Department of Ocean Engineering
Massachusetts Institute of Technology Mechanical Engineering

2.20 (13.021) Marine Hydrodynamics

Supplemental Problems

A. ADDED MASS

Aa1. A sphere of volume V in a fluid of density ρ is located at a point (0,L,0) with respect to a certain coordinate system. *In terms of this coordinate system,* (a) identify whether each of the 6 × 6 added mass coefficients m_{ij} are zero (0), or non-zero (×) (do not work out any values).

(b) If the sphere has *generalized* velocity of (0,U_2,0,0,0,U_6), the total kinetic energy of the surrounding fluid is ________________.

Aa2. A sphere is located at $(0, 0, L)$ relative to a given coordinate system as shown. In a table for the added mass coefficients m_{ij}, $i, j = 1, 2, \ldots, 6$, mark all the values: "+" if it is positive, "−" if it is negative, and "0" if it is zero.

Aa3. A circular cylinder has radius a=1cm and length L=1m. Its added mass in water can be estimated as m_{22}=________________; m_{55}=________________; and m_{44}=________________. If the cylinder is translating through water with velocity U_2=0.5m/s, assuming potential flow and ignoring the mass of the cylinder itself, the total amount of work required to bring the cylinder to rest is ________________.

Aa4. A certain body with added mass coefficients m_{ij} has constant velocities $U_1, U_2 \neq 0$ and all other U_i, $\dot{U}_i$, $\Omega_k = 0$. In terms of the added mass coefficients m_{ij}: (a) the forces and moments on the body (in the body coordinates) are F_1=________________; F_2=________________; F_3=________________; and M_3=________________; (b) the linear momentum of the surrounding fluid in the x_1 direction is ________________; and (c) the total kinetic energy in the fluid is ________________.

Aa5. A certain body has nonzero added mass coefficients only on the diagonal, i.e., m_{ij}=$m_i\delta_{ij}$. For a body motion given by U_1=t and U_2=$-t$, and all other U_i, Ω_i=0, the forces and moments on the body in terms of m_i are F_1=__________, F_2=__________, F_3=__________, M_1=__________, M_2=__________, M_3=__________. The total kinetic energy in the fluid at time t=1 is __________.

Aa6. A sphere of volume 1 m^3 accelerates at $\dot{U}_1$= 2m/s^2 while at the same time the surrounding fluid (density ρ= 1 kg/m^3) is accelerated at $\dot{V}_1$= 1 m/s^2. The horizontal force on the sphere is F_1=________________. If $\dot{V}_1$ remains the same, this force will vanish if $\dot{U}_1$=________________.

Aa7. A 2D circular cylinder of radius $1m$ moves in an unbounded fluid at $U = 15m/s$. Assuming potential flow, the total amount of work done to bring this cylinder to rest is ________________per unit width.

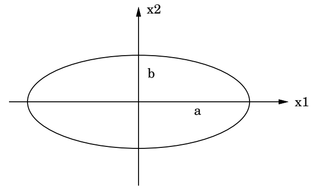
Aa8. A two-dimensional square box of width $2m$ accelerates to the right at an acceleration of $1m/s^2$ while at the same time the surrounding fluid also accelerates to the right at $1m/s^2$ (both with respect to a fixed coordinate system). The total horizontal force on the square is ________________N/m to the [right] [left].

Aa9. A sphere of radius $R = 1m$ and density $\rho_s = 2.5\rho_{water}$ is released in a current of velocity $U(t) = At$ where $A = 10m/s^2$. (a) In the absence of gravity, calculate the *initial* ($t = 0$) *horizontal* acceleration $\dot{u}$ of the sphere. (b) If the current is absent but there is gravity, calculate the *initial vertical* acceleration $\dot{v}$ of the sphere. (c) If both the current and gravity are present, calculate the angle θ (relative to the vertical) the sphere will tend to move initially.

Aa10. A 2D square body of dimension $2m \times 2m$ in water travels at velocity $\vec{U} = (1, 2)m/s$, acceleration $d\vec{U}/dt = (2, 3)m/s^2$, and no rotation ($\Omega = 0$). Per unit ($1m$) depth of this body: (a) calculate the forces F_1, F_2 and moment M_3 on this body; (b) obtain the linear momentum $\mathcal{L}$ of the fluid around the body in the x_1, and x_2 directions; (c) find the kinetic energy KE of the fluid.

A1. A two-dimensional ellipse has the following added mass coefficients:

$$m_{11} = \pi\rho b^2$$
$$m_{22} = \pi\rho a^2$$
$$m_{66} = \frac{1}{8}\pi\rho(a^2 - b^2)^2$$



Find the hydrodynamic forces F_1 and F_2 and moment M on the ellipse, per unit span, when its translational and angular velocities through an infinite inviscid fluid are the following functions of time t:

(a) $U_1 = 1 \quad U_2 = 0 \quad \Omega = 0$

(b) $U_1 = 0 \quad U_2 = 0 \quad \Omega = t$

(c) $U_1 = 4t^2 \quad U_2 = 0 \quad \Omega = 0$

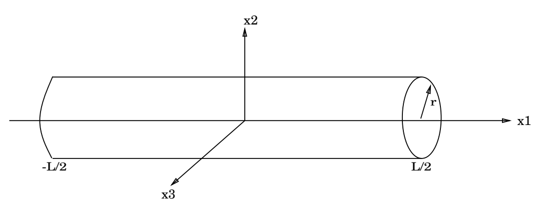
(d) $U_1 = 0 \quad U_2 = 4t \quad \Omega = 0$

(e) $U_1 = 3t \quad U_2 = 2t \quad \Omega = 0$

(f) $U_1 = 3t \quad U_2 = 2t \quad \Omega = t$

(g) If the ellipse is now stationary, but the infinite fluid is moving past it in the $+x_2$ direction at a velocity $U = 4t$, find the hydrodynamic forces and moments on the ellipse.

A2. A slender vehicle operating in an infinite fluid of density ρ can be modeled as a circular cylinder of length L and radius r.



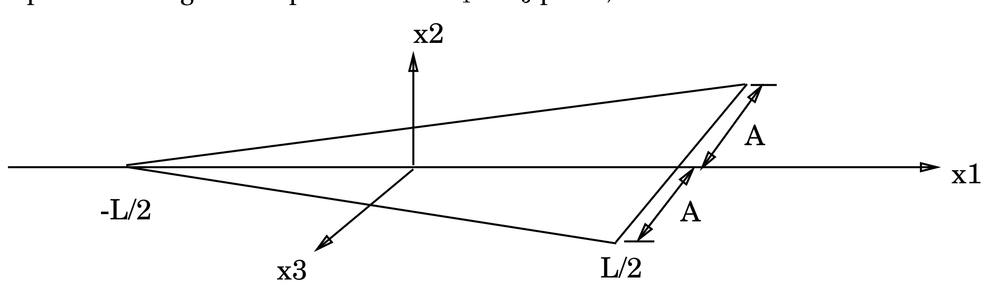
Using strip theory, estimate

(a) the added mass coefficient m_{22}

(b) the added moment of inertia m_{55}

(c) the added moment of inertia m_{44}

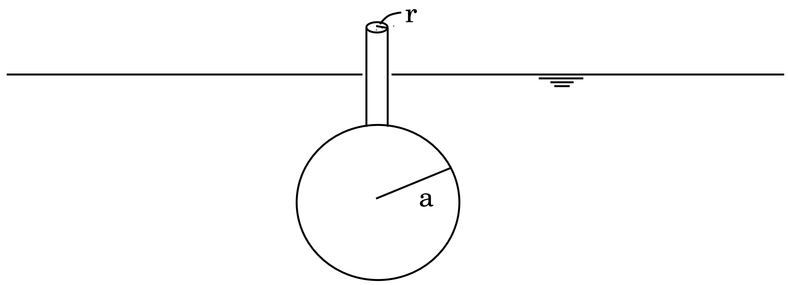
A3. A flat plate of triangular shape lies in the $x_1 - x_3$ plane, as shown:



(a) Assuming that $A/L << 1$, use strip theory to find m_{22} and m_{66}.

(b) What other elements of the added mass matrix are non-zero?

A4. A buoy consists of a large sphere under a circular cylinder, as shown:

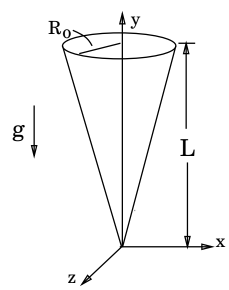


The volume and added mass of the cylinder are negligible compared to those of the sphere.

(a) Write the equation of motion for heave.

(b) Estimate the buoy's natural frequency in heave.

A5. A cone of negligible density is pivoted about the apex in a fluid of density ρ. The length L is much larger than its largest radius R_0.



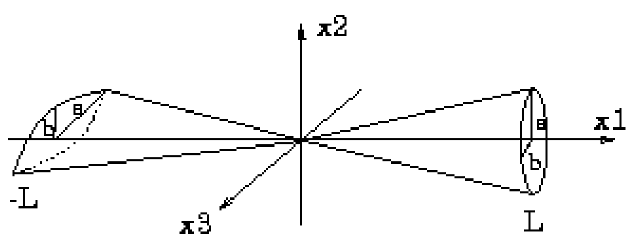
(a) Using strip theory, find the added moment of inertia about the apex (m_{66}).

(b) Calculate the center of buoyancy .

(c) Write the equation of motion for roll.

(d) Find the natural roll frequency.

A6. A body is composed of two cones of elliptical cross-section. The cones are aligned as shown along the x_1-axis. Each section has minor axis of length $2b$ and major axis of $2a$. The cones are arranged so that the major axis of the elliptical section is parallel to the x_2-axis for $x_1 > 0$ and parallel to the x_3-axis for $x_1 < 0$. The ratio a/b is constant at all sections and $a(x_1) = c|x_1|$. Each cone is of length L so that the composite body is of length $2L$. $L >> a$ and $a > b$.



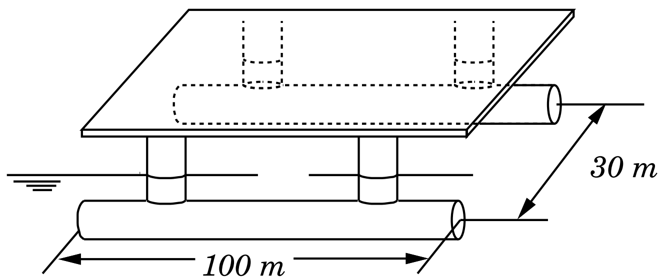
Calculate:

(a) m_{33}; (b) m_{53}; (c) m_{44}; (d) m_{66}.

(e) Indicate schematically which m_{ij} can be obtained by means of the slender-body approximation and which cannot.

(f) Comment on the limits of applicability of the slender-body approximation for this object. In particular, if $L \approx a$ but $a >> b$, which m_{ij} would be suspect?

A7. A semi-submersible platform has the configuration shown:

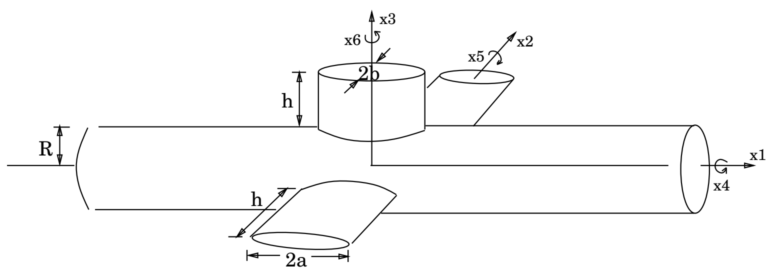


The diameter of the uprights is 5m, and that of the pontoons is 10m. The volume displaced by the uprights is negligible compared to that of the pontoons.

(a) Estimate the added mass in heave, neglecting the effects of the free surface, the uprights and the interactions between the pontoons.

(b) Use the result of (a) to estimate the natural frequency in heave of the platform.

A8. A new class of submarine can be modeled by a cylinder of length L and radius R, with a vertical sail and horizontal elliptical wings of major and minor axis radii a and b and length h, as shown.



Assuming that these main members are slender so that their longitudinal added mass may be ignored, and neglecting also the interactions among the members, find

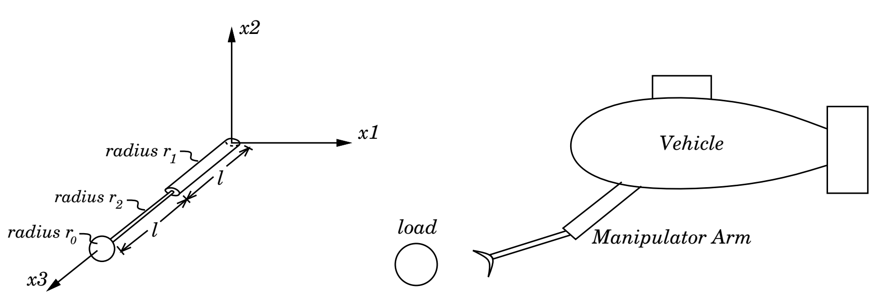
(a) m_{33}; (b) m_{35}; and (c) m_{55}.

(d) Find the instantaneous force and moment $\vec{F}$, $\vec{M}$ on the submarine at an instant when its 6 degree-of-freedom motions are: velocity [1,2,3,1,2,3] and acceleration [3,2,1,3,2,1]. You may leave your answers in terms of m_{ij}.

A9. An underwater vehicle is to have a manipulator arm mounted on it. The designers of the vehicle must know the forces and moments acting on the arm and the objects it manipulates so that they can select appropriate actuator motors. You may assume the following simplifications for this problem:

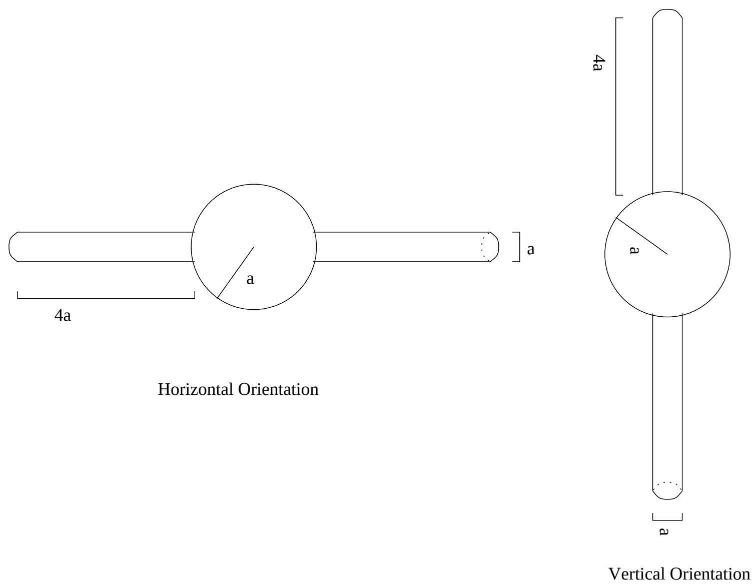
Idealize the arm as two circular cylinders each of length l having radii r_1 and r_2 respectively, with $r_1 = 2r_2$.

Idealize the arm's load as a spherical package of radius r_0. The radius r_1 is small compared to r_0, and r_0 is small compared to l.



In the coordinate system shown, estimate m_{11} and m_{44}.

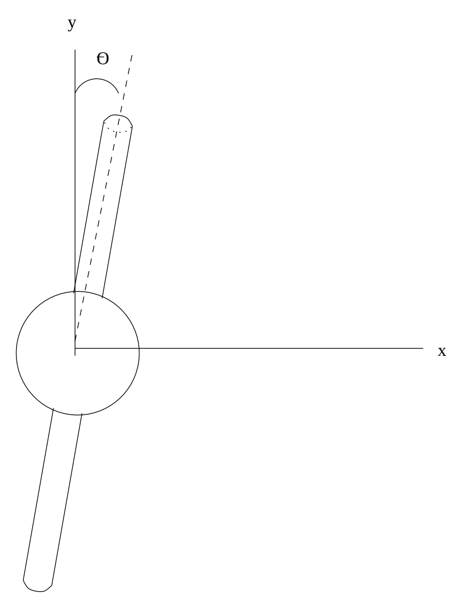
A10. Housing for certain underwater sensor equipment has a geometry shown below. The sphere has radius a, and the cylinders have radius $0.5a$ and length $4a$. The density of the device can be assumed to be uniform and have a value of twice that of water.



(a) To get it to the sea floor, the device is lowered into the water and then released, find its *initial* acceleration for (i) a vertical orientation; and (ii) a horizontal orientation.

(b) Assuming deep water, calculate the terminal drop velocity V_3 for the device falling in a *horizontal* orientation for (i) very small a; and (ii) large a.

(c) An engineer is concerned that if dropped in a vertical orientation, the device may become "unstable" and reach the bottom in an unpredictable manner. If the equipment is falling with steady downward velocity U at a small angle θ_6 from its vertical position, estimate, based on potential flow effects only, the overturning moment M_3 on the device as a function of U and θ_6.



B. BASIC EQUATIONS

Ba1. A fluid flow is steady if [$\partial/\partial t$] [D/Dt] [both $\partial/\partial t$ and D/Dt] [either $\partial/\partial t$ or D/Dt] of the flow variables is/are zero.

Ba2. The two common ways of describing fluid flow are the so-called __________________ and the __________________ descriptions.

Ba3. Although fluids, such as water, are really made up of discreet molecules, we are able to describe their behavior by differential equations by virtue of a __________________ hypothesis. Fluids differ from solids in that a fluid at rest cannot sustain __________________.

