

## 13.024 (2003) Problem Set 3 Solution:

### Review of Fluid Dynamics

1. *Steady flow around a circular cylinder: The 2-D potential flow past a cylinder may be evaluated by considering a dipole in a free stream. Derive expressions for the potential and dynamic pressure in the fluid around a cylinder of radius  $R_1=1m$  in a free stream of speed  $U=1m/s$ . Integrate the pressure around the body to find the hydrodynamic force. Also, check that the normal velocity is zero on the surface of the cylinder and compute the tangential velocity. Note: for comparison, you will also solve this problem by various numerical methods later in the term.*

In polar coordinates, the potential of a free stream of strength  $U$  plus a dipole of strength  $m$  is given by:

$$\Phi = Ur \cos \theta + \frac{m \cos \theta}{2\pi r}$$

The radial and tangential velocities are then given by:

$$u_r = \frac{\partial \Phi}{\partial r} = U \cos \theta - \frac{m \cos \theta}{2\pi r^2}$$
$$u_\theta = \frac{1}{r} \frac{\partial \Phi}{\partial \theta} = -\sin \theta \left( U + \frac{m}{2\pi r^2} \right)$$

We can find the dipole strength by requiring that  $u_r=0$  at  $(r, \theta) = (R, 0)$ . Hence,

$$m = 2\pi R^2 U$$

Substituting the value of  $m$ , we get the following expressions for the velocities on the surface of the body ( $r=R$ ):

$$u_r = U \cos \theta - U \cos \theta = 0$$

$$u_\theta = -2U \sin \theta$$

The dynamic pressure may be found from Bernoulli's theorem:

$$p = -\frac{1}{2} \rho (u_r^2 + u_\theta^2)$$

And on the surface of the body the above expression becomes:

$$p = -2\rho U^2 \sin^2 \theta$$

The force on the body is given by:

$$\vec{F} = \int_0^{2\pi} p(\hat{i} \cos \theta + \hat{j} \sin \theta) R d\theta = -2R\rho U^2 \left( \hat{i} \int_0^{2\pi} \sin^2 \theta \cos \theta d\theta + \hat{j} \int_0^{2\pi} \sin^3 \theta d\theta \right) = 0\hat{i} + 0\hat{j}$$

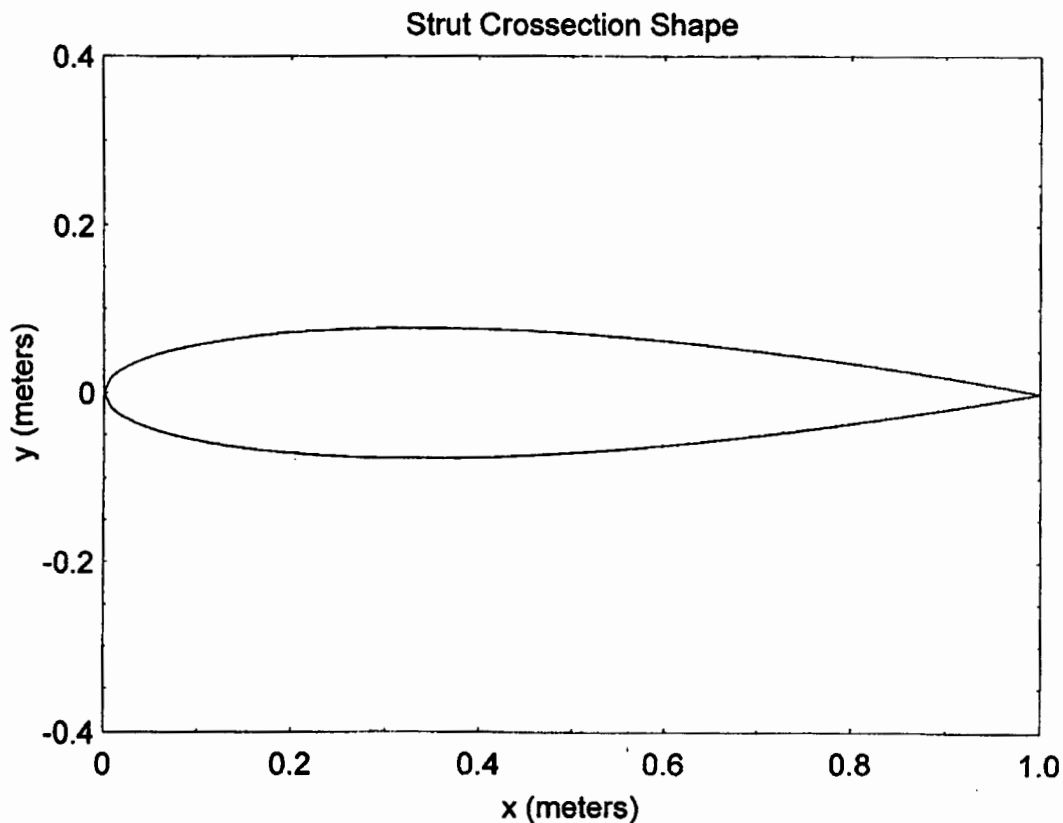
Of course we expected zero force since there is no viscosity or circulation.

2. Suppose you have a 2-dimensional strut whose cross section is given by the shape

$$y = \pm 0.2\sqrt{x}(1-x)$$

The + sign in the  $\pm$  is for the upper surface and the - sign is from the lower surface.

Here is what it looks like:



This strut is of interest because pairs of them in a V shape could support the aft most shaft bearing in a twin screw ship, amongst other uses.

A flow of 10 m/s is coming from left to right in the above figure. It is “nose-on” to the strut so the symmetric strut’s angle of attack is zero and there is no lift and no wake in the inviscid flow approximation which is to be used here.

Your job is to determine the flow field on the surface of the strut using Green’s Theorem. For the Green Function, use  $G = -\ln 1/r$ . Calculate the velocity potential and the pressure distribution on the strut. Plot these functions vs  $x$  using MATLAB.