Final Exam -- 2.087 Fall 2014

The final exam will take place during lecture time (9:30-11:00AM) on Wednesday 22 October.

The final exam will cover the material of Chapter 1, 2, 3, 4, 5, and 6 in Strang's Differential Equations and Linear Algebra, especially topic addressed in the lectures and practiced on Homeworks #1 through #5.

In the final exam, you are permitted to use a computer and you are allowed to implement and run MATLAB® code to find solutions. You can run other applications as you see fit such as Mathematica. The quiz is intended to be completely "software neutral". You will NOT NEED to interpret MATLAB code or write code, but you can do so if it is helpful for solving a problem.

The quiz is open-book: you may refer to the textbook, to any class materials, to old solutions from past semesters, and to your own notes. In fact, you can refer to any material whatsoever, but you may not communicate with anyone via voice, text, or any other means except with exam proctors to be sure you understand the questions.

The following topics are "fair game" as subject matter for the final exam in 2.087.

General topics

How do you determine the order of a differential equation?
Categorizing linear versus non-linear equations.
What does superposition enable you to infer about solutions to linear differential equations?
What is a null solution? How do you solve for a null solution?
What is a particular solution?
How can you solve initial value problems?
Constructing differential equations based on a description of an engineering system.
What is an equilibrium or steady-state solution?
What does it mean for an equilibrium solution to be stable or unstable?

First order equations in a single variable

Finding solutions to equations of the form $y' = ay + q(t)$ in general and most especially for the following source functions:
constant source,
unit step function (Heaviside),
delta function (Dirac),
exponential function
sine and cosine

Heaviside step functions and Dirac delta functions (including time delays).
Differentiation and integration of Heaviside step functions and Dirac delta functions.
Representing piecewise functions using Heaviside step functions and Dirac delta functions.

Finding solutions to non-linear, separable first order differential equations.

Second order equations in a single variable

Finding solutions for second order equations – both null (homogeneous) and particular solutions including initial value problems and especially emphasizing the following source functions:
constant source,
polynomial source,
unit step function (Heaviside),
delta function (Dirac),
exponential function
sine and cosine

The impulse response and the convolution integral as a means to find the system response to other source functions.

Critical damping, under-damping, over-damping.
Natural frequency, damped frequency.
Resonance.

Systems of differential equations

Finding solutions for systems of first order equations – both null (homogeneous) and particular solutions including initial value problems.

Eigenvalues and eigenvectors. How to compute them.
Use and interpretation of eigenvalues and eigenvectors in engineering contexts.

Formulating differential equations from descriptions of physical systems in general and especially for systems of masses, springs, and dampers, RLC circuits, predator-prey models, simple heat transfer, and simple fluid systems.

Understanding the behaviors of non-linear differential equations and systems of non-linear differential equations. Direction fields. Phase portraits. Euler’s method. Finding critical points. Assessing stability of critical points. Sinks, circles, spirals, and saddles.

Linear algebra

Matrix and vector multiplication. Inner product of two vectors. Outer product of two vectors.
Rotation, Scaling. Projection.
Solution of systems by Gaussian elimination and back substitution.
Matrix inverse – definition, conditions for existence, when to use it, when not to use it.
Existence and uniqueness of solutions of linear systems.
Null solutions. Rank. Row space. Column space. Dimension. Basis.
Construction of linear systems to represent specific cases of mechanical engineering interest.
Interpretation of matrix elements mathematically and physically for specific cases of mechanical engineering interest.
Symmetric positive definiteness. Definition, Relationship to eigenvalues. Implications for existence and uniqueness of solutions. Physical interpretation of SPD matrices.

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