Ba4. If the velocity field of a flow is given by $\vec{v}(\vec{x},t)$, the acceleration of any fluid particle is given by $\vec{a}(\vec{x},t)$= ______________________________. If a vector 'acceleration' meter moves around the flow at a prescribed velocity $\vec{U}$, the acceleration it records at a point $\vec{x}$ is given by __________________.

Ba5. The temperature at any point in a room is given by $\varphi(x,y,z,t)$. A small fly flies around the room with a velocity $\vec{U}$. The time rate of change of temperature experienced by the fly is given by ______________________________.

Ba6. An ROV measuring water salinity is moving with velocity $2xt\hat{\imath} + 4y^2\hat{\jmath} - 3t\hat{k}$. The salinity S of the water changes with the tidal currents, and is given by $S(x,y,z,t) = 2xcos(at)$. Find the rate of change of salinity of the water as measured by the ROV. ______________________________

Ba7. In a certain river with a velocity field $\vec{v}(\vec{x},t)$, the concentration of dissolved oxygen is given by the function $\vec{f}(\vec{x},t)$. A fish is in the river, with (absolute) velocity $\vec{U}(t)$. The (time) rate of change of dissolved oxygen concentration f experienced by the fish is given by __________________. The fish lays eggs which are very small and neutrally buoyant, the rate of change of f as experienced by the eggs is __________________. Some of the eggs eventually become trapped among rocks and remain there, the rate of change of f experienced by those eggs is then given by __________________.

Ba8. To determine the temperature Θ in a certain part of the ocean, a *fixed* probe measures a rate of change of temperature given by bt, while a heavy probe dropped from the surface and reaching a constant downward velocity of $-W$ records a rate of change of temperature in that area given by $-aW + bt$. If the temperature in that region is known to be independent of the horizontal coordinate, *i.e.*, $\Theta = \Theta(z,t)$, then the temperature there is given by $\Theta(z,t)$=______________________________+ constant. If the fluid velocity there is given by the previous problem, a temperature probe drifting freely with the flow will record a rate of change given by __________________.

Ba9. A small probe moves with (absolute) velocity $\vec{U}$ in water which has a velocity field given by $\vec{v}(\vec{x},t)$. If the probe measures relative velocity, the measured velocity is $\vec{V}(t)$=__________________. If $\vec{U}$= constant, the measured "acceleration" at the probe is $\vec{A}(t) \equiv d\vec{V}/dt =$ __________________. If $\vec{U} = \vec{U(t)}$, $\vec{A}(t)$= __________________. If the probe is fixed, $\vec{U}$=0, $\vec{A}(t)$= __________________.

Ba10. If ρ is the density and $\vec{v}$ the velocity of a fluid, the condition of *incompressibility* is expressed mathematically as __________________. In marine hydrodynamics, incompressibility is often a valid assumption because the [velocity, pressure, shear stresses, temperature, gravity] is [much greater, comparable, much smaller] than that of sound waves.

Ba11. The *differential* equation governing the conservation of mass of a fluid, whether compressible or not, is ______________________________. If the fluid is incompressible, then the density ρ satisfies ______________________________.

Ba12. The conservation of mass equation depends on the assumption(s) of [constant density] [irrotationality] [inviscid fluid] [incompressibility] [Newtonian fluid] [matter cannot be created]. For incompressible flow, the density ρ satisfies the equation __________________.

Ba13. The velocity field in a certain part of the ocean is given by $u = 3\cos(3x+4y)e^{5z}$, $v = 4\cos(3x+4y)e^{5z}$ and $w = c\sin(3x+4y)e^{5z}$. The constant c must be c=__________________.

Ba14. Dye is injected into a flow at a given point. In *steady* flow, the locus of all dyed particles forms a [pathline] [streamline] [streakline] [none of the above]. In *unsteady* flow, the locus of all dyed particles forms a [pathline] [streamline] [streakline] [none of the above]. A small neutrally-buoyant particle is released into the flow at another point. In *steady* flow, the line that the particle makes forms a [pathline] [streamline] [streakline] [none of the above]. In *unsteady* flow, this line forms a [pathline] [streamline] [streakline] [none of the above].

Ba15. A tanker grounds on a reef and begins to leak a neutrally buoyant chemical into the ocean. The trace of the chemical as shown in a picture taken from the sky forms a ________________. At some time the captain of the vessel abandons ship and jumps into the ocean. Assuming that he drifts with the current, the trajectory he moves through describes a ________________.

Ba16. In a certain rescue operation, a marker is dropped onto the water from a helicopter. The trajectory traced by the drifting marking forms a [streamline, pathline, streakline, none of the above]. Eventually, the marker is observed to again pass the point where it was dropped. The flow must be [rotational, irrotational, can't tell]. To learn more about the flow, the helicopter pilot releases a large number of similar markers at the same point in quick succession. The line connecting these markers at a later instant is a [streamline, pathline, streakline, none of the above]. It is now observed that all of the markers have identical trajectories and all eventually pass through the drop-off point after equal time intervals. The flow is most likely [rotational, irrotational, can't tell].

Ba17. Of the following: (a) velocity; (b) pressure; (c) shear stress; (d) density; (e) vorticity; (f) velocity potential; (g) mass flux; (h) momentum flux.

The scalar quantities are (write only the letters):______________________________

The vector quantities are:____

The tensor quantities are:____

Ba18. A certain fluid flow has a velocity field $\vec{v}(\vec{x},t)$ and density $\rho(\vec{x},t)$. The rate of change of buoyancy experienced by a small fish swimming with (absolute) velocity $\vec{U}$ and maintaining constant volume V is ____________________________. After a while, the fish stops swimming but instead tries to adjust its volume to remain neutrally buoyant (and therefore freely drifting). The rate of change of its volume dV/dt must be proportional to ________________.

Ba19. A small thin flat disk of surface area A is oriented in a fluid with directional cosines given by (n_1, n_2, n_3). If the stress tensor there is τ_{ij}, $i,j = 1,2,3$, the net force on the disk is given by F_i=________________, $i = 1,2,3$.

Ba20. The stress tensor in a flow is given by $\tau_{ij} = i + j$; i,j=1,2,3. The force acting on a small surface, area δA, with unit normal (into the surface) given by $\hat{n} = (1,-1,-1)/\sqrt{3}$ is $\delta\vec{F}$=(________________, ________________, ________________).

Ba21. The basic assumption of a Newtonian fluid is that there is a linear relationship between ____________________________and ____________________________. The constant of proportionality is given the name ____________________________.

Ba22. The velocity of a Newtonian fluid is two dimensional and given by $\vec{v} = (x^2y, -xy^2)$. The stress tensor has components: τ_{xx}= ________________, τ_{xy}= ________________, τ_{yy}= ________________, τ_{yx}= ________________.

Ba23. The velocity v_i and the total stress τ_{ij} of a fluid is related by the Euler equations which are given in Einstein notation form as: ____________________________. If the fluid is Newtonian, the shear stresses are related to ________________by ________________relationships, and can be eliminated to give the following momentum equations for v_i and the pressure p ____________________________.

Ba24. The Navier-Stokes equations govern the conservation of ________________ of a fluid and assumes that the fluid is ________________ and ________________. For a flow which is *steady, one-dimensional* ($u = u(x)$ and $\partial/\partial y = \partial/\partial z = v = w = 0$), in the presence of a body force given by the potential $-\rho g(x \sin\alpha + y\cos\alpha)$ (where α is a constant), reduce these equations (in Cartesian form).

Ba25. If p is the pressure and τ_{ij} the shear stress in a fluid, the force on a fluid volume $\mathcal{V}$ bounded by a closed surface $\mathcal{S}$ which has normal n_j, j=1,2,3, is given by $F_i = \int\int_{\mathcal{S}}$ ________________ dS. This can also be expressed as $\int\int\int_{\mathcal{V}}$ ________________ dV.

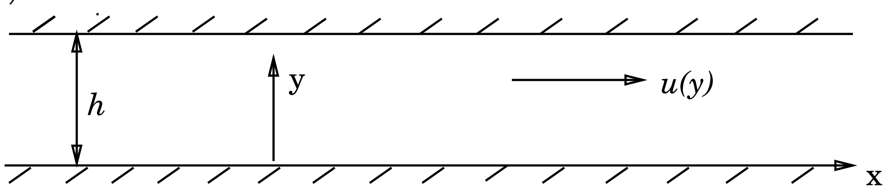
Ba26. For a viscous flow with velocity $\vec{v}$, the kinematic boundary condition on a body with surface velocity $\vec{U}$ and surface normal $\hat{n}$, specifies that ________________. The dynamic condition in this case requires the continuity of ________________. For ideal flow, the kinematic condition becomes ________________ and the dynamic condition specifies the continuity of ________________. In the absence of dynamic boundary conditions, the body force due to gravity g can be eliminated from the momentum equations by replacing the total pressure p with ________________.

Ba27. A fish swims by moving its body $B(\vec{x}, t)$ with velocity $U(\vec{x}, t)$. The proper boundary condition on the fish assuming *viscous* fluid is ____________________________. If the fluid is *inviscid*, the appropriate boundary condition on the fish would be ________________.

Ba28. A spherical gas bubble of radius R and internal pressure p_0 is formed in water. If the surface tension coefficient between the gas and water is Σ, the pressure outside the bubble must be p=____________________________. In terms of (kg-m-s) the units of Σ is ________________.

Basic Equation problems continued next page

B1. Consider a two-dimensional steady viscous flow between two parallel walls where $u = u(y)$, $P = P(x)$ and $v = w = 0$:



(a) Determine whether or not this flow satisfies continuity.

(b) Find the simplified Navier-Stokes governing equation for u.

(c) Given $P(x) = -2x + 3$. Show that $u(y) = C_1 + C_2 y - y^2/\mu$, satisfies the resulting equation of (b). (where C_1 and C_2 are constants.)

(d) If the lower and upper walls are stationary and located at $y = 0, h$ respectively, apply suitable boundary conditions to determine C_1 and C_2.

(e) Calculate the shear stress τ_{xy} anywhere in the fluid. What is its value τ_0 at the lower wall?

(f) By using a simple control volume for momentum conservation, show that given $P(x)$ only (and *not* $u(y)$), τ_{xy} can be obtained in terms of τ_0.

B2. The x-component of the Navier-Stokes equations is:

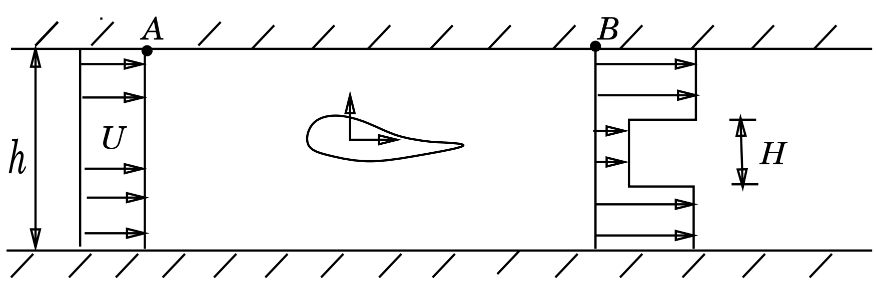
$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = f_x - \frac{1}{\rho}\frac{\partial P}{\partial x} + \nu\left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right] + \frac{1}{3}\nu\frac{\partial \Theta}{\partial x}$$

(a) Reduce this to the special case of steady 2-dimensional incompressible flow between two infinite parallel plates in the absence of body forces. In doing this, give a physical interpretation of each term and carefully state the reason for keeping or eliminating it.

(b) What boundary condition(s) would need to be imposed in order to solve the problem?

(c) What additional assumption(s) would one need to obtain Poiseuille flow in a channel?

B3. A rudder is tested in a two-dimensional flow tunnel of height $h = 1$m, as shown:



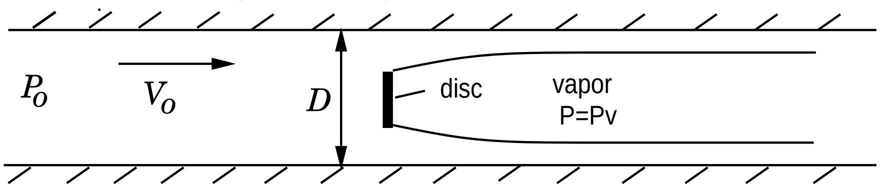
After the flow has become STEADY, the horizontal velocity far ahead of the rudder is found to be uniform and given by $u = U = 2$m/s. Some distance downstream, the velocity profile can be (very) roughly approximated by a region inside the wake with $u = U/2$ and an exterior region with $u = 3U/2$. You may assume that the total pressure at the downstream station isuniform across the tunnel and given by the value at the wall.

(a) Determine the height of the wake region H.

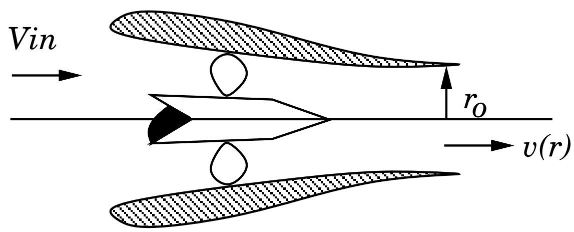
(b) Assuming ideal fluid flow (no shear stress), calculate the pressure difference $P_A - P_B$.

(c) Calculate the drag force D on the body.

B4. A circular disc is mounted in a circular water tunnel of diameter D. The velocity far upstream is V_0. When the upstream pressure is dropped to P_0, a long vapor cavity forms behind the disc. The pressure in the cavity is $P_v < P_0$. The velocity in the fluid surrounding the cavity can be assumed constant far downstream. Ignoring gravity and letting the vapor density be zero, find the drag on the body in terms of V_0, P_0, P_v, D and the fluid density ρ.



B5. A ducted propeller (such as a so-called Kort nozzle) receives water at a velocity of 3 m/s. The water has a density of 1000 kg/m^3. The inflow velocity is uniform and the cross-sectional area of the upstream end of the duct is 0.5 m^2. The velocity at the duct exit is not uniform but is given by $v = 2V_0(1 - \frac{r^2}{r_0^2})$, where r_0 is the radius of the circular cross-section and $V_0 = 5$m/s.

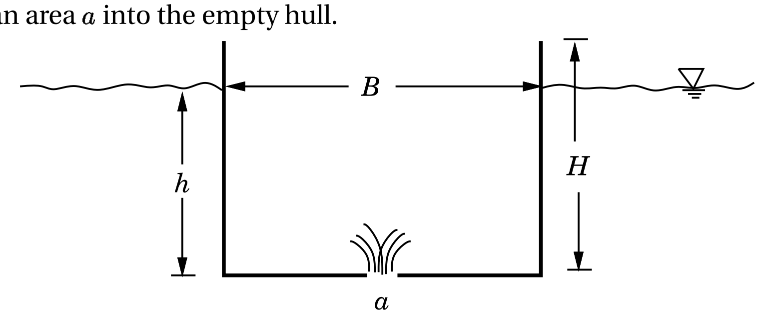


(a) Show that V_0 is the average exit velocity.

(b) Find the thrust.

(c) If the exit velocity was uniformly equal to V_0, what would the thrust be then?

B6. A two-dimensional barge with dimensions as shown in the figure is holed in its bottom. The hold opens an area a into the empty hull.



The top of the barge is open to the atmosphere. The hole is small compared to the dimensions of the barge so the barge sinks very slowly. Therefore, you may treat this as a quasi-steady problem.

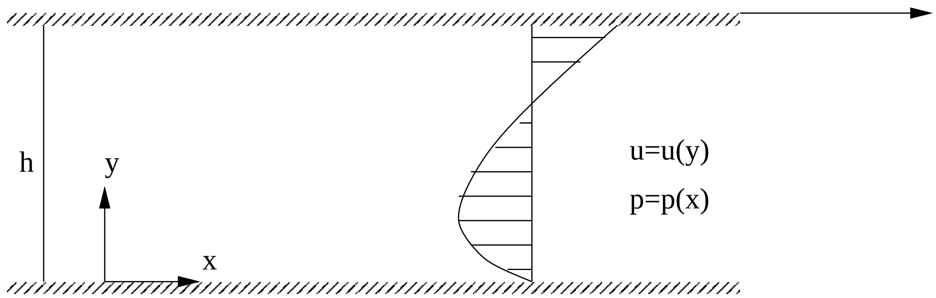
(a) What does the barge initially weigh?

(b) What quantity of water must enter the barge for it to sink?

(c) Show that the velocity of water entering the tank is constant in time. You may assume the velocity is uniform across the area of the hole.

(d) How long will it take the barge to sink?

B7. Consider a steady, incompressible flow of a viscous fluid between two infinite plates separated by a distance h:



A constant shear stress τ_t is applied on the wall at $y = h$ which induces a constant shear stress at the bottom τ_b, a pressure variation $p(x)$, and a flow profile $u(y)$ sketched above.

(a) Write down the (simplified) governing equation(s) and the boundary condition(s) for this problem.

(b) Using a control volume argument, write down an equation for $\frac{dp}{dx} = f(\tau_t, \tau_b)$.

(c) Using the $\frac{dp}{dx}$ found in (b), solve the governing equations for $u(y)$ in terms of τ_t and τ_b.

(d) If $u(h) = U$, find τ_b as a function of τ_t and U.

(e) If it is given that the volume flow rate Q=0, find an equation for the velocity profile $u(y)$ as a function of the applied shear stress τ_t and h only.

(f) What is the position of the point Y where $u(Y)$=0 for the profile you obtained in (e)?

