec{A}}_{\text{scattered out} > 0,\ W_s} + \int \langle \vec{S_e} \rangle \cdot d\vec{A}$$

$$-\int \langle \vec{S_e} \rangle \cdot d\vec{A} = W_a + W_s = \underbrace{W_e}_{\text{extinction power}}$$

Extinction — due to absorption & scattering, ~~from~~

after:  ~~E~~ Detector $U = I_i(A - C_e)$

Cross-section:
- Extinction cross-section $C_e = W_e / I_i$
- absorption .. $C_a = W_a / I_i$
- scattering .. $C_s = W_s / I_i$

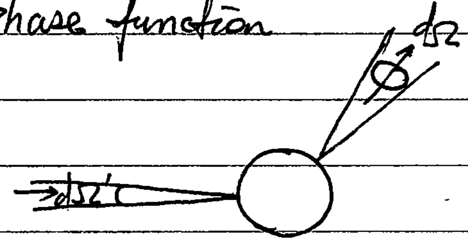
efficiency:
- extinction $Q_e = C_e / G$
- absorption $Q_a = C_a / G$
- scattering $Q_s = C_s / G$

G – projected cross-section.

Albedo:

$$\omega_0 = \frac{Q_s}{Q_e} = \frac{Q_s}{Q_s + Q_a}$$

* phase function



$\Phi(\Omega' \to \Omega) =$ ~~power scattered into $d\Omega$ / incident power in $d\Omega'$~~

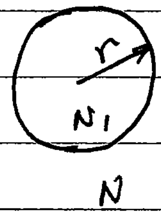
$$= \frac{\text{power scattered from } d\Omega' \text{ into } \Omega \text{ direction}}{\text{power scattered from } \Omega' \text{ in } \Omega \text{ if scattering is isotropic}}$$

$\Phi = 1$ isotropic scattering

取 $\int_{4\pi} \Phi(\Omega' \to \Omega)\, d\Omega = 1$

* Scattering of a spherical particle

Mie theory. (Gustav Mie, 1908)



$$Q_e = \frac{2}{x^2} \sum_{n=1}^{\infty} (2n+1)\, \mathrm{Re}\{a_n + b_n\}$$

$$Q_s = \frac{2}{x^2} \sum_{n=1}^{2} (2n+1) \left(|a_n|^2 + |b_n|^2 \right)$$

$$a_n = \frac{\psi_n'(mx)\psi_n(x) - m\psi_n(mx)\psi_n'(x)}{\psi_n'(mx)\xi_n(x) - m\psi_n(mx)\xi_n'(x)}$$

$$b_n = \frac{m\psi_n'(mx)\psi_n(x) - \psi_n(mx)\psi_n'(x)}{m\psi_n'(mx)\xi_n(x) - \psi_n(mx)\xi_n'(x)}$$

ψ_n, ξ_n — Ricatti-Bessel function

$\psi_{n+1}(x) = \frac{2n+1}{x}\psi_n(x) - \psi_{n-1}(x)$

$\chi_{n+1}(x) = \frac{2n+1}{x}\chi_n(x) - \chi_{n-1}$

$\xi_n = \psi_n - i\chi_n$

$\psi_{-1}(x) = \cos x \qquad \psi_0 = \sin x$

$\chi_{-1}(x) = -\sin x \qquad \chi_0 = \cos x$

$x = \frac{2\pi r}{\lambda_0} \qquad m = \frac{N_1}{N}$

└ particle size parameter

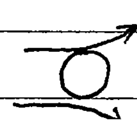
Show ~~exar~~ example of Mie theory.

Comment: (a) In some region

$Q_a > 1$ emissivity > 1.

(b) for large particles

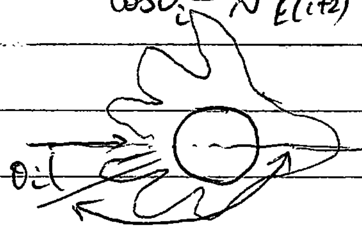
$Q_e = 2$



shadow $Q_e = 1$
another 1 edge 1

$\cos\theta_i = \sqrt{\frac{n^2-1}{i(i+2)}}$ $\quad i=1 \quad \cos\theta_1 = \sqrt{\frac{n^2-1}{3}}$

water $n=1.33 \Rightarrow \theta = 138$



but this edge ~~effect~~ may be only small deflecti.
difficult to distinguish from ~~scattered~~ forward propagating wave.
Rain-bow

(C) Rayleigh Scattering

$$Q_{sca} = \frac{8}{3}\left|\frac{m^2-1}{m^2+2}\right|^2 x^4$$

$$Q_e = +4\,\mathrm{Im}\left(\frac{m^2-1}{m^2+2}\right)x$$

$$\approx Q_a$$

$$C_s = Q_s \pi r^2 = \frac{8}{3}\pi r^2 \left|\frac{N^2-1}{N^2+2}\right| \frac{16\pi^4 r^4}{\lambda_0^4}$$
$$= \frac{24\pi^3 V^2}{\lambda_0}\left|\frac{N^2-1}{N^2+2}\right|$$

$$C_e = +4\,\mathrm{Im}\,\frac{2nki + n^2-1+k^2}{n^2+k^2+2+2nki}\,x$$

$$= 4x\left\{\frac{2nk(n^2+k^2+2)-(n^2-1+k^2)2nk}{(n^2+k^2+2)^2+(2nk)^2}\right\}\pi r$$

$$= 4\frac{2\pi r}{\lambda_0}\frac{(3\pm k^2)2nk}{(n^2+k^2+2)^2+(2nk)^2}\pi r^2$$

$$= \frac{2\pi\, 6(3\pm k^2)nk}{(n^2+k^2+2)}V$$

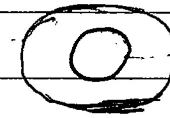
$C_s \sim V^2 \sim r^6$
$C_a \sim V \sim r^3$.

blue sky.
difficult to optically ~~measure~~ detect nanoparticle ~~size~~.

(d) ~~Rayleigh~~ dipole scattering: $\quad \vec{p} = \left(\frac{\varepsilon_r-1}{\varepsilon+2}\right) r^3 \vec{E}_{inc}$

phase function: $\quad \varphi(\theta) = \frac{3}{4}(1+\cos^2\theta)$

$$= \begin{cases} TE & \text{isotropic} \\ TM & \frac{3}{4}\cos^2\theta \end{cases}$$



(d) Rayleigh-Gans: when $|m-1| \ll 1 \quad x|m-1| \ll 1$

(e) Geometric limit $\quad x \gg 1, \quad x|m-1| \gg 1$

Ray tracing: $\quad Q_e = 1$
(but diffract: $Q_e = 2$)
specular vs diffuse