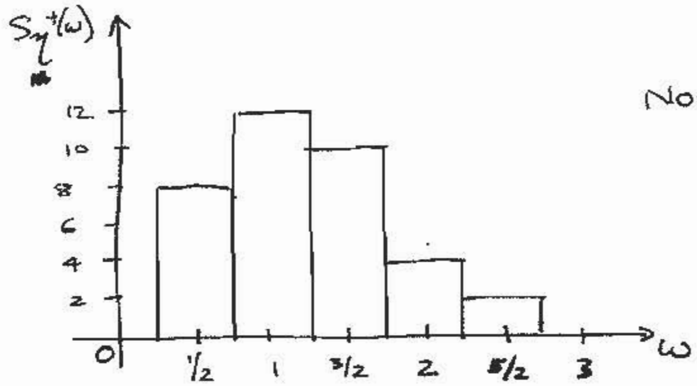
13.42 HW5 SOLUTIONS SPRING 2005

1)



Note: "+" was inadvertently omitted from assignment. Okay if you treated it either as $S_\eta^+(\omega)$ or $S_\eta(\omega)$ as long as it was treated correctly.

a) $\text{VARIANCE} = \sigma_\eta^2 = \sum_{i=1}^{5} S_\eta^+(\omega)\,d\omega$

$$\sigma_\eta^2 = (8)(\tfrac{1}{2}) + (12)(\tfrac{1}{2}) + (10)(\tfrac{1}{2}) + (4)(\tfrac{1}{2}) + (2)(\tfrac{1}{2}) = \underline{\underline{18\ m^2}}$$

b) FIND THE AVERAGE UPCROSSINGS OF THE PLANE $z = 3m$.

$$\bar{\eta}(3) = \frac{1}{2\pi}\sqrt{\frac{M_2}{M_0}}\, e^{\frac{-3^2}{2M_0}}$$

where $\text{VARIANCE} = M_0 = 18\ m^2$

$$M_2 = \sum_{i=1}^{5} \omega_i^2 S_\eta^+(\omega_i)\,d\omega = (\tfrac{1}{2})^2(8)(\tfrac{1}{2}) + (1)^2(12)(\tfrac{1}{2}) + (\tfrac{3}{2})^2(10)(\tfrac{1}{2})$$

$$+ (2)^2(4)(\tfrac{1}{2}) + (\tfrac{5}{2})^2(2)(\tfrac{1}{2}) = 37.5\ m^2$$

$$\bar{\eta}(3) = \frac{1}{2\pi}\sqrt{\frac{37.5\,m^2}{18\,m^2}}\, e^{\frac{-9}{2\cdot 18}} = \underline{\underline{0.1789 \text{ upcrossings / second}}}$$

c) FIND h, MINIMUM DECK CLEARANCE TO BE FLOODED $\leq$ ONCE PER HOUR.

From part b) $M_0 = 18\,m^2$, $M_2 = 37.5\,m^2$

$$\bar{\eta}(h) \leq 1 \text{ upcrossing per hour} = \frac{1 \text{ upcrossing}}{3600 \text{ seconds}}$$

$$\bar{\eta}(h) \leq 0.000278\,s^{-1}$$

$$\bar{\eta}(h) = \frac{1}{2\pi}\sqrt{\frac{37.5}{18.0}}\, e^{-h^2/2\cdot 18} \leq 0.000278$$

$$e^{-h^2/36} \leq 0.001209$$

SIMPLIFIES TO:

$$h \geq 15.55\ m$$

2) GIVEN: $\sigma_\eta^2 = 18m^2$ $\varepsilon = 0.6$

a) WITH A SEA SPECTRUM BANDWIDTH OF 0.6, USE THE APPROXIMATION:

$$P(\eta \geq \eta_0) \approx \frac{2\sqrt{1-\varepsilon^2}}{1+\sqrt{1-\varepsilon^2}} e^{-\eta_0^2/2}$$

$$P(\eta \geq \eta_0) \approx \frac{2\sqrt{1-0.6^2}}{1+\sqrt{1-0.6^2}} e^{-\eta_0^2/2}$$

$$P(\eta \geq \eta_0) \approx 0.8\overline{888} e^{-\eta_0^2/2}$$

TO FIND $P(\eta \geq 5m)$, $A = 5m$. AND η_0 IS A NONDIMENSIONALIZED NUMBER.