C. CALCULUS

The following problems are intended to refresh some concepts of vector calculus needed for this course. For all problems, the gradient operator in the Cartesian coordinate system is:

$$\vec{\nabla} \equiv \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$$

In cylindrical polar coordinates it is defined as:

$$\vec{\nabla} \equiv \vec{e_r}\frac{\partial}{\partial r} + \vec{e_\theta}\frac{1}{r}\frac{\partial}{\partial \theta} + \vec{e_z}\frac{\partial}{\partial z}$$

We define $D/Dt \equiv \partial/\partial t + \vec{v}\cdot\vec{\nabla}$ as the material derivative.

C1. Given the scalar function $\phi(x,y,z)$ expand the following:

(a) $\vec{\nabla}\phi$

(b) $\vec{\nabla}\cdot\vec{\nabla}\phi \equiv \vec{\nabla}^2\phi$

(c) $\vec{\nabla}\phi\cdot\vec{\nabla}\phi \equiv |\vec{\nabla}\phi|^2$

(d) $\vec{\nabla}\times\vec{\nabla}\phi$

(e) for any $\phi(r,\theta,z)$, $\vec{\nabla}^2\phi$

C2. Given the scalar function $\phi(x,y) = yx^3 + \sin x$. Evaluate the following:

(a) $\vec{\nabla}\phi$

(b) $\vec{\nabla}^2\phi$

(c) $\vec{\nabla}\times\vec{\nabla}\phi$

C3. Given the vector functions $\vec{v} = u\hat{i} + v\hat{j} + w\hat{k}$ and $\vec{U} = \xi\hat{i} + \psi\hat{j} + \zeta\hat{k}$, where u, v,w,ξ, ψ, and ζ are scalar functions of (x,y,z,t) Evaluate the following:

(a) $\vec{\nabla}\cdot\vec{v}$

(b) $\vec{\nabla}\times\vec{v}$

(c) show $\vec{v}\times\vec{U} = -\vec{U}\times\vec{v}$

(d) $\vec{v}\cdot\vec{\nabla}P$ where P is a scalar function of (x,y,z,t)

(e) $\vec{v}\cdot\vec{\nabla}\vec{v}$

(f) $\vec{v}\vec{\nabla}\cdot\vec{v}$

C4. For $\vec{v}$ and P above, with constants ρ and ν

(a) Expand the following equation into its three components in the Cartesian coordinate system:

$$\frac{D\vec{v}}{Dt} \equiv \left(\frac{\partial}{\partial t} + \vec{v}\cdot\vec{\nabla}\right)\vec{v} = -\frac{1}{\rho}\vec{\nabla}P + \nu\vec{\nabla}^2\vec{v}$$

(b) Let $\vec{\omega} = \vec{\nabla}\times\vec{v}$. Take the curl ($\vec{\nabla}\times$) of the equation in part (a) to yield the following vector equation:

$$\frac{D\vec{\omega}}{Dt} - \nu\vec{\nabla}^2\vec{\omega} = \vec{f}$$

HINT: Use the vector identity:

$$\vec{\nabla}\times(\vec{a}\times\vec{b}) = (\vec{b}\cdot\vec{\nabla})\vec{a} - (\vec{a}\cdot\vec{\nabla})\vec{b} + \vec{a}(\vec{\nabla}\cdot\vec{b}) - \vec{b}(\vec{\nabla}\cdot\vec{a})$$

(c) Determine $\vec{f}$ as a function of $\vec{v}$ and $\vec{\omega}$.

(d) Simplify $\vec{f}$ for the two dimensional case, i.e.

$$\vec{v} = u\hat{i} + v\hat{j},\ \frac{\partial}{\partial z} = 0.$$

C5. Gauss Theorem states that for a volume V enclosed by a surface S the following applies for any vector whose derivatives exist in V and on S:

$$\int_V \vec{\nabla} \cdot \vec{F} dV = \int_S \vec{F} \cdot \vec{n} dA$$

If $\vec{F} = -(z^2 - 1 - 2y^2)\hat{\imath} + (z^2 - 1 - 2x^2)\hat{\jmath} + (x^2 + y^2 + z^2)\hat{k}$ and S is a negative hemisphere centered at the origin, verify this theorem.

C6. Stokes Theorem states that for a Surface s enclosed by a curve C the following applies for any vector whose derivatives exist on S and C:

$$\int_S \vec{n} \cdot (\vec{\nabla} \times \vec{F}) dS = \oint_C \vec{F} \cdot d\vec{\ell}$$

If $\vec{F} = (1 + 2y^2)\hat{\imath} - (1 + 2x^2)\hat{\jmath}$ and S is a circle centered at the origin, verify this theorem.

C7. Given the vector function $\vec{v} = x^3\hat{\imath} + xy^2\hat{\jmath} + xz\hat{k}$, find the circulation of $\vec{v}$ around the rectangle with corners (in (x, y, z) Cartesian coordinates) at (2,1,0), (-2,1,0), (-2,-1,0) and (2,-1,0), where circulation is defined as:

$$\oint_C \vec{v} \cdot d\vec{\ell}$$

where C is the bounding path and $d\vec{\ell}$ is the unit tangent vector to C.

HINT: Recall Stokes' Theorem: $\int\int_S (\vec{\nabla} \times \vec{v}) \cdot \hat{n} dS = \oint_C \vec{v} \cdot d\vec{\ell}$
where S is any surface bounded by C and $\hat{n}$ is the unit normal vector to the surface.

C8. Given a vector field in polar coordinates $\vec{v}(r, \theta) = v_r \hat{e_r} + v_\theta \hat{e_\theta}$ where

$$\begin{array}{lcll} v_r & = & 0 & \text{for any } r \\ v_\theta & = & K\frac{r}{a^2} & \text{for } r < a \\ & & K\frac{1}{r} & \text{for } r > a \end{array}$$

(a) What are the units of K?

(b) Find $\oint_C \vec{v} \cdot d\vec{l}$, where C is a circle of radius of $r < a$ and $d\vec{\ell}$ is the unit tangent vector to C.

(c) Find $\oint_C \vec{v} \cdot d\vec{l}$, where C is a circle of radius of $r > a$ and $d\vec{\ell}$ is the unit tangent vector to C.

(d) Does there exist a scalar potential ϕ such that $\vec{v} = \vec{\nabla}\phi$ for $r < a$? For $r > a$? If so, find $\oint_C \vec{v} \cdot d\vec{l}$ in terms of ϕ.

C9. Show that the "material derivative" defined by:

$$\frac{D}{Dt} \equiv \frac{\partial}{\partial t} + \vec{V} \cdot \vec{\nabla}$$

when applied to a vector $\vec{V} = \vec{V}(x, y, z, t)$ (with $\vec{\nabla} \times \vec{V} = 0$) yields:

$$\frac{D\vec{V}}{Dt} = \frac{\partial \vec{V}}{\partial t} + \frac{1}{2}\vec{\nabla}(\vec{V} \cdot \vec{V}) .$$

If $\vec{V}$ is given by $\vec{V} = \vec{\nabla}\phi(x, y, z, t)$, and the material derivative of $\vec{V}$ is zero, i.e., $D\vec{V}/Dt = 0$, show that the following quantity must be a function of time only:

$$\frac{\partial \phi}{\partial t} + \frac{1}{2}|\vec{\nabla}\phi|^2 = f(t)$$

C10. Consider the integral: $I(t) = \int_{a(t)}^{b(t)} f(x, t)\, dx$.

Given: $a(t) = 4t$, $b(t) = t + 5$, and $f(x, t) = \sin x \cos 2t$. (a) Evaluate $dI(t)/dt$ (Hint: recall Leibniz rule in calculus).

C11. A spherical balloon is being filled with air.

(a) If the radius of the balloon is given by $r(t) = \ln(t + 2)$, calculate the time dependent volume of the sphere $V(t)$ by means of a *volumetric integral* using a spherical coordinate system. Differentiate to obtain the time rate of change of the volume $dV(t)/dt$. Do not employ simple expressions or formulas for the volume of the sphere. Rederive the expressions by integrating in the spherical coordinate system, as described.

(b) Calculate $dV(t)/dt$ again by performing a surface integration of the normal velocity $U_n = dr(t)/dt$ over the surface of the sphere. Again, rederive all expressions for the surface area of a sphere by integration, and do not use known formulas.

(c) Guided by the problem above, write down a form of Leibniz rule in three dimensions (assuming spherical symmetry).

I. IDEAL FLUID FLOW

Ia1. Kelvin's theorem is a statement of the conservation of _________________and its statement and conditions are: ______________________________.

Ia2. Consider a rotating fluid between two circles radii R_1 and R_2 ($R_2 > R_1$). If the circulation at R_1 is Γ_1 and at R_2 is Γ_2, the average vorticity of the fluid is ω= _________________, and the average angular velocity is Ω= _________________.

Ia3. In a certain flow, a small marker particle released at point P is observed to pass the same point again after some time. The flow can NOT be: [steady] [unsteady] [rotational] [irrotational] [steady and irrotational] [unsteady and irrotational] [steady and rotational] [unsteady and rotational].

Ia4. The definition of the circulation along a contour $\mathcal{C}$ is Γ=_________________. For Γ to be meaningfully defined, it is necessary for $\mathcal{C}$ to be [closed] [material] [fixed in space] [not changing in time] [on a two-dimensional plane] [in an ideal fluid] [none of the above].

Ia5. A testing tunnel with circular cross sections has a radius of r_1=10m at the inflow and r_2=1m at the test section. A small vortex ring forming along the wall near the inflow has a cross-sectional area of A_1=0.05m^2. If the longitudinal velocity profile there is given by $u_1(r)$=$(1-(r/r_1)^2)$ m/s, the average (circumferential) vorticity in the ring is ω_1=_________________. At the test section, this ring will have a cross-sectional area of A_2=_________________; and an average vorticity of ω_2=_________________.

Ia6. A smoke ring is modelled as a vortex tube of diameter (across the tube) of $1cm$ and a circumference (length of the tube) of $1m$. If the vorticity inside the tube is constant and given by ω=$1s^{-1}$, the total circulation around the smoke ring is Γ=_________________. If the smoke ring expands to a circumference of $2m$, the diameter will become _________________cm, the circulation Γ=_________________, and the vorticity ω=_________________.

Ia7. In an ideal flow, a vortex tube spanning two side walls of a tank 1m apart has a cross-sectional area that varies linearly from 0.1 m^2 at one end to 0.2 m^2 at the other end. If the circulation around the tube is Γ= 1m^2/s, the average vorticity at the small end of the tube is ω_1=_________________. After some time, the tube area becomes a constant across the tank. The (constant) tube cross-sectional area is now _________________ m^2 and the average vorticity inside the tube is ω=_________________. Further downstream, the tank width increases to 2m. The average vorticity inside the tube (still of constant cross-sectional area) is now ω=_________________.

Ia8. A three-dimensional free vortex tube, having components of circulation $\vec{\Gamma} = [\Gamma_x, \Gamma_y, \Gamma_z]$, is in an infinite fluid which has steady translational velocity $\vec{U} = [u, v, z]$. The force on the vortex tube is given by $\vec{F} = [F_x, F_y, F_z]$ which has components $F_x/\rho =$_________________, $F_y/\rho =$_________________, and $F_z/\rho =$_________________.

Ia9. By rotating the wall of a circular tube, the water inside is made to rotate as a rigid body with angular velocity Ω. Assuming 2D flow, the radial and circumferential velocities (with respect to the axis of rotation z) is v_r=_________________and v_θ=_________________. The vorticity is ω=ω_z=_________________. At a given instant, the tank wall stops and the fluid motion eventually also stops starting from the outer circumference. This decay in motion can be described by the equation governing the vorticity ω: _________________.

If the radius of the tube is 1cm, the decay time is *of the order of* _________________sec.

Ia10.

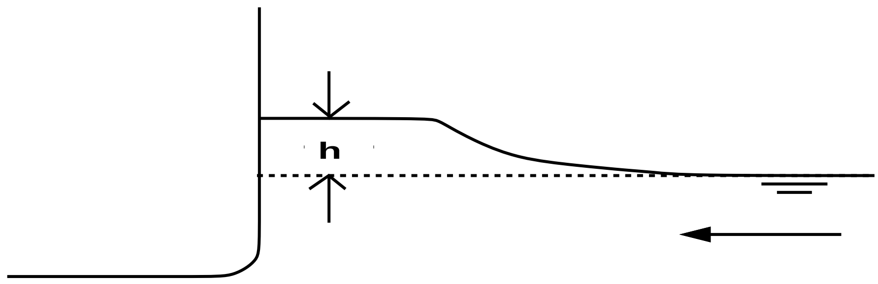
$$\frac{D\vec{\omega}}{Dt} = (\vec{\omega} \cdot \nabla)\vec{v} + \nu\nabla^2\vec{\omega}$$

This is the so-called ______________________________equation and the first, second and third terms represent respectively
first term:____.
second term:____.
third term:____.
In two-dimensional flows, this equation becomes (in vector form): ______________________.
For this flow, [mass] [linear momentum] [angular momentum] [translational velocity] [angular velocity] [energy] is/are conserved.

Ia11. In a vorticity equation, the term expressing the change of vorticity due to vortex elongation and rotation is __________________. In two dimensions, this term reduces to __________________.

Ia12. A tanker of draft H_w=4m carries oil of density 80% of sea water to a height H_o= 6m. If a small puncture hole is made on the bottom of the hull, the initial velocity of the leaking oil is __________________. (Assume ideal flow and open tanks.)

I1. As a ship advances through calm water at 6 knots, the water level at the bow is observed to be higher than the undisturbed free surface. How large is this change in water level?



I2. Consider the three-dimensional inviscid incompressible flow with velocity components given by:

$$
\begin{aligned}
u(x,y,z,t) &= -A(t)y - Bx \\
v(x,y,z,t) &= A(t)x - By \\
w(x,y,z,t) &= w(z)
\end{aligned}
$$

(a) Find $w(z)$ given $w(0) = 0$.

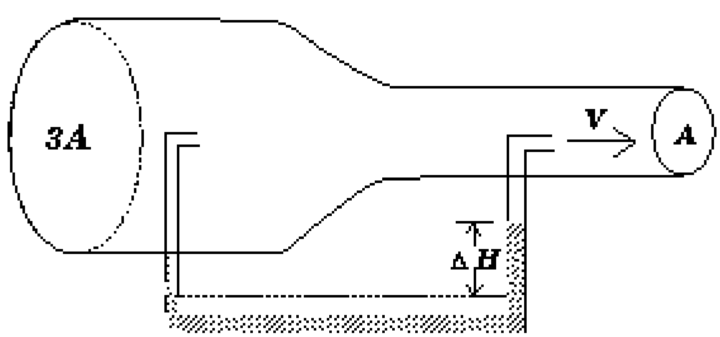
(b) Describe this flow in words and/or pictures.

(c) Determine the vorticity vector.

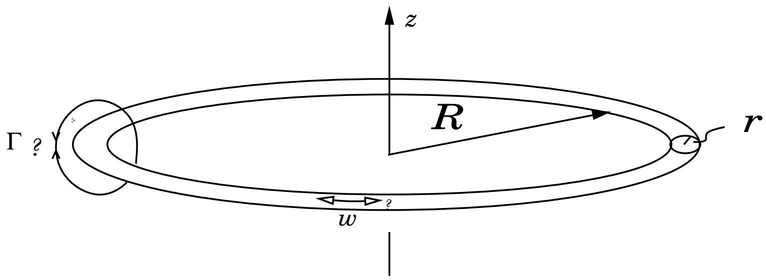
(d) Find the function $A(t)$ from the vorticity equation given $A(0) = 1$.

(e) Find the circulation $\Gamma(t)$ around a path of fluid particles initially (i.e. at $t = 0$) located in a circle of radius 1 on the plane $z = 1$.

I3. In order to determine the velocity through the test section of a propeller tunnel, a manometer is placed across the upstream contraction, which has an area ratio of 3 to 1. The manometer fluid has a density of 1.2 times that of water. Assuming that the velocity is uniform over any section, what velocity corresponds to a ΔH of 1 meter?



I4. A rising smoke ring is to be represented as a toroidal vortex filament of constant vorticity ω across the ring, and major and minor radii R and r. The flow is otherwise assumed to be ideal and irrotational.



The ring expands linearly with vertical distance travelled, z, as it rises, with radius given by $R(z) = R_0 + Az$ for some positive constant A.

(a) Determine the direction of the vorticity (clockwise or counterclockwise with respect to the positive z axis) within the filament. Explain your reasoning.

(b) What are the dependencies on z of (i) the circulation Γ around the ring; (ii) the radius r; and (iii) the vorticity ω given their initial values at $z = 0$ are respectively Γ_0, r_0 and ω_0?

I5. Consider the steady two-dimensionsal flow given in Problem B1, part (b). You may leave your answers in terms of the constants C_1 and C_2.

(a) Calculate the vorticity ω_z. Is the flow irrotational?
(b) Write down the vorticity equation for ω_z for the case of steady, two-dimensional flow. Does your answer in (a) satisfy this equation?
(c) Can you find a velocity potential ϕ and/or stream function ψ to describe the flow? Justify your answer and find ϕ and/or ψ if possible.
(d) Based upon the vorticity equation of (b), show that the velocity profile $u(y)$ can at most be a quadratic function for any pressure variation $P(x)$ or wall boundary conditions.

L. LIFTING SURFACES

La1. A two-dimensional hydrofoil has symmetric profiles given by (y_L, y_U)=$(-,+)a\cos(\pi x/\ell)$, $-\ell/2 \le x \le \ell/2$. According to thin-wing theory, the potential flow around the foil moving with longitudinal speed U can be modeled by a line source of strength distribution $m(x)$=________________. The velocity potential is then ϕ=________________.

