Interpolation Cheat Sheet

Piecewise-Constant, Left-Endpoint

say $f(x) = mx + c$:

$m = f'(x)$

$x = a$ $x = b$
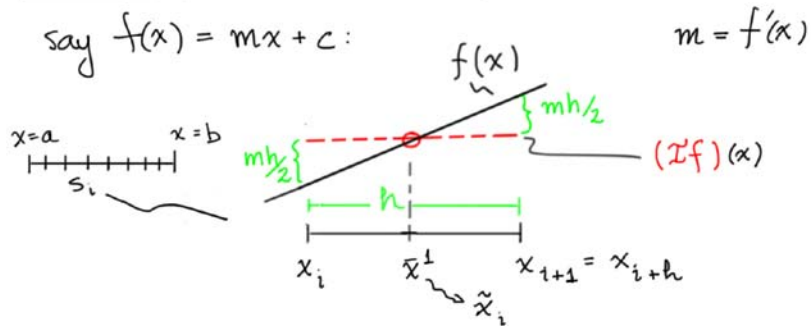
$f(x)$

e_i

$(\mathcal{I}f)(x)$

x_i S_i $x_{i+1} = x_i + h$

$\bar{x}^1 \rightsquigarrow \tilde{x}_i$
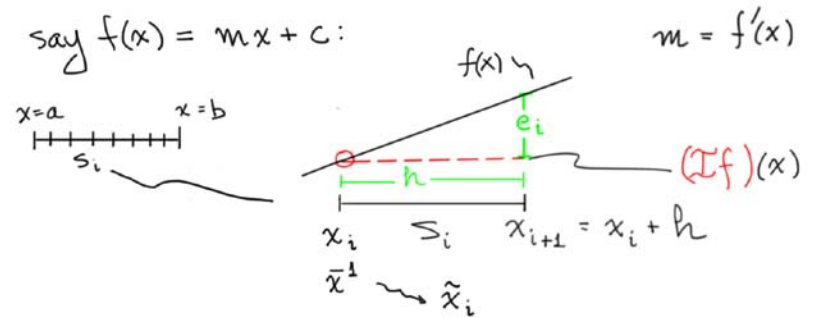
$$e_{max} = e_i = m\,h$$
$$= \max_{a \le x \le b} |f'(x)| \cdot h$$

in fact, valid for any f f'

Piecewise-Constant, Midpoint

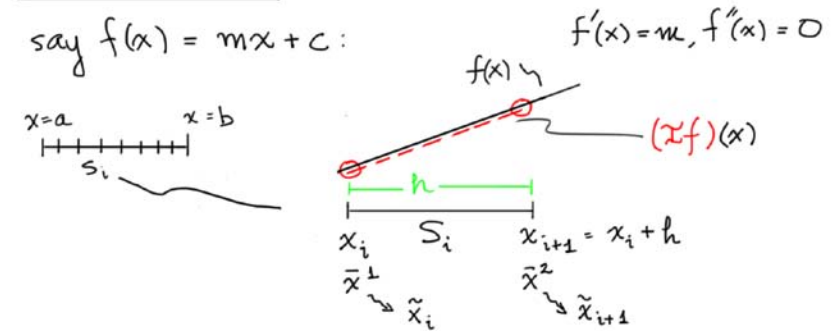say $f(x) = mx + c$:

$m = f'(x)$

$f(x)$

$x = a$ $x = b$

$mh/2$ $mh/2$

$(\mathcal{I}f)(x)$

S_i

h

x_i $\bar{x}^1 \rightsquigarrow \tilde{x}_i$ $x_{i+1} = x_i + h$

$$e_{max} = e_i = m\,h/2$$
$$= \max_{a \le x \le b} |f'(x)| \cdot h/2$$

in fact, valid for any f f'

Piecewise-Linear

say $f(x) = mx + c$:

$f'(x) = m, \; f''(x) = 0$

$f(x)$

$x = a$ $x = b$

$(\mathcal{I}f)(x)$

S_i

h

x_i S_i $x_{i+1} = x_i + h$

$\bar{x}^1 \rightsquigarrow \tilde{x}_i$ $\bar{x}^2 \rightsquigarrow \tilde{x}_{i+1}$

$$e_{max} = e_i = 0 \qquad f \text{ linear}$$
$$e_{max} \le \frac{h^2}{8} \max_{a \le x \le b} |f''(x)| \qquad f \text{ general } \; f''$$

"exact + 1"

Summary

I	exact for	order	bound
piecewise-constant, left-endpoint	constant: cx^0	$p = 1$	$\max \|f'\| \cdot h$
piecewise-constant, midpoint	constant: cx^0	$p = 1$	$\max \|f'\| \cdot \dfrac{h}{2}$
piecewise-linear	linear: $mx^1 + cx^0$	$p = 2$	$\max \|f''\| \dfrac{h^2}{8}$

2.086 Numerical Computation for Mechanical Engineers
Spring 2013