$$\eta_0 = \frac{A}{\sqrt{M_0}} = \frac{5m}{\sqrt{18m^2}} = 1.17851$$

$$\therefore P(\eta \geq 1.17851) \approx 0.8\overline{888}\, e^{-(1.17851)^2/2}$$

$$\boxed{P(\text{WAVE MAXIMA EXCEEDING } 5m) \approx 44.4\%}$$

b) 10m?
$$\eta_0 = \frac{10M}{\sqrt{18m^2}} = 2.357$$

$$\therefore P(\eta \geq 2.357) \approx 0.8\overline{888}\, e^{-(2.357)^2/2}$$

$$\boxed{P(\text{WAVE MAXIMA EXCEEDING } 10m) \approx 5.5\%}$$

c) FIND THE REQUIRED DECK HEIGHT TO HAVE 1% CHANCE OF FLOODING:

$$0.01 \approx 0.8\overline{888}\, e^{-\eta_0^2/2}$$

$$0.01125 \approx e^{-\eta_0^2/2}$$

$$2(\ln 0.01125) \approx -\eta_0^2$$

$$\eta_0 \approx 2.99579 = \frac{A}{\sqrt{M_0}} \quad \therefore A \approx (2.99579)\sqrt{18m^2}$$

$$\boxed{A \approx 12.71m}$$

3) $$f(t) = \sum_{i=1}^{N} f_i \cos(\omega_i t + \phi_i)$$ GAUSSIAN WITH ZERO MEAN

a) $$(m + a_{33})\ddot{x}(t) + (b_{33})\dot{x}(t) + (c_{33})x(t) = f(t)$$

where m = ship's mass

a_{33} = added mass coefficient

b_{33} = damping coefficient

c_{33} = restoring coefficient

$x(t)$ = heave motion

b) $$H(\omega) = \frac{1}{-\omega^2(m + a_{33}) + i\omega b_{33} + c_{33}} e^{i\phi_i}$$

c) INPUT TO LTI SYSTEM IS GAUSSIAN WITH ZERO MEAN, THEREFORE OUTPUT IS GAUSSIAN WITH ZERO MEAN

$$f(t) \longrightarrow \boxed{\text{LTI}} \longrightarrow x(t)$$

d) VARIANCE OF THE HEAVE IS THE SAME AS THE 0^{th} MOMENT OF THE SPECTRUM OF THE HEAVE $\sigma_x^2 = \int S_x(\omega)\,d\omega$,

AND FROM WIENER-KINCHINE, $S_x(\omega) = S_F(\omega)\,|H(\omega)|^2$

so $\sigma_x^2 = \int S_F(\omega)\,|H(\omega)|^2\,d\omega$ where $H(\omega)$ is in part b).

e) THINK OF ACCELERATION OF HEAVE AS AN LTI SYSTEM WITH HEAVE.

$$x(t) \longrightarrow \boxed{\text{LTI}} \longrightarrow \ddot{x}(t)$$

From part c), HEAVE IS GAUSSIAN WITH ZERO MEAN,

$\therefore$ HEAVE ACCELERATION, $\ddot{x}(t)$, IS ALSO GAUSSIAN WITH ZERO MEAN.

3) f)

$$f(t) \longrightarrow \underset{\text{LTI}}{\boxed{H(\omega)}} \longrightarrow x(t) \longrightarrow \underset{\text{LTI}}{\boxed{H_1(\omega)}} \longrightarrow \ddot{x}(t)$$

$$S_x(\omega) = S_f(\omega)\,|H(\omega)|^2 \qquad S_{\ddot{x}}(\omega) = S_x(\omega)\,|H_1(\omega)|^2$$

$$S_{\ddot{x}}(\omega) = S_f(\omega)\,|H(\omega)|^2\,|H_1(\omega)|^2$$

where $H(\omega) = \dfrac{1}{-\omega^2(m+a_{33}) + i\omega\, b_{33} + c_{33}}\, e^{i\phi_i}$

and $H_1(\omega) = -\omega^2$

g) YES. THE OUTPUT FROM AN LTI SYSTEM WITH A GAUSSIAN INPUT IS ALSO GAUSSIAN.