La2. The following problems refer to the 2 foils of chord length ℓ =1 m, centered at $x = 0$. They operate in an oncoming stream of $U = -10$m/s with geometries described by:

(I)	(II)
$y_u = 0.05(1 - 4x^2)$	$y_u = 0.10(1 - 4x^2)$
$y_l = -0.05(1 - 4x^2)$	$y_l = 0$

(a) The flow over foil (I) is given by a source distribution $m(x)$= ________________m/s.
(b) The flow over foil (II) is given by a vorticity distribution $\gamma(x)$= ________________m/s.
(c) Foil (I) at an angle of attack of ________________° will produce the same lift L= ________________N/m as foil (II) at a 3° angle of attack. In this case the center of pressure is at x= ________________m for foil (I) and x = ________________m for foil (II).
(d) Foil (II) will least likely cavitate at an angle of attack of ________________°.

La3. At the leading edge of a foil at an ideal angle of attack α_i, [velocity is finite] [velocity is infinite] [streamlines enter smoothly] [there is a stagnation point] [there is separation] [slope of dividing streamline is equal to the camber slope]. For a foil with symmetric parabolic camber, α_i =__________________________.

La4. A two-dimensional hydrofoil of chord length ℓ =1 m operating in water at a speed of 10m/s generates a lift of 50,000 N/m at a 5° angle of attack. At ZERO angle of attack, it will generate a lift of ________________N/m. At this speed, the foil is expected to stall at a 10° angle of attack. The maximum lift the foil can produce is ________________N/m.

La5. The total circulation around a lifting surface is determined by specifying the ________________ condition which states ________________.

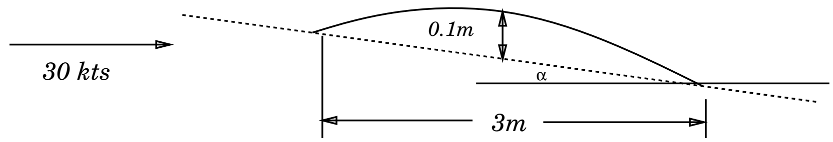
La6. A two-dimensional hydrofoil of chord length ℓ operates at a speed U. If the circulation distribution along the foil is given by $\gamma(x)$=$U(1/2 + x/\ell)$ (x=0 is mid-chord), the difference between the tangential velocities on the bottom and top surfaces of the foil is $u_-(x) - u_+(x)$=________________, the pressure difference is $p_-(x) - p_+(x)$= ________________, and the total lift on the foil is L=________________. If the half thickness of the foil is given by $t(x)$=$h\cos(\pi x/\ell)$, $(-\ell/2 < x < \ell/2)$, the source distribution along the center line is $m(x)$=________________.

La7. A two-dimensional hydrofoil of chord length ℓ=1m has a parabolic camber distribution with maximum camber of 5%. The lift coefficient of the foil as a function of the angle of attack α is C_L=________________. If the foil has a span of S=10m and supports a vessel of deadweight (‘displacement’) 10^6N ($\approx$ 100 tons), at a fixed angle of attack α=0.1rad, the vessel will be lifted out of the water at a speed of U=________________. At that point, the overturning moment on the vessel due to the hydrofoil is ________________.

Lifting Surfaces problems continued next page

L1. A parabolic camber hydrofoil has a maximum camber of 5% of the chord. If it is freely pivoted at its leading edge, what angle of attack will it assume?

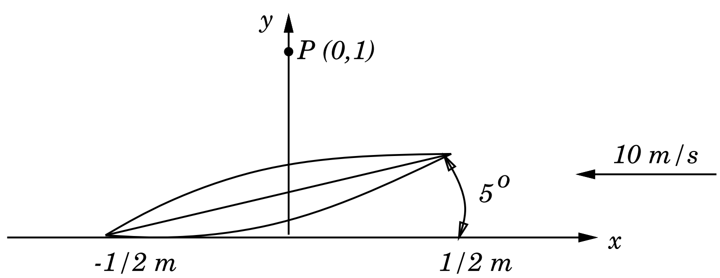
L2. A parabolically cambered 2-D hydrofoil has a chord of 3m and is operating in 15° C sea water at 30 knots. The maximum camber is 0.1m.



(a) Determine the angle of attack α if the foil is to generate a lift of 200,000 Newtons per meter of span.

(b) Where is the center of lift at this angle of attack?

L3. A 2-D hydrofoil has a chord length of 1 meter and operates at an angle of attack of 5 degrees at a speed of 10 m/s in sea water at 15° C. It has no camber, but has the following thickness distribution: $t(x) = 0.1(1 - 4s^2)$



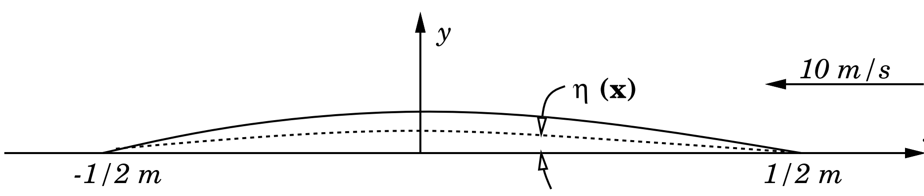
(a) How much lift per unit span does the hydrofoil produce?

(b) Calculate the moment about the midchord.

(c) Calculate a source distribution $m(x)$ and vorticity distribution $\gamma(x)$ to represent this foil.

(d) Calculate the total vertical velocity at the field point $P = (0, 1m)$. You may leave the answer in terms of definite integrals.

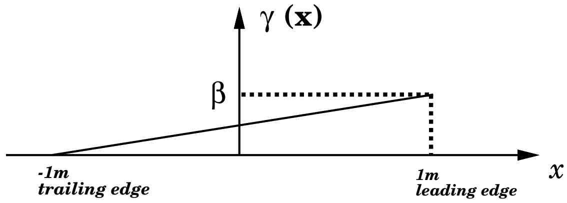
A second hydrofoil has the same chord length, the same thickness distribution and enough camber to make one surface flat. The camber distribution is: $\eta(x) = t(x) = 0.1(1 - 4s^2)$



(e) Calculate the lift if this foil operates at an angle of attack of 0°.

(f) Calculate the lift if this foil operates at an angle of attack of 5°.

L4. A two-dimensional hydrofoil with a chord of 2 meters is designed to produce 1000 Newtons of lift per meter of span when moving through 5° C fresh water at a speed of 1 m/s. The desired distribution of vorticity is:



(a) Find β.

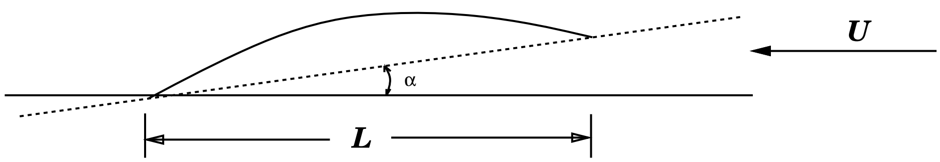
(b) Locate the center of pressure, x_{cp}

(c) Find the slope of the camber line, $\frac{d\eta}{dx}$.

Note that:

$$\int_{-1}^{1} \frac{1}{\zeta - x} d\zeta = ln \frac{1-x}{1+x}$$
$$\int_{-1}^{1} \frac{\zeta}{\zeta - x} d\zeta = x ln \frac{1-x}{1+x} + 2$$

L5. A 2-D hydrofoil has parabolic camber and no thickness. The maximum camber is 4% of the chord. Use *linearized* theory to answer the following:



(a) At what angle of attack α is there no lift on this foil?

(b) What is the maximum horizontal perturbation velocity when $\alpha = 0$?

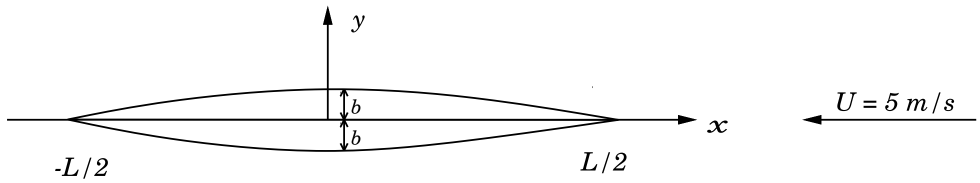
(c) What is the inception cavitation number when $\alpha = 0$?

(d) What is the inception cavitation number when $\alpha = 5°$?

Lifting Surfaces problems continued next page

L6. The centerboard of a certain yacht is modeled as a two-dimensional hydrofoil. If the upper and lower surface of this symmmetric foil are parabolas given by:

$$y_{\pm} = \pm b[1 - (2x/l)^2]; -l/2 \leq x \leq l/2; l = 1m, b = 0.02m;$$

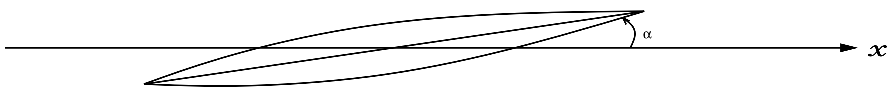


Using $\rho = 1000kg/m^3$,

(a) Find the source strength distribution $m(x)$ along the centerline for the above flow, $U = 5m/s$.

(b) Determine a formula (in terms of an integral) for the vertical velocity v at any point in the fluid.

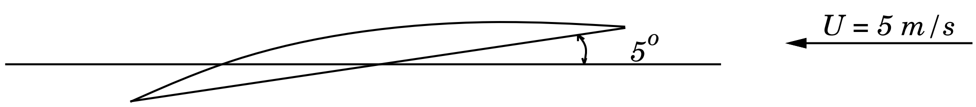
When sailing upwind, the centerboard is found to move at a non-zero angle of attack α to the flow:



(c) Calculate the lift force per unit span for the foil for (i) $\alpha = 5°$ and for (ii) $\alpha = 45°$.

An enterprising member of the MIT sailing team suggests that better performance can be obtained by removing the lower half of the foil:

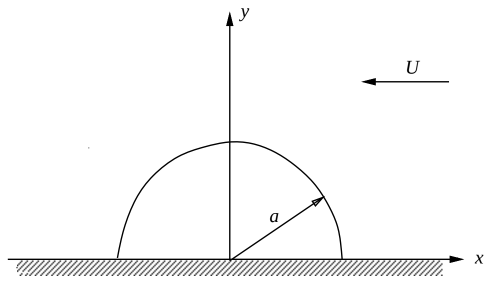
$$y_+ = b[1 - (2x/L)^2]; y_- = 0 \qquad \text{for } \alpha = 0$$



(d) (i) Repeat (c) (i) for this case.
(ii) At what angle α will the lift force begin to be negative?
(iii) At what angle α is separation least likely to occur around the foil?

Lifting Surfaces problems continued next page

L7. A two-dimensional marine vehicle has a cross-section in the form of a half circle of radius a=1 m:



(a) Assuming potential flow, find the horizontal F_x and vertical F_y forces (per unit width) on the vehicle when it is travelling on a flat bottom with velocity U=10 m/s. You may assume that the clearance between the body and the bottom is small and that stagnation pressure is maintained throughout this gap.

Assume the validity of two-dimensional linearized potential-flow lifting theory for the next three parts.

(b) The vehicle is designed also to travel away from the bottom. Estimate F_x and F_y for U=10 m/s and zero angle of attack (α=0°).

(c) To prepare for "landing" the vehicle slows down to U=8 m/s. What angle of attack α must it now have to obtain the same lift as in (b)?

(d) The vehicle is also capable of "flying" upside down (say after a maneuver). Estimate the angle of attack α the vehicle must attain to obtain the same lift at U=10 m/s.

(e) Discuss qualitatively how your answers in (a) would be affected under *real* fluid flow conditions. Indicate how F_x or F_y might change but do not perform quantitative calculations for the new values.

(f) Discuss qualitatively how your answers for the lifting problems above would be affected under *real* fluid flow conditions. Indicate how F_x or F_y might change but do not perform quantitative calculations for the new values.

Lifting Surfaces problems continued next page

M. MODEL TESTING

NOTE: For all of the following problems, assume, where necessary, that prototypes operate in 15° C salt water (SW) and model tests are conducted in 15° C fresh water (FW).

Ma1. The two similarity parameters most often of concern to people at the MIT Propeller Tunnel are the ________________# and the ________________#. The two similarity parameters most often of concern to people at the MIT Towing Tank are the ________________# and the ________________#.

Ma2. An engineer studying the launch of a ship (sliding the hull down an incline into water) would be interested at least in what following *three* dimensionless flow parameters? During the initial instant of water impact, the most important parameter is probably the ________________. After some time, it is known that wave effects are of primary concern and the experiment must maintain ________________similitude. In a 1:10 length scale experiment ($L_r \equiv L_P/L_M$=10); the entry velocity of the model must scale with U_r=________________. The time scale is T_r=________________; the peak impact pressure on the hull is scaled by p_r=________________; and the force by F_r=________________.

Ma3. A floating body of dimension L undergoes oscillatory motion of amplitude A and frequency ω. If the fluid density is ρ, its kinematic viscosity ν, and the gravitational acceleration is g, identify by name and expression (in terms of the variables above) the relevant similarity parameters for this problem.

Ma4. The volume flow rate Q of a certain pump is governed *only* by the fluid density ρ, the angular velocity of the shaft Ω, and the power rating P of the motor. The independent dimensionless parameter(s) governing this problem is(are) ________________________________. Keeping all else fixed, an increase in Q by a factor of 2 will require an increase in the power P by a factor of ________________.

Ma5. The pressure difference Δp created by a certain pump is governed *only* by the volume flow rate Q, the fluid density ρ, and the angular velocity of the motor Ω. Determine the independent dimensionless parameter(s) governing this problem. Keeping all else fixed, an increase in Q by a factor of 8 will obtain a change in Δp by a factor of ________________.

Ma6. Both Froude and Reynolds similitude for flow past a ship can in principle be obtained if experiments can be performed in superfluids ($\nu_M \neq \nu_P$) or on spaceships ($g_M \neq g_P$). For a 20:1 model scale ($L_r \equiv L_P/L_M$=20), we need (a) ν_r=________________ if $g_r = 1$, or (b) g_r=________________ if $\nu_r = 1$. For (a), the velocity scale U_r=________________ and the force scale F_r=________________; for (b), the velocity scale U_r=________________ and the force scale F_r=________________.

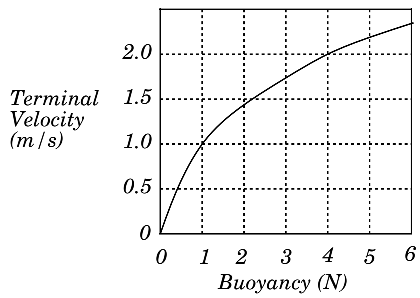
Ma7. A 6:1 ship model is used to determine the resistance due to waves. (a) If the prototype ship speed is U_P=10 m/s, the model speed should be U_M=________________m/s. (b) If the power required to drive the model is P_M=10^3 W (W = Nm/s), the power requirement for the prototype is P_P=________________W. (c) If the maximum wave pressure on the model is measured to be p_M=10^4N/m^2, the prototype maximum pressure would be p_P=________________N/m^2.

Ma8. A computer programmer wants to create a replication of the flow around a ship ("P") for an animated movie ("M"). She has taken 13.021 and knows that Froude and Reynolds numbers can not both be scaled easily in the real world, but she wants to do this in the computer. If $\nu_r \equiv \nu_P/\nu_M = 10^{-6}$ and $g_r \equiv g_P/g_M$=1, then U_r= ________________, and L_r= ________________. Keeping all other parameters the same, the water droplets in her movie appear too big relative to the ship. To fix this, and assuming ρ_r=1, she needs to change the value of the fluid property ________________("X") in the movie and X_r=________________.

M1. A low-speed, unstaffed research submarine is designed in such a way as to maintain a laminar boundary layer over as much of the hull as possible. It is 3 meters long and designed to operate in 15° C sea water at 0.5 m/s. A 1 m model is build to determine the drag characteristics.

(a) What speed should the model be tested at in 15° C sea water?

By adjusting the ballast, the model is given various amounts of buoyancy and released at the bottom of the ocean. After terminal velocity is reached, the speed of ascent is measured. The results are as follows:



(b) What is the drag of the full-scale submarine at design speed?
(c) What is the advantage of this "pop-up" test over a conventional test in a propeller tunnel?

M2. A 1/20 scale model hydrofoil has a span of 0.5m and a planform area of 0.05m^2. It is being tested in a variable pressure propeller tunnel (FW, 15° C) at its design angle of attack with a flow speed of 7 m/s.

(a) The measured lift is 400 Newtons. Find the lift generated by the full-scale prototype (SW, 15° C) at 9 m/s.
(b) The measured drag is 36 Newtons. What is the best estimate for the drag of the full-scale prototype at 9 m/s?
(c) When the static pressure in the test section is reduced to 1.5 x 10^4 N/m^2, the foil begins to cavitate. If the full-scale prototype is to operate 4 meters below the surface, at what speed will cavitation occur? (Use $P_v = 2330 N/m^2$ for FW at $15°C$; $P_v = 1700 N/m^2$ for SW at $15°C$.)

M3. A hydrofoil is to operate at 40 knots. A 1/30 scale model is tested in a propeller tunnel at 10 m/s.

(a) If the measured lift on the model at design angle of attack is 5000 Newtons, what is the lift on the full-scale prototype?
(b) The maximum achievable lift coefficient on the model is 2.2. Would you expect the maximum lift coefficient at full-scale to be less than, greater than or equal to 2.2? Explain.

M4. A propeller operating at 120 rpm powers a container ship at 22 knots. The propeller diameter is 7 meters, and the shaft centerline is immersed 5 meters below the surface. A model propeller is to be tested in a cavitation tunnel at 1200 rpm. The model's diameter is 0.3m. In order to match the cavitation number at the shaft centerline, what should be the pressure in the propeller tunnel at that location?

M5. A naval architect suspects that a ship's propeller will cavitate due to the ship's wake field. In order to investigate this possibility, he conducts a "self-propelled" test on a 1/100 scale model in a vacuum towing tank (15° C FW). The ship is 200 meters long and has a design speed of 20 knots in 15° C sea water. Its wetted surface area is 10000 m^2.

(a) What is the correct towing force? (Assume that simulator drag is negligible.)
(b) What should be the "atmospheric pressure" above the towing tank?

M6. A submarine is extremely deeply submerged. Its propeller operates at 100 rpm when the ship is running at a cruise speed of 20 kts.

(a) If the shaft speed is increased to 200 rpm, will the sub's speed be greater or less than 40 knots? Explain briefly.
(b) If the submarine were less deeply submerged, is there any effect which might modify this result? Explain.

M7. The resistance of a 300 m oil tanker moving at 17 knots is to be determined by towing a 1.5 m model in the MIT towing tank. The ship's wetted surface area is 20000 m^2.

(a) Determine the speed at which the model should be tested.
(b) The measured drag is 1 Newton. What is the predicted full-scale drag?

M8. An over-ambitious yacht designer wishes to scale surface tension forces, inertial forces and gravitational forces simultaneously.

(a) Form a non-dimensional number which represents the ratio of surface tension forces to inertial forces. This number should be expressed in terms of a velocity U, a length L, surface tension coefficient T and density ρ.
(b) The designer wishes to measure the forces on a 1 meter model of a 10 meter sailing yacht. The yacht is to sail in uncontaminated 15° C sea water. The model is to be tested in 15° C sea water which is contaminated with a surfactant which changes the surface tension, but not the density. What is the correct value of model-scale surface tension?

M9. The proposed design for a knotmeter consists of a device which measures the frequency of the vortices shed from a 10 mm diameter circular cylinder placed normally in the flow.

(a) Determine the frequency signal that would correspond to a speed of 10 knots.
(b) Over what flow regime is this device approximately linear (speed proportional to frequency)?
(c) If this device is used at speeds beyond this range, will the linear calibration still be approximately correct? Explain.

M10. A circular cylinder of diameter 1 meter and length 10 meters oscillates normal to its axis in 20° C fresh water. Its velocity is given by $V = 0.3cos(\omega t)$ m/s.

(a) Estimate the hydrodynamic force exerted on the cylinder when $\omega = 3/s$.
(b) Estimate the hydrodynamic force exerted on the cylinder when $\omega = 0.03/s$.
(c) If this cylinder is held stationary in a steady current of 0.3 m/s, what is the frequency of the unsteady lift force?

M11. To stay within her budget, the producer of "The Poseidon Adventure" wants to film the storm scene in a towing tank. If her model is built to a 1/100 scale, and she wishes to show the film at 24 frames per second, at what speed should she run her camera?

M12. A ship designer wishes to investigate the seakeeping performance of a 200 meter ship by testing a 2 meter model in a towing tank.

(a) The full scale waves have a wavelength of 100m and a wave height of 3m. What should the wavelength and wave height of the waves used in the model test be?
(b) An accelerometer is mounted on the bow of the model. If the amplitude of the measured acceleration is 1/4 G, what is the acceleration of the full scale ship?
(c) What is a good estimate of the ship's acceleration in waves 100m long and 1.5m high?

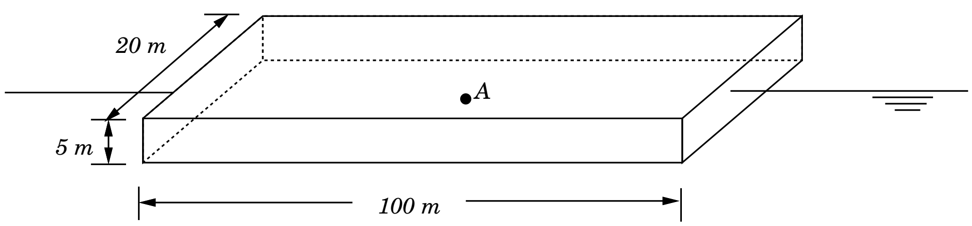
M13. A 250m surface ship is to operate at 20 knots in 15° C sea water. Resistance testing is to be performed in 15° C sea water using a 7m scale model. The ship's wetted surface area is 10000 m^2.

(a) Determine the speed at which the model should be tested.
(b) The measured drag is 70 Newtons. What is the predicted full-scale drag?
(c) A shaft bossing on the model experiences vortex shedding at a frequency of 5 Hz. Determine the frequency of vortex shedding on the prototype ship.

M14. A new submarine design has to be tested by means of a model (M) one-fourth the size of the prototype (P), i.e., $L_r \equiv L_M/L_P = 1/4$. Assume that a proper design has eliminated the concern for cavitation.

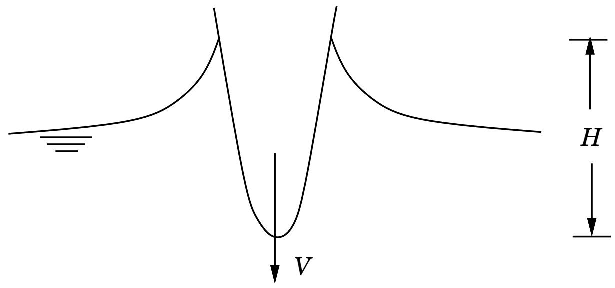
(a) When the vessel is operating near the free-surface and wave effects are important, what similitude must be observed? For the same model and prototype fluid properties and gravity g, determine how the velocity V, time T and forces F should be scaled.
(b) Repeat (a) for the case where the submarine is deeply submerged.
(c) In general, for *exact* dynamic similitude, both (a) and (b) must be observed and a different model fluid must be used. Find the required $\nu_r = \nu_M/\nu_P$.
(d) Can you think of a fluid that may have the proper ν_r?

M15. A rectangular barge with submerged dimensions 100m x 20m x 5m is to operate in the North Atlantic Ocean (water temperature 5° C) at a speed of 10 knots. A 100:1 scale model is tested at the MIT towing tank (water temperature 25° C).



(a) At what speed should the model be towed?
(b) If the drag force on the model at that speed is 0.80 Newtons, determine the drag force (in N) and the "EHP" (in Watts = Nm/s) of the barge. You may ignore roughness of correlation allowances for this calculation.
(c) To determine the structural load, a pressure gauge is mounted under the hull (at A). If the measured pressure is 500 N/m^2, what is the full scale pressure at that point?
(d) When the model speed increases beyond 2m/s, cavitation is observed to begin under the sharp edge of the bow. At what speed do you expect this to happen for the barge? ($P_v \approx 0$.)

M16. To evaluate the transient slamming behavior of a ship bow (typical draft $H = 5m$, vertical velocity $V = 5m/s$), a 2-D 1:10 scale model is tested in the laboratory.



(a) Identify the two most important similarity parameters for this study. Justify your answer.
(b) How should the model and prototype velocity V and time T be scaled?
(c) In a certain test, a transducer on the bottom of the model records a maximum pressure of $10^5 N/m^2$. What is the expected maximum pressure on the actual ship?
(d) When dropped at a given oblique angle, a restoring moment per unit length of hull of 100 Nm/m is measured. Calculate the prototype value for this quantity.

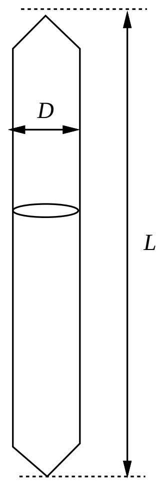
M17. One occasionally sees model ships used in the movies. Despite the best efforts of the special effects people it is always clear that models are being used. This can be detected by observing the spray associated with breaking waves (e.g. the bow wave of the ship). Let L_f be the length of the full scale ship and V_f be its speed. The available model ship has length $0.01L_f$.

(a) What speed is appropriate for the model if the wave pattern is to scale correctly? Why did you select this scaling rule?
(b) The formation of spray is governed by the balance between inertial and surface tension forces. The fluid surface tension is σ (in N/m) for both full-scale and model. What is the ratio of *apparent* model surface tension to full scale surface tension if the scaling is done as in (a)?
(c) According to your result in (b), will the model droplets be too large or too small? Why?

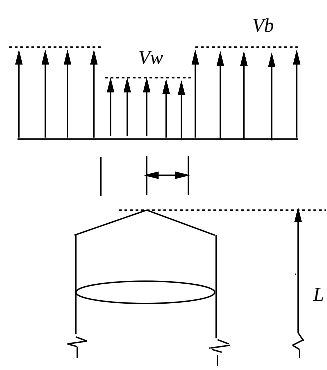
M18. It is proposed that the steady longitudinal drag force on an automobile be measured using a geometrically-similar 1:5 length-scale (*i.e.*, $L_r \equiv L_P/L_M$=5) model in a towing tank filled with fresh water.

(a) State the condition(s) required to ensure dynamic similitude between the prototype and the model.
(b) If the prototype speed is U_P=36 km/hr $\approx$ 10 m/s, at what speed U_M should the model be towed?
(c) At the speed in (b), the drag force on the model is measured to be F_M=10^4 N. What is the force F_P on the prototype automobile (in air)?
(d) Due to a soft suspension, the prototype heaves noticeably at a frequency near ω_P=2 rad/s. At what frequency ω_M should the model be moved up and down to achieve *fluid dynamic* similitude for this effect?
(e) It is pointed out that the presence of surface waves in the towing tank (an effect absent in the prototype) may distort the force measurements. In order to keep this effect due to gravity less than $O(25)$%, how deep should the tank be for a model that 'runs' on the tank bottom?

M19. An offshore construction company thinks there is an easy way to start the driving of a pile into the seafloor in deep water. The idea is to simply drop it vertically from the surface. The geometry of the pile is approximately a cylinder, tapered at both ends, length L and diameter D (L/D=5) (surface area $S \approx \pi DL$; cross-sectional area $A = \pi D^2/4$; $S/A \approx 20$), as shown. The average density of this (concrete) pile is $\rho_c = 3\rho_{\text{water}}$. The key issue for the success of this concept is the vertical velocity V_b at which the pile hits the bottom. To test this idea, a model of length L_M=5 cm is used and the bottom velocity V_{bM} is measured.



(a) Obtain a formula for the total drag coefficient (based on S) C_D in terms of V_b.
(b) The experiment is repeated for the model with and without a turbulence stimulator ring on the "nose". Despite observed turbulent conditions along the entire model for the tripped case, the measured V_{bM} is found to be almost the same for both cases, with $V_{bM} \simeq 2$ m/s. Explain why this can be so from the *fluid mechanics point of view* (*i.e.*, assume the measurements are correct).
(c) Using the measured V_{bM} value above, find the total drag coefficient C_{DP} of a prototype pile of length L_P= 5 m. For this calculation, you may assume that the prototype Reynolds number (based on L and V_b) is $\sim O(10^8)$. State the key assumption(s) used.
(d) What is the bottom velocity V_{bP} of the prototype pile?
(e) In order to understand the physics involved, a laser system is set up to measure the velocity profile in the wake of the *prototype* pile (shown below in a frame fixed with the pile). It is found that pressure gradient effects are small for this slender geometry, and that the extent of the wake is approximately given by the momentum thickness $\theta(L)$ at the end. What is the cross-sectional area A_w of this wake?



(f) It is found that the velocity profile inside A_w can be approximated by a constant V_w (see figure). For any given value of the total drag coefficient C_D (defined above), the ratio of $\gamma \equiv V_w/V_b$ can be calculated. Obtain a formula/equation for γ in terms of C_D and the ratio S/A_w.

P. POTENTIAL FLOW

Pa1. A body in a uniform stream is modeled by sources and sinks of strengths m_i, i=1,2,... The body will be closed [if] [only if] [if and only if] $\sum_i m_i$=0.

Pa2. For a given (2D) flow, if a ϕ does not exist, the flow must be [rotational, irrotational, compressible, incompressible, impossible]; if a ψ does not exist, the flow must be [rotational, irrotational, compressible, incompressible, impossible]; if neither a ϕ nor a ψ exists, the flow must be [rotational, irrotational, compressible, incompressible, impossible].

Pa3. A certain potential flow is given by $\phi(x, y)$=$ax^2 + xy - y^2$. The constant a must be equal to __________________. and the stream function for this flow is $\psi(x, y)$=__________________.

Pa4. For the following 2D 'flows,' complete the missing entries. Mark "n/a" if an answer is not applicable or does not exist: (don't forget +C!)

u	v	ϕ	ψ
			$x^2 + xy + y^2$
		$e^x \sin y$	
$\sin(x - y)$	$\sin(x - y)$		
$\sin(x + y)$	$\sin(x + y)$		

Pa5. When neither a velocity potential nor a stream function for a flow exists, the flow must be [impossible] [irrotational] [rotational] [viscous] [inviscid] [compressible] [incompressible] [turbulent] [undefined].

Pa6. A 2D potential flow is constructed with:

1. a uniform stream of (U, V)=$(1, 1)$;
2. a point source of strength π at $(0, 0)$;
3. a dipole of moment 2π oriented in the $+x$ direction at $(1, 1)$;
4. a dipole of moment 2π oriented in the $-x$ direction at $(-1, 1)$;
5. a point vortex of circulation 4π at $(0, 2)$; and
6. a point vortex of circulation 4π at $(0, -2)$.

The velocity potential is given by $\phi(x, y)$ = __________________.
The *horizontal* velocity at A=(2,0) is u_A=__________________.
The *vertical* velocity at B=(0,1) is v_B=__________________.

Pa7. In an interior corner flow, a particle travelling along the wall has a speed which increases as the fourth root of its distance from the corner ($u \sim r^{1/4}$). The angle of the corner is θ_o =__________________radians.

Pa8. The *exterior* flow around the corner of a rectangular house with two walls located at 90° and 180° meeting at the origin may be modeled by potential flow with ϕ=__________________.

Pa9. In a certain corner flow, a particle travelling along the wall has a speed which increases as the square root of its distance from the corner (i.e. $u \sim r^{1/2}$), the angle of the corner is θ_0=__________________.

Pa10. Two walls are placed along the the positive x and the negative y axes to partition the flow into an inner and an outer region. The general expression for the potential for the interior corner flow is __________________, and for the exterior corner flow is __________________.

Pa11. If a two-dimensional Rankine half-body placed in a flow of 10m/s is to have an ultimate height far downstream of 1m, the source inside the body must produce a volume flux of __________________m^2/s.

Pa12. A small rupture in an underwater storage tank causes an insoluble neutrally-buoyant fluid to be discharged at a volume rate of Q=$4\pi\mathrm{m}^3$/s. If the current at that location is U=1m/s, the three-dimensional plume can be detected __________________m upstream of the leak. Far downstream, the affected region will be of the form of a circular cylinder of radius __________________m.

Pa13. A pipe discharges an insoluble pollutant at a volume rate of Q=πm^3/s into the middle of a river with a flow rate of U=4m/s. If the river is deep, a three-dimensional plume will be created which extends a distance of _________________m upstream and has a radius of _________________m far downstream. If the river is shallow, a two-dimensional plume will be created. If the river depth is 1m, the 2D plume will extend a distance of _________________m upstream and has a width of _________________m far downstream.

Pa14. The velocity potential ϕ at (x, y, z) due to a three- dimensional source of strength M located at (ξ, η, ζ) is:_.____________________________If the velocity at a point 0.5m from the source has a magnitude of 1m/s, the value of M=_________________.

Pa15. The flow around a two-dimensional body in a uniform stream (U, V) can be modeled by the following internal singularities: a dipole of moment μ oriented in the $+x$ direction located at $(0, 0)$, a source of strength m at $(-a, a)$, a sink of strength m at $(-a, -a)$, and three vortices of circulation Γ each located respectively at $(a, 0)$, $(-a, 0.5a)$, and $(-a, -0.5a)$. The vertical velocity at the point (-a,0) is v=_________________.

Pa16. A sphere of radius a is placed a distance b above a wall. In order for the method of images to apply, the following must hold: [$b \ll a$] [$b \gg a$] [$b \approx a$] [any a or b]. If the same sphere is centered vertically between two infinite horizontal flat plates, the number of image spheres needed to satisfy *exactly* the boundary conditions on the flat plates is _________________.

Pa17. A pair of two-dimensional point vortices both of circulation Γ are placed at points (x, y)=$(1, 1)$ and $(-1, 1)$. If there is a wall on y=0, the total vertical force on the vortex at $(1, 1)$ is F_y=___ρ.

Pa18. A pair of point vortices, both of circulation Γ are placed at points $(x, y) = (-1, 1)$ and $(1, 1)$. If there is a wall on $y = 0$, the total vertical force on the vortex at (1,1) is $F_y/\rho =$___________.

Pa19. A two-dimensional dipole of moment μ and oriented in the negative x direction and a two-dimensional point vortex of circulation Γ are both placed at (L, L) in a corner formed by two walls $x = 0$ and $y = 0$. Draw a picture (only) to symbolically identify the orientations and locations of all the singularities in order to satisfy the necessary kinematic boundary conditions. Identify (with an × in the picture) the point of maximum pressure in the flow.

Pa20. A sphere of radius a and origin at $(0, 0, 0)$ is placed in a uniform stream $(U, 0, 0)$. If p_∞ is the hydrodynamic pressure far away, the maximum pressure on the surface of the sphere is p_{max}=_________________. The minimum pressure on the surface of the sphere occurs where the tangential velocity is equal to _________________and is given by $p_{min} =$_________________. At a point directly upstream $(X, 0, 0)$, the magnitude of the velocity is $U/2$, so X/a=_________________.

Pa21. A flying device with speed $\vec{U}$=(U_x, U_y, U_z) deploys a combination of wings and flaps to generate the following three circulations: $\vec{\Gamma_1}$=$(\Gamma_{1x}, \Gamma_{1y}, \Gamma_{1z})$; $\vec{\Gamma_2}$=$(\Gamma_{2x}, \Gamma_{2y}, \Gamma_{2z})$; $\vec{\Gamma_3}$=$(\Gamma_{3x}, \Gamma_{3y}, \Gamma_{3z})$. Assuming otherwise potential flow, the transverse force on the plane is given by F_y=______.

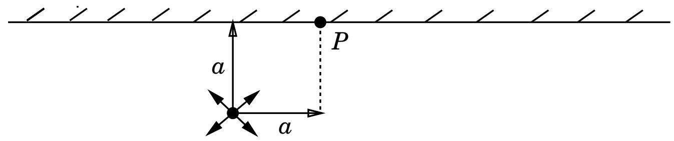
Pa22. A 2D potential flow past a body is represented by a uniform stream (U, V)=$(1, 2)$ and a system of internal singularities:

type	strength	location	orientation
source/sink	1	$(0, 0)$	—
source/sink	-1	$(1, -1)$	—
dipole	2	$(1, 0)$	$+x$
vortex	3	$(0, 1)$	—
vortex	-2	$(1, 1)$	—

The force on the body is given by F_x/ρ=_________________; F_y/ρ=_________________.

Pa23. A circular cylinder of radius a is fixed in an accelerating fluid, density ρ, with horizontal velocity given by $U(t)$. The force on the cylinder (per width) is F_x=_________________, and F_y=_________________. If a horizontal wall is placed some distance below the cylinder, F_x should [increase] [decrease] [not change], and F_y should [increase] [decrease] [not change].

P1. A 2-D source of volume flux m in a fluid of density ρ is a distance a from a rigid wall of infinite extent.

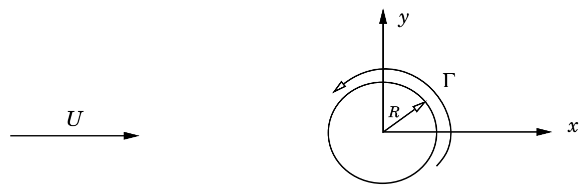


(a) Calculate the velocity at point P.

(b) How does the pressure at point P differ from that at an infinite distance from the source?

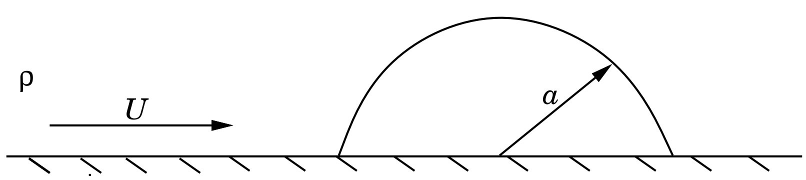
(c) If there is now a freestream of velocity $(U,0)$, what would the volume flux of the source have to become for the same results in (a)?

P2. The Flettner Rotorship was propelled by cylindrical rotors which were turned about their axes. The flow about these rotors can be approximated by two-dimensional flow past a dipole/vortex combination as shown:

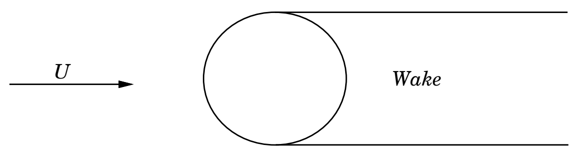


(a) Write down the appropriate velocity potential for a rotor of radius R with circulation Γ in a streaming flow of velocity U.
(b) Find the angular position of the stagnation points on the cylinder surface.
(c) What is the magnitude and direction of the total aerodynamic force on a rotor of length L?
(d) Find the stream function for this flow.
(e) Find the positions of the dividing streamline when at a distance of $10R$ away from the origin.
(f) A certain marker particle is observed to pass directly under the cylinder at a distance of $2R$ (i.e., through the point $(x, y) = (0, -2R)$). At what distance will the particle cross the x-axis?

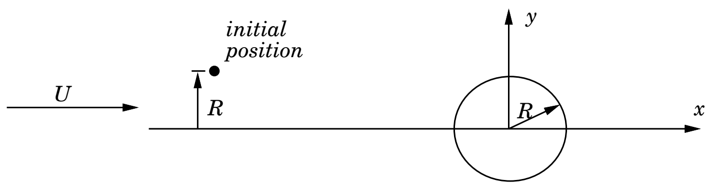
P3. A hemi-*spherical* oil storage tank of radius a is located on the ocean bottom in the presence of a uniform current U, as shown. The pressure beneath the tank is equal to the hydrostatic pressure at that depth. Calculate the total vertical force exerted on the tank by the fluid. The answer may be left in terms of a definite integral.



P4. A rough estimate of the drag of a bluff body in real fluid flow may be obtained by assuming that the pressure on the body forward of the separation point is that predicted by potential flow theory, whereas the surface pressure downstream of the separation point is constant and equal to either (A) its predicted value at the separation point, or (B) the value at infinity. Assuming that separation occurs at the midpoint of a circular cylinder, use this theory to estimate its drag coefficient C_D using the two assumptions.

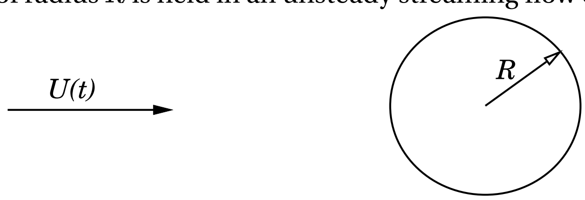


P5. A circular cylinder of radius R is in an invsicid steady streaming flow of speed U. A small, neutrally buoyant “marker” particle is placed in the flow far upstream of the cylinder at a distance R above the dividing streamline.



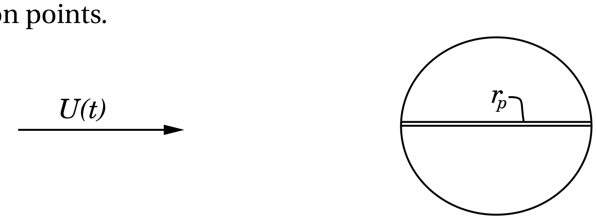
(a) How far from the cylinder is this particle when it passes the y-axis?
(b) What is the speed of the particle at that point?
(c) If a circulation of strength Γ is imposed on the cylinder, what are the answers for (a) and (b)?

P6. A stationary sphere of radius R is held in an unsteady streaming flow of velocity $U(t)$.



(a) Assuming potential flow, find the pressure differential between the forward and aft stagnation points.

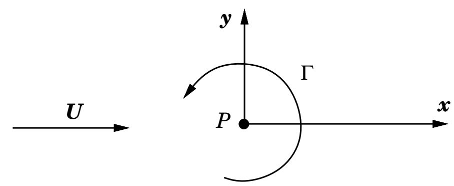
In order to measure the acceleration of the flow, a straight small-diameter pipe is run between the stagnation points.



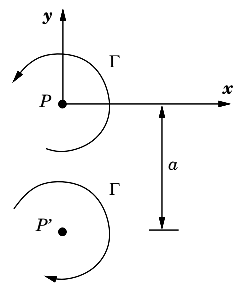
(b) Assuming that the flow within the pipe is laminar and assuming that the presence of the pipe does not affect the potential flow outside the sphere, find the coefficient of proportionality between the volume flux Q through the pipe and the acceleration dU/dt.

P7. The flow around a rotating cylinder may be approximated as that near a 2D point vortex of circulation Γ at its origin P.

(a) Given the fluid density ρ, determine the magnitude and direction of the force on the vortex P.

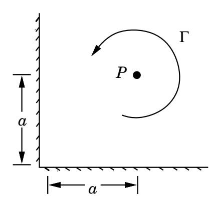


(b) For the two vortices below, determine the magnitude and direction of the velocity induced by P' at P.



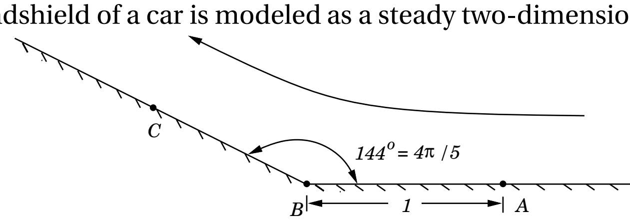
(c) Determine the magnitude and direction of the force on the vortex P in the situation of part (b).

Consider the vortex inside a 90° corner:



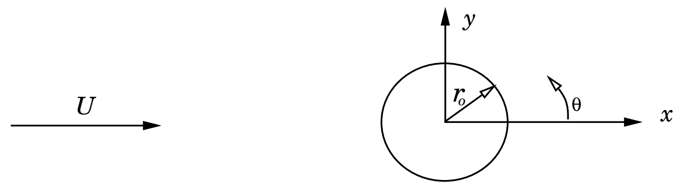
(d) Write down the potential $\phi(x, y)$ for this flow.
(e) Calculate the magnitude and direction of the force on P.

P8. The lift on the windshield of a car is modeled as a steady two-dimensional potential flow:



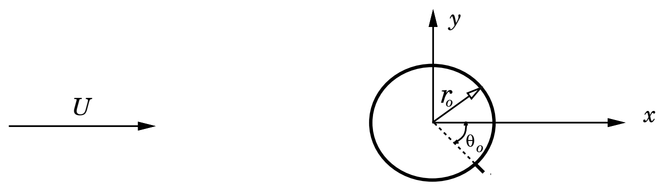
(a) Write down the general solution for the velocity potential ϕ for this flow.
(b) If the horizontal velocity at A is $u = -1.25$, calculate the radial velocity v_r along $\bar{BC}$ as a function of radial distance r.
(c) Find the pressure distribution along $\bar{BC}$ in terms of the stagnation pressure P_s at B. (Ignore gravity.)

P9. A steady uniform stream of fluid density ρ and velocity U flows past a 2D circular cylinder of radius R:



(a) Calculate the potential ϕ and force F on the body.
(b) Calculate the tangential velocity $v_\theta(\theta)$ on the surface of the cylinder. Find the locations and values of the maximum and minimum values of $|v_\theta|$.

In order to create lift, a small sharp fin is attached to the cylinder at $\theta = \theta_0$:



(c) Repeat parts (a) and (b) for this flow (give your answers as a function of θ_0). Sketch the streamlines for this flow.

P10. A "flow generating device" placed at the origin in a free stream creates a two-dimensional flow given by the following velocity potential:

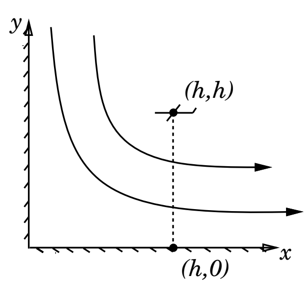
$$\phi = Ux + \frac{m}{2\pi}\ln(r) + \frac{\Gamma}{2\pi}\theta$$

Calculate the force exerted by the fluid, density ρ, on the "flow generating device."

P11. A plane with very long wing span (i.e. you can assume two-dimensional flow) is designed to create a constant amount of circulation Γ_0 regardless of the forward speed.

(a) What is the lift L on the plane when moving at a horizontal speed of U?
(b) What lift L will the plane generate when flying at horizontal velocity U at a distance h above the ground?

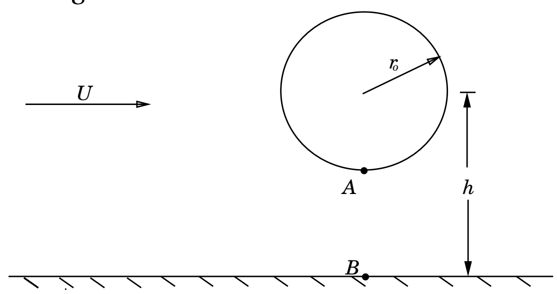
A dare-devil pilot flies the plane towards a cliff and finds that by controlling the throttle he can hold stationary at a point (h, h) with respect to the base of the cliff due to a strong draft down and away from the cliff(i.e. corner flow):



(c) If the corner flow (only) produces a wind which has a *horizontal* component of U_c in the vicinity of the plane, what is the ground wind velocity at a point directly below the plane, i.e. at $(h, 0)$?
(d) Find the weight of the plane W in terms of U_c, Γ_0 and h.

P12. We saw in lecture that a dipole is the result of combining a source and a sink in a special way. Show by a similar process that, in two dimensions, a pair of point vortices can also be used to make a dipole.

P13. A circle of radius R is placed a distance h from an infinite wall; $h \gg R$. There is a streaming flow from left to right of magnitude U.



(a) Write the potential function for this flow.
(b) Show that the flux of fluid, Q between points A and B is :

$$Q = \tfrac{2h+R}{2h-R} U(h-R)$$

P14. A circle of radius a with its center a distance b ($\epsilon \equiv a/b \ll 1$) above a horizontal wall ($y = 0$) is placed in a uniform stream $(U, 0, 0)$.

(a) Write down the potential ϕ for this flow in terms of x, y (not r, θ).
(b) Calculate the *leading order* velocity due to the image (u_i, v_i) in the vicinity of the circle ($x = O(a)$, $y = O(b)$).
Note: For $\epsilon \ll 1$, *leading order* means dropping all higher-order terms in ϵ. Thus if the answer is $O(1)$, then drop all terms of $O(\epsilon, \epsilon^2, \ldots)$; if the answer is $O(\epsilon)$, then keep that and drop all terms of $O(\epsilon^2, \epsilon^3, \ldots)$; etc. (`Hint:` Keeping *every* term dimensionless will make the order of these terms obvious.)
(c) From (b), show that the image velocity at the circle is (i) effectively horizontal ($v_i \ll u_i$); and (ii) u_i is almost constant and can be represented to leading order by its value at the center of the circle.
(d) Using the result of (c) and evaluating the total horizontal velocity,(i) show that $(\pm a, b)$ are no longer stagnation points; and (ii) find the new stagnation points' positions $\pm x_s$.
(e) Using the result from (b) and examining the horizontal image velocity u_i at $(0, b \pm a)$, deduce *qualitatively* whether the vertical force on the circle due to the presence of the wall would be zero, positive or negative.

P15. A two-dimensional ideal flow has a stream function: $\psi(r, \theta) = r \sin\theta - \frac{\sin\theta}{r} + r - 1$.

(a) Show that the stream function satisfies the boundary condition for a fixed body coinciding with the circle $r = 1$.
(b) Find the positions and values of the maximum and minimum velocities on this body.
(c) Find the maximum pressure difference Δp between any two points on this body.
(d) What is the volume flux between the two points ($r = 1, \theta = 90°$) and ($r = 2, \theta = 90°$)?
(e) At what point C represents a streamline that gives half of the flux rate obtained ini part (d)?

V. VISCOUS FLOWS

Va1. A two-dimensional cylinder of radius $1m$ is accelerated in an infinite ideal fluid at $2m/s^2$. At the same time, the fluid is accelerated against the cylinder (in the opposite direction) at $1m/s^2$. Assuming potential flow, the total force on the cylinder is ________________. If both the fluid and the cylinder have ceased accelerating, and the flow is moving steadily against the cylinder at a constant velocity of $1m/s$, the drag on the cylinder per unit length is approximately (a) assuming potential flow ________________and (b) considering real fluid effects ________________.

Va2. The drag coefficient of a sphere [increases] [decreases] as its velocity increases past a critical value corresponding to a Reynolds number of roughly ________________because the boundary layer becomes ________________. The exact critical value depends on ________ and ________________so that in a crude laboratory test that value is usually [lower] [higher] than the theoretical value.

Va3. A smooth solid metal sphere of radius 1 m and density ρ_s=$2\rho_{\text{water}}$ is released in the ocean. The *initial* acceleration of the sphere is ________________m/s^2. After a long time, the velocity of the sphere would be ________________. If the body is flattened into a circular disk of radius 1 m and thickness 1 mm, the final velocity would be ________________, if it falls perpendicularly (horizontal disk). If the body is in the form of a square plate of side 1 m and thickness 1 mm and is forced to fall parallel to the flow (vertical plate), the final velocity would be ________________.

Va4. A person drinks from a 300 cc can of soda (density ≅ water) through a straw 4 mm in diameter and 20 cm in length. If he has to finish the soda in 1 minute, the pressure difference Δp he needs to maintain (ignoring gravity) is ________________. If the diameter is now only 2 mm, for the same Δp, the time required to finish the drink becomes ________________. (Assume laminar flow throughout.)

Va5. A ship has a heaving velocity given by $U_0 \cos(\omega t)$ where U_0=2 m/s and ω=2 rad/s. Considering a large flat plate on the side of hull and assuming laminar flow, the fluid velocity will have an amplitude of 1% that of the ship a distance ________________ away from the hull. The maximum skin friction drag on the hull occurs at ωt = ________________and amounts to ________________N per square meter of plate. The flow can be considered to be 'separated' from the plate at $\omega t \pm 2n\pi =$ ________________.

Va6. The shell of a space capsule of thickness $L = 10cm$ is punctured leaving a long cylindrical hole of radius $a = 10^{-4}m$. (a) If the interior of the capsule is *air* pressurized at $P = 10^5 Pa$, and the exterior is vacuum; calculate the volume rate Q at which air leaks out of this capsule. (b) After some time, the hole increases to a radius of $2a$, if the flow rate remains the same, find the interior pressure P at this time.

Va7. A flat plate 1m long by 0.1m wide (a "snowboard") slides at a velocity of U=1 m/s on a fixed flat surface trapping a layer of water of thickness 0.001m between them. Assuming laminar flow of the water in the gap, the total frictional force on the sliding plate is F=________________. It is argued that F can be changed by applying a force on the front or back of the plate thus inducing a pressure gradient in the trapped fluid. F can thus be completely cancelled when the pressure difference in the fluid $p_{\text{front}} - p_{\text{back}}$=________________.

Va8. A designer can smooth a flat plate, length L, and (try to) keep the boundary layer LAMINAR as long as possible, or add a tripwire at the front edge to make sure that the boundary is TURBULENT over the entire plate. To minimize the friction drag D on the plate, it is useful to add the tripwire only if the Reynolds number based on L is [greater] [less] than $\mathcal{R}_L$ = ________________.

Va9. A flat plate of dimensions 1m × 1m by 0.01m thickness is dropped vertically (parallel to the flat sides) in water. If the density of the plate is twice that of water, the maximum velocity the plate will attain is U= ________________. At that speed, the displacement thickness near the trailing edge of the plate is δ^* = ________________. It is found that the plate is welded together horizontally in the middle. This should not affect your answer above if the weld height k is much less than ________________.

Va10. To model hydroplaning of a car on a wet road, it is assumed that the tire has a contact area of 0.2m × 0.2 m gliding over a layer of water of constant thickness of 10^{-4} m. If the wheels are locked and the speed of the skidding car is 30m/s, assuming laminar flow and otherwise no pressure gradients, the total 'frictional' force on each wheel is ________________.

Va11. A horizontal pipe of length L and radius a ($a \ll L$) drains water into open air from a reservoir which is filled to a depth h above the pipe. The flow rate is Q= ________________. The total horizontal force on the pipe due to shear stress is D= ________________. To double Q, but keeping L constant, the radius of the pipe must be changed by a factor of ________________. In this case D will change by a factor of ________________.

Va12. A vertical pipette (small hollow cylinder) of radius a is filled with a fluid (density ρ, kinematic viscosity ν) to a height of h. If the gravitational acceleration is g, the flow rate is given by Q=________________. The rate of change of h is given by dh/dt=________________.

Va13. To lubricate the gliding motion of a large box on a (horizontal) surface, a thin layer of oil (density ρ, dynamic viscosity μ) is applied between the gliding surfaces. The velocity of the box is U, and the thickness of the oil film is h. The velocity profile, $u(y)$, of the oil film is [parabolic] [hyperbolic] [constant] [linear] [can't tell]. The frictional drag on the box is ________________per unit area of the bottom.

Va14. Turbulent fluctuations affect the equations governing the mean flow in the form of Reynolds stresses which have the general form τ_{ij}^R=________________in terms of the fluctuation velocities u_i'. For a certain *two-dimensional* turbulent flow, $u_1' = \cos(kx + ky - \omega t)$ where k, ω correspond to scales much smaller than the mean flow. It follows then that u_2'=____________ so that τ_{12}^R=________________.

Va15. If k is the height of the typical roughness on a ship hull length, L, boundary-layer thickness δ: [k], [k/L], [k/δ], [Reynolds #], [Froude #], [hull shape] must be kept the same to produce the *same* drag coefficient. In a *Froude* experiment, model friction coefficient are usually [lower] [higher] [the same] compared to the prototype due to roughness effect.

Va16. A large flat plate in water starts impulsively from rest to a steady speed. The velocity at a point $1mm$ from the plate reaches some value U after 4 seconds. The same velocity will be attained at a point $2mm$ away after ________________seconds. If the same experiment is repeated in a fluid with twice the kinematic viscosity, the time to reach U at $1mm$ is ________________seconds.

Va17. A 'single-gear' transmission for a certain car consists of two concentric cylinders of length L, and radii R and $R - h$ ($h \ll R$) respectively filled with a fluid of dynamic viscosity μ in between. If the outer cylinder has an angular velocity of Ω and the inner cylinder is stationary, the torque of the transmission in terms of L, R, h, Ω and μ is ________________. If the transmission is suddenly engaged, the torque will not be felt by the inner cylinder until after a time delay measured in terms of h, ν=μ/ρ, . . . by $T \sim$________________ (give only the *form* of the dependence, ignore constants). If an oscillatory motion of frequency ω is applied to the outer cylinder instead, the gap height has to be of the order of $h \sim$________________ (give only the *form* of the dependence in terms of ν=μ/ρ, ω, . . ., ignore constants) or less for the transmission to be useful.

Va18. Flow separation usually occurs when the velocity near the body surface is [large] [small] [constant] [decreasing] [increasing] under ________________pressure gradient. At the point of separation (on the body), the velocity must be [positive] [zero] [negative], the shear stress must be [positive] [zero] [negative], and the vorticity must be [positive] [zero] [negative].

Va19. In an old-fashioned jukebox, phonographic records (thin circular disks of radius a) are dropped vertically onto a turntable. Assuming steady-state and moderately large Reynolds number, the drag coefficient for downward velocity V should be roughly C_D=$drag/(.5\rho V^2 \pi a^2)$ $\sim$________________. If the weight of the record is W, its terminal velocity is given by ________________.

Va20. For laminar flow between two infinite plates a distance h apart driven by a pressure gradient, the velocity profile is [constant] [linear] [parabolic] [hyperbolic] [elliptic] [error function], and the flow rate Q is proportional to h to the power ____________. If the flow is driven by the top plate moving at a speed U in the absence of any pressure gradient, the velocity profile is [constant] [linear] [parabolic] [hyperbolic] [elliptic] [error function], and the flow rate Q is proportional to h to the power ____________.

Va21. If instead of the exact solution for steady laminar flow U_0 over a flat plate with distance x measured from the leading edge, we assume an approximate profile given by $u(y;x)/U_0=y/\delta(x)$ for $y \leq \delta(x)$ and by $u(y;x)/U_0=1$ for $y \geq \delta(x)$; in terms of δ, the displacement thickness is given by $\delta^*=$____________, the momentum thickness by $\theta=$____________, and the wall shear stress by $\tau_0=$____________. The dependence of δ on x is expected to be $\delta \sim x^{**}$____________; also $\delta \sim U^{**}$____________and $\delta \sim \nu^{**}$____________.

Va22. The apparent increase in the diffusivity of momentum due to turbulent fluctuations (u_i') is given the name ____________which is given by the formula ____________. In general, for turbulent flows, this effect is [greater than] [comparable to] [smaller than] that due to viscosity except in a small region near the body called the ____________.

Va23. In a turbulent flow, the rate of kinetic energy loss (per unit mass) ϵ $(= \partial(v^2)/\partial t)$ depends (ultimately) on the kinematic viscosity of the fluid ν. Using dimensional analysis, the size (say diameter) η of the typical eddy responsible for this viscous dissipation is given in terms of ϵ and ν by $\eta \sim$____________ (η is the so-called Kolmogorov turbulent length scale). Similarly, the typical velocity v_η of that eddy can be obtained in terms of ϵ and ν to be $v_\eta \sim$____________.

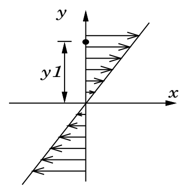
Va24. A shear flow given by $u = y$ and $v = w = 0$ becomes turbulent with velocity fluctuations given by $u_1'=-u_2'=\varepsilon y \cos(t/\tau')$, where τ' is a small turbulent time scale. In this case $u_3'=$____________. The viscous shear stress is $\tau_{xy}(y)=$____________ and the Reynolds turbulent shear stress is $\tau_{xyR}(y)=$____________. In order for these two stresses to be of comparable magnitude at $y=1$, the order of magnitude of the constant ε is ____________.

Va25. Two submarines, one twice the length of the other, are manufactured using the *same* kind of rivets which protrude a small distance above the body surface. In estimating the friction coefficients for the two vessels operating at the same speed, the longer submarine should be considered overall to be [rougher than] [smoother than] [same roughness as] the shorter submarine. If instead the shorter vessel is scaled *exactly* (including rivet heights) from the longer vessel, the longer submarine should be considered overall to be [rougher than] [smoother than] [same roughness as] the shorter submarine.

Viscous Flows problems continued next page

V1. A two-dimensional steady velocity field of a viscous incompressible fluid of density ρ and viscosity μ is specified by :

$$u(x,y) = Ay \qquad A = constant$$
$$v(x,y) = 0$$



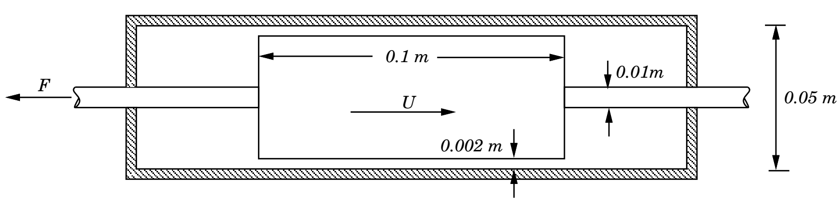
(a) What is the shear stress $\tau_{xy}(x,y)$? $\tau_{yx}(x,y)$?
(b) What is the pressure gradient $\nabla P(x,y)$?
(c) What is the vorticity field $\omega(x,y)$?
(d) What is the circulation Γ around a circular contour of radius R centered at the origin?
(e) What is the velocity potential $\phi(x,y)$ and stream function $\psi(x,y)$? If either or both of these quantities cannot be defined, explain why.
(f) What is the momentum flux across the y-axis between the origin and the point $(0, y_1)$?

V2. A viscous steady incompressible fluid flow is given by:

$$\begin{aligned} u &= a + bz + z^2 \\ v &= c + dz + z^2 \qquad a, b, c, d \text{ constants} \\ w &= 0 \end{aligned}$$

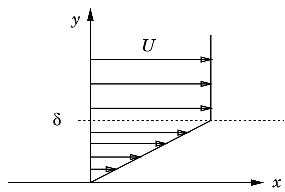
(a) Show that the equation for continuity is satisfied.
(b) Determine the constants so that the boundary conditions on stationary horizontal rigid walls at $z = 0$ and $z = 1$ can be satisfied.
(c) Find the pressure field $P(x,y,z)$ associated with this flow.
(d) What is the wall shear stress τ_0 in the x and y directions on $z = 0$?
(e) Calculate the vorticity (vector) field for this flow.

V3. A dashpot is designed to produce a force F which is proportional to its velocity U. It consists of a piston in a closed cylinder, as shown. The cylinder is completely filled with hydraulic oil of viscosity $\mu = 10 kg/m - s$ on both sides of the piston. The cylinder diameter is 0.05m, the piston length 0.1m and the gap between the piston and cylinder wall is 0.002m. Approximating the flow through the gap as locally two dimensional, what is the damping coefficient F/U?



V4. A very simple model of a laminar boundary layer on a flat plate with zero pressure gradient is the following:

$$u/U = y/\delta \qquad y < \delta$$
$$u/U = 1 \qquad y \geq \delta$$



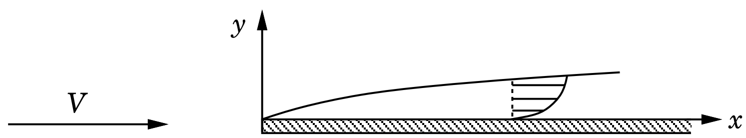
(a) Find expressions for τ_{xy}, δ^* and θ in terms of δ, U and μ.
(b) Apply Eqn. 72 of JNN chapter 3 (which satisfies the boundary layer equations in an integral sense) to find an expression for δ/x in terms of $\mathcal{R}_x = Ux/\nu$.
(c) Compare this result with $\delta_{.99}$ given by the Blasius solution.

V5. An approximation of the velocity profile within a two-dimensional laminar boundary layer is:

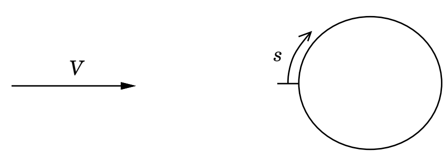
$$\frac{u}{U} = 1 - e^{-y/\delta}\,,$$

where U is the local velocity outside the boundary layer and δ is a measure of its thickness.

(a) Find expressions for the shear stress at the wall τ_{xy}, the displacement thickness δ^* and the momentum thickness θ in terms of δ, U and the dynamic viscosity μ.
(b) Applying von Karman's momentum integral equation, find an expression for δ/x in terms of $\mathcal{R}_x = Ux/\nu$ for the case of a flat plate held in a stream of velocity V at zero angle of attack.



(c) Again applying von Karman's momentum integral equation, find a differential equation governing the growth of δ vs. arc length s for a steady streaming flow of velocity V past a 2D circular cylinder.



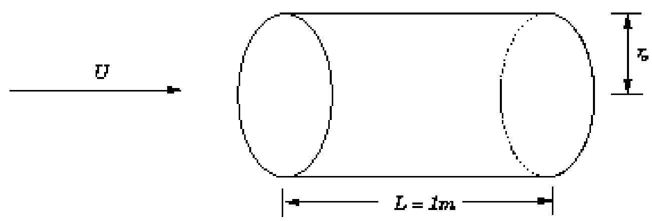
Hint: First deduce an expression for the tangential velocity U outside the boundary layer as a function of s. Do *not* try to solve the final differential equation.

(d) Repeat parts (a) through (c) using the linear approximation for the velocity profile in the laminar boundary layer given by:

$$\frac{u}{U} = a + b(y/\delta)$$

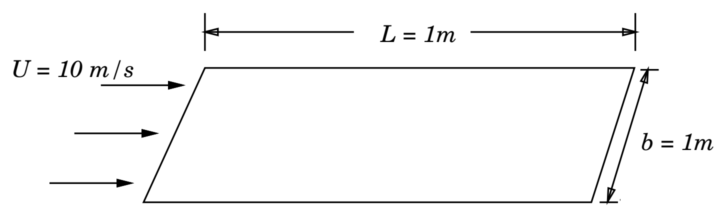
Be sure to first apply appropriate boundary conditions to determine a and b.

V6. A smooth thin circular pipe (radius r_0 and length L) is placed parallel to a steady uniform stream of velocity $U = 0.05m/s$. For the following problems, use $\nu = 10^{-6}m^2/s$ and $\rho = 10^3 kg/m^3$ and ignore any ambient turbulence.



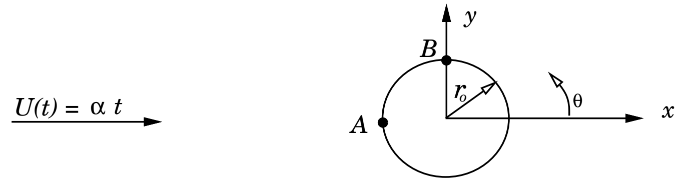
(a) For $r_0 = L = 1m$, estimate the displacement thickness δ^* at the exit end.
(b) Calculate the drag force on the pipe. (Do not forget the exterior surface!)
(c) Assuming that the assumptions of (a) and (b) are still valid and the longitudinal velocity remains constant at $U = 0.05m/s$ at the center of the pipe, determine how small r_0 must be so that Poiseuille flow may be assumed in 90% of the pipe's length.
(d) What is the difference between the pressures at the two ends of the tube in (c)? You may assume Poiseuille flow *throughout.*

V7. Consider the top surface (only) of a smooth flat plate (length L, width B, $L = B = 1m$) under a uniform flow of velocity $U = 10m/s$. For the following problems, use $\nu = 10^{-6}m^2/s$ and $\rho = 10^3 kg/m^3$ and ignore any ambient turbulence.



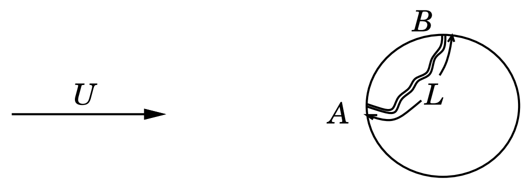
(a) At what range of lengths x do you expect the boundary layer to become turbulent?
(b) If the flow is tripped at the tip of the plate, estimate (i) the displacement thickness δ^* at the end of the plate; (ii) the total drag D; and (iii) the friction coefficient C_f. (Use the 1/7 power law.)
(c) To model roughness, spherical grains of diameter $k = 5mm$ are glued evenly over the plate at a density of $1600/m^2$ (average spacing = 2.5 cm). Assuming that the flow around each grain can be approximated as that past an isolated sphere, calculate C_f for this rough plate. How does this model compare with observed data for a sand-roughened plate?

V8. A sphere of radius R is placed in an accelerated flow with horizontal velocity $U(t) = \alpha t$ and density ρ:



(a) Assuming potential flow:
(i) Find the hydrodynamic force (F_x, F_y) on the sphere.
(ii) Calculate the pressures P_A and P_B at the points A and B on the sphere.

In an attempt to relieve some pressure on the sphere, a thin tube (radius r_T and length L) is used to connect A and B:



(b) If Poiseuille flow can be assumed in the tube throughout and dynamic viscosity is μ (you may assume $U = constant$ for the following):
(i) Find the flow rate Q and its direction through the tube.
(ii) Obtain formulas (but do not evaluate any integrals) for the force (F_{T_x}, F_{T_y}) on the sphere due to the presence of the tube.

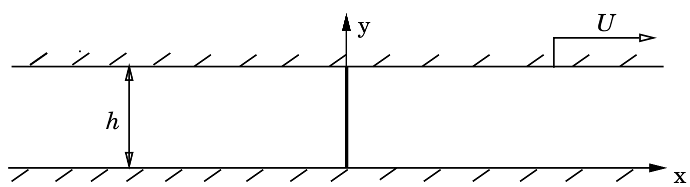
V9. A solid sphere of radius a and density ρ_s is held in a fluid of density ρ, where $\rho < \rho_s$, and kinematic viscosity ν under a gravitational field g. At time $t = 0$, the sphere is released.

(a) Find the initial acceleration $dU/dt(t = 0_+)$ of the sphere. [Hint: Do not forget added mass.]
(b) At $t = 0_+$, the velocity U of the sphere is still zero. Find the dynamic pressure $P_d(\theta)$ on the surface of the sphere at this instant.
(c) Determine the maximum velocity V_{max} that the sphere will eventually attain as a function of the drag coefficient C_D of the sphere. Estimate V_{max} for (i) a very small sphere; (ii) for a large sphere. Explain how large/small the radius should be in (i) and (ii) for the results to be valid.

V10. Radioactive water is leaking out through a long $3\mu m$ high hairline crack in a 10cm thick wall of a pressurized vessel. If the pressure inside is $10^6 N/m^2$ higher than atmospheric pressure outside:

(a) Calculate the maximum flow velocity U_{max} inside the crack.
(b) Calculate the volume rate per length of crack, Q at which the water is leaking out.
(c) Some time later, the crack becomes twice as high but the pressure difference has dropped by half. Find the new flow rate Q.
(d) In the aftermath of the accident, an expert witness speculates that the shear stress due to the flow can contribute to the widening of the crack. Under the conditions of (a) and (b) above, what is the shear stress τ_0 on the sides of the crack? What is the force per unit length of crack, F, due to the shear stress?

V11. Plane Couette flow occurs between two infinite planes located a distance h apart. One plane moves with a speed U with respect to the other, but a blockage causes the net flux of fluid to be zero.



(a) Formulate the governing equations used for this problem, stating clearly the assumptions used.
(b) Find an expression for the pressure gradient for $x/h \gg 1$.
(c) Sketch the velocity profile for $x/h \gg 1$.

V12. For ordinary Newtonian fluids, we have a good fit to experimental data using the turbulent velocity profile:

$$\frac{u}{u_\tau} = 8.7(\frac{u_\tau y}{\nu})^{1/7}$$

There has been interest in additives which alter the character of the fluid (making it non-Newtonian) for the purpose of reducing drag. For some additives, we find the velocity profile may be fit reasonable well by:

$$\frac{u}{u_\tau} = 8.7(\frac{u_\tau y}{\nu})^{1/7} + B$$

where B is a constant. Derive an equation giving friction coefficient vs. Reynolds' number for this fluid. You needn't solve the equation but verify that it recovers the usual Newtonian result for $B = 0$.

V13. A certain body is shaped so that the potential flow solution for velocity tangent to its surface is given by:

$$U = 2U_0 \sin \frac{x}{a}$$

where a is a constant characterizing the curvature of the body and U_0 is the free stream flow velocity. It is proposed that the laminar boundary layer profile be approximated as:

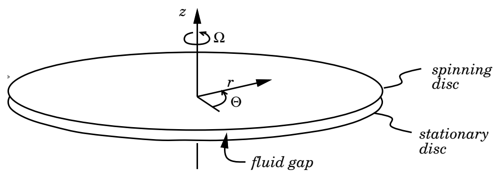
$$u = Uy\frac{2\delta - y}{\delta^2}$$

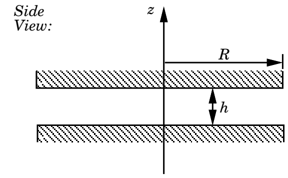
(a) Is this a feasible approximation for the profile? Why/why not?

Suppose the given profile is used.

(b) Find an expression for the displacement thickness as a function of $\delta(x)$.
(c) Find an expression for the momentum thickness as a function of $\delta(x)$.
(d) Find a differential equation for the growth rate of δ with x in terms of U_0, δ, x, a and ν only. (Do not try to solve this equation.)

V14. A simple model for a hydraulic clutch is as a pair of parallel discs, one rotating with respect to the other with angular velocity Ω. They are separated by a thin gap filled with fluid. The geometry is sketched below:





The Navier-Stokes equations in cylindrical coordinates are:

$$\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} + v_z\frac{\partial v_r}{\partial z} - \frac{v_\theta^2}{r} = -\frac{1}{\rho}\frac{\partial P}{\partial r}$$
$$+\nu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_r}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2} + \frac{\partial^2 v_r}{\partial z^2} - \frac{v_r}{r^2} - \frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta}\right],$$

$$\frac{\partial v_\theta}{\partial t} + v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + v_z\frac{\partial v_\theta}{\partial z} + \frac{v_r v_\theta}{r} = -\frac{1}{\rho r}\frac{\partial P}{\partial \theta}$$
$$+\nu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_\theta}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial \theta^2} + \frac{\partial^2 v_\theta}{\partial z^2} - \frac{v_\theta}{r^2} + \frac{2}{r^2}\frac{\partial v_r}{\partial \theta}\right],$$

$$\frac{\partial v_z}{\partial t} + v_r\frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta} + v_z\frac{\partial v_z}{\partial z} = -\frac{1}{\rho}\frac{\partial P}{\partial z}$$
$$+\nu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2} + \frac{\partial^2 v_z}{\partial z^2}\right].$$

If the radius R of the discs is large compared to the separation distance h, state the equations governing the steady fluid flow in the interior of the device. Give reasons for each term that you drop.

State the appropriate kinematic boundary conditions.

V15. A styrofoam sphere of diameter 0.5m has negligible density. It is held in sea water ($\rho = 1000kg/m^3, \nu = 10^{-6}m^2/s$) in the presence of gravity ($g = 10m/s^2$). At time $t = 0$ the sphere is released.

(a) What is the initial acceleration of the sphere?
(b) Assuming a drag coefficient of $C_D = 0.4$, find the terminal velocity of the sphere.

W. WAVES

Wa1. The equations governing surface waves can be linearized if $[A]$ $[\lambda/h]$ $[\lambda]$ $[h]$ $[kA]$ $[\omega T]$ $[V_g/V_p]$ $[u/V_p]$ $[A/\lambda]$ is (are) small.

Wa2. Waves and currents are created as winds blow over the ocean surface. Define the proper dynamic boundary condition(s) to be applied on the water surface.

Wa3. A plane wave propagates from deep water towards the shore over mildly varying slope. At a point P, the depth $h = 30m$, period $T = 10s$ and amplitude $A = 1m$.

(a) At P, calculate kh, Vg and $(pmax - pmin)_{bottom}$.
(b) In deep water, calculate T, λ and Vp/Vg.
(c) The average energy density of the wave at P is $\bar{E}$= ________________Joules/m^2 of which ________________% is due to potential energy. The power delivered (energy flux) there (per unit width of wave front) is ________________Watts/m. This is [larger] [smaller] [the same] [unrelated] [can not be determined] compared to the power delivered in deep water where the amplitude is $Adeep$ = ________________m.

Wa4. A wave tank 100m long, 1m wide and 4m deep has a wavemaker at one end.

(a) If the wavemaker begins to oscillate at $\omega = 4rad/s$, the disturbance will be felt at the opposite wall after ________________s.
(b) After steady conditions are reached, there are ________________wave crests in the tank at any instant.
(c) If the maximum and minimum wave heights in the tank are 20cm and 10cm respectively, the reflection coefficient of the far wall is $|R|$= ________________.

Wa5. A plane progressive wave has a free-surface elevation given by $h(x, y, t) = acos(px + qy + rt)$. Its period T is ________________. Its wavelength in the x and y directions are respectively λ_x= ________________, λ_y= ________________. The relationship between p, q and r in deep water is ________________.

Wa6. A deep-water plane progressive wave of frequency ω is incident onto a wall at $x = 0$ at an angle of 45°. Assuming perfect reflection from the wall, the wavelength of the wave travelling on the wall is λ_y= ________________.

Wa7. A plane progressive wave travelling from left to right in deep water has wavelength λ and amplitude A. (a) Qualitatively sketch the wave profile and the streamlines beneath it. Indicate the relevant dimensions. If the wave hits a vertical wall and is totally reflected. (b) At the instant when the wave elevation is maximum on the wall, qualitatively sketch the wave profile and the streamlines beneath it. Indicate the relevant dimensions.

Wa8. A wave tank 100m long, 10m wide, and 5m deep, has a wavemaker at one end and an absorbing wall (beach) at the other end. At time t=0, the wavemaker begins to oscillate at a frequency $\omega = \pi$ rad/s creating a train of waves.

(a) The wavelength of the waves is λ=________________, and
(b) the front of the wave will reach the other end at t=________________.
(c) If the average power delivered by the wavemaker is 1kW (1W = 1J/s), the amplitude of the waves in the tank is A=________________.
(d) If the water depth is reduced to only 0.2m, the corresponding answers are λ=__________, t=__________, and A=__________.
(e) For this depth, the maximum shear stress on the bottom is $\tau_{0\max}$=________________.

Wa9. For wave propagation in a certain medium, the dispersion relationship has the form $\omega \sim k^{5/4}$. The ratio of the group to phase speed in this medium is V_g/V_p=________________.

Wa10. A two-dimensional ship model of length 1m is towed in a deep tank at a speed of 5m/s.

(a) The waves behind the ship have V_p= ________________m/s.
(b) If the wave-making resistance on the model is 104 Joules/m2, the amplitude of the waves is A= ________________m.
(c) As the speed of the model is decreased by a factor of 2, λ will [increase] [decrease] [remain the same] by a factor of ________________.

Wa11. When a log blocking the entire width of a towing tank is towed at a speed of 1m/s, it generates a two-dimensional wave of amplitude 0.1m behind it. The wave drag on the log is ________________N, and the effective power expended by the towing carriage due to wave generation is ________________. If the wave amplitude is doubled at twice the speed, the effective power is changed by a factor of ________________.

Wa12. A two-dimensional ship model is towed in a deep-water tank at a speed U. The steady waves behind the ship have phase velocity V_p=__________, group velocity V_g=__________, and wavelength λ=________________. If the waves have amplitude A, the wave resistance of the model is given by D=________________.

Wa13. An observer on a fixed buoy in deep water measures a wave period of 10 seconds while a ship moving in an opposite direction to that of wave propagation encounters a wave every 6 seconds. The wavelength of the wave is ________________m and the speed of the ship is ________________m/s. If the ship turns around and follows the wave at the same speed, an observer on the ship will measure a wave period of ________________seconds.

Wa14. In an experiment, waves are created by heaving a buoy up and down with (sinusoidal) frequency ω and amplitude A. The relevant Froude and Reynolds numbers for gravity g, kinematic viscosity ν, and density ρ for this experiment are $F_r \sim$________________, and $R_e \sim$________________. Depending on the physics one is interested in, the force on the buoy can be normalized/nondimensionalized by $F^*=F/$__________,$F/$__________, or $F/$__________.

Wa15. A towing tank has a partially reflecting beach at the far end. Equipped with a wave elevation (H) gauge, the minimum amount of data one needs in order to calculate the reflection coefficient $|R|$ are: ________________. In terms of this data, $|R|$ is given by: ________________.

Wa16. The resonant frequency of a certain cylindrical cable in air is 1 rad/s. If the cable is neutrally buoyant, its resonant frequency in water should be ________________rad/s. In deep water, a wave which has this resonance frequency will have a wavelength of λ= ________________m.

Wa17. A wave tank is 100m long and 1m deep. At time t=0, the wavemaker begins to oscillate at a period of T =10s. The wavelength of the waves is λ=________________, and the front of the wave will reach the other end at t=________________. If the wave amplitude is A=1cm, the magnitude of the velocity on the bottom is U_0=________________. The boundary layer on the tank bottom is a ________________ boundary layer and has a thickness given by $\delta_{1/e}$=________________. The maximum shear stress on the bottom due to this boundary layer is τ_0=________________. *Assuming* that the average power loss per unit area is given by $U_0\tau_0$ and remains constant in time, with no more power input from the wavemaker the time it takes for the boundary layer dissipation to reduce the wave amplitude A by 50% would be ________________.

Waves problems continued next page

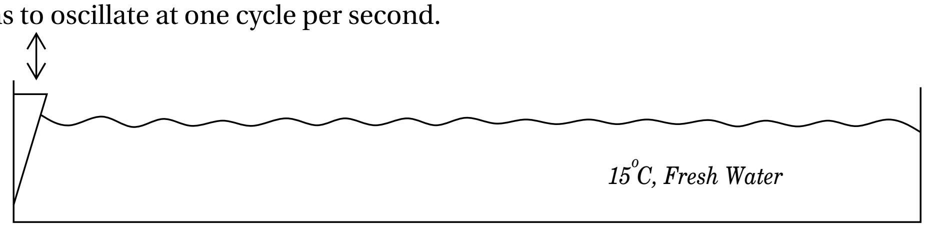
W1. Deep water waves with an amplitude of 1 meter are observed to pass a fixed buoy at a frequency of 1 wave every 5 seconds.

(a) What is their wavelength?
(b) What is the velocity of the crests?
(c) What is the amplitude of the particle velocity on the free surface?
(d) At what depth is the particle velocity reduced to 10% of its value on the surface?

W2. A plane progressive wave has a wavelength of 50m in deep water (15° C SW) and a wave height of 4m.

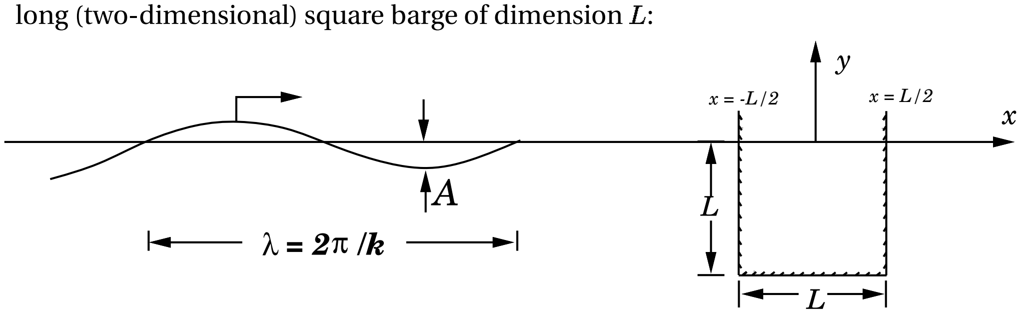
(a) What is the frequency at which wave crests pass a fixed point?
(b) What is the magnitude of the particle velocity at the surface?
(c) What is the magnitude of the particle velocity at depth of 10m below the free surface?
(d) What is the magnitude of the pressure variation at a depth of 10m?
(e) What is the velocity at which the crests move?
(f) What is the average energy density?
(g) What is the rate of energy flux per meter of crest length?

W3. A wave tank 50m long, 2m wide and very deep has a wave maker at one end and a vertical wall at the other. The tank contains fresh water at 15° C. At time $t = 0$, the wave maker begins to oscillate at one cycle per second.



(a) What is the wave length of the generated waves?
(b) The average power delivered to the waves is 3 Joules/s. What is the amplitude of the waves?
(c) When does the wave train reach the opposite end of the tank?
(d) If the waves are perfectly reflected, what is the magnitude of the wave force on the far wall of the tank?

W4. A deep water wave of amplitude A and wave number $k = 2\pi/\lambda$ is incident upon a stationary long (two-dimensional) square barge of dimension L:

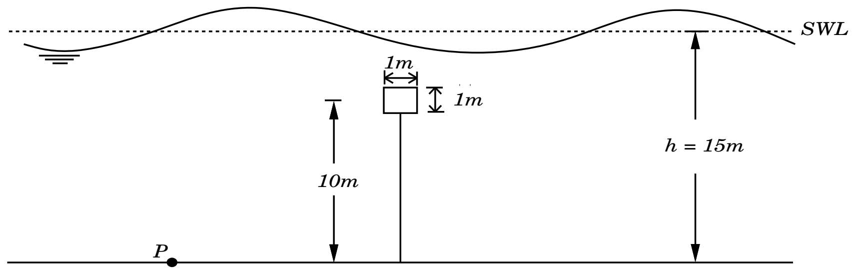


(a) Assuming $A/L \ll 1$ and $kL \ll 1$ and ignoring both viscous and diffraction effects, calculate the horizontal wave force F_x on the barge using a Froude-Krylov approximation.
(b) An alternative approach is to use Morison's equation and write:

$$F_x = C_m \rho L^2 \frac{dU(t)}{dt}$$

where dU/dt is the (Eulerian) fluid acceleration at the center of the waterplane ($x = y = 0$). By comparing the resulting formulas from (a) and (b), find an expression for C_m as a function of the dimensionless parameter kL. What is the asymptotic value of C_m for $kL \ll 1$?

W5. It is proposed that an underwater storage tank be constructed by anchoring a long (two-dimensional) square section of width = height = $L = 1m$ (using $\rho = 1000kg/m^3, g = 10/s^2$):



(a) If the mass of the empty tank can be ignored, find the natural swaying period T_n of the tank.
(b) As a plane progressive wave passes by, a pressure probe at P on the ocean floor detects a dynamic pressure of $P_d(t) = P_0 \cos \omega t$ where $P_0 = 10000N/m^2$ and $\omega = 1rad/s$. Find the wavelength λ and amplitude A of the wave.
(c) When the body is small compared to the wavelength ($L \ll \lambda$), it is correct to assume that the wavefield is largely unaffected by the presence of the body. Using this assumption, calculate the horizontal force (per unit width) F on the box and the dynamic tension force T in the anchor line due to the wave motion.
(d) During this "storm," the anchor breaks and the tank is flooded and becomes neutrally buoyant. How close will the tank come to (i) touching the ocean floor; and (ii) the mean water line?