2.086 Numerical Computation for Mechanical Engineers

MINI-QUIZ 6 Fall 2014

You may refer to the textbook, lecture notes, Matlab® tutorials, and other class materials as well as your own notes and scripts.

You may use a calculator (for simple arithmetic operations and function evaluations). However, laptops, tablets, and smartphones are not permitted.

You have 30 minutes of recitation to complete the mini quiz. When you are finished, you can hand in your quiz and start working on your assignment.

NAME _____

There are a total of 100 points: four questions, each worth 25 points.

All questions are multiple choice; ***circle one and only one answer***. Make sure to fully erase or indicate "Retracted" on any other circles not associated with your single final answer.

We provide two blank pages at the end of the quiz which you may use for any derivations, but note that we will *only grade your multiple choice selections*.

This (same) quiz will be administered in all recitation sections. *You may not discuss this quiz with other people until the graded quizzes are returned to the class.*

Note that in this quiz we do not give any units for the various (physical) quantities. You may assume that all units are consistent, or that we have nondimensionalized the equations. In short, no need to worry about units.

Question 1 (25 points). The figure below shows the numerical result obtained from an attempt to solve the initial value problem

$$\frac{dw}{dt} = -5w, \quad 0 < t < t_f$$
$$w(t = 0) = 1$$

You are asked to identify the numerical method used to obtain this solution as well as the size of the timestep, Δt, used. Which of the following is the correct choice?

(a) Euler Forward, $\Delta t = 0.05$

(b) Euler Forward, $\Delta t = 0.2$

(c) Euler Backward, $\Delta t = 0.05$

(d) Euler Backward, $\Delta t = 0.2$

(e) None of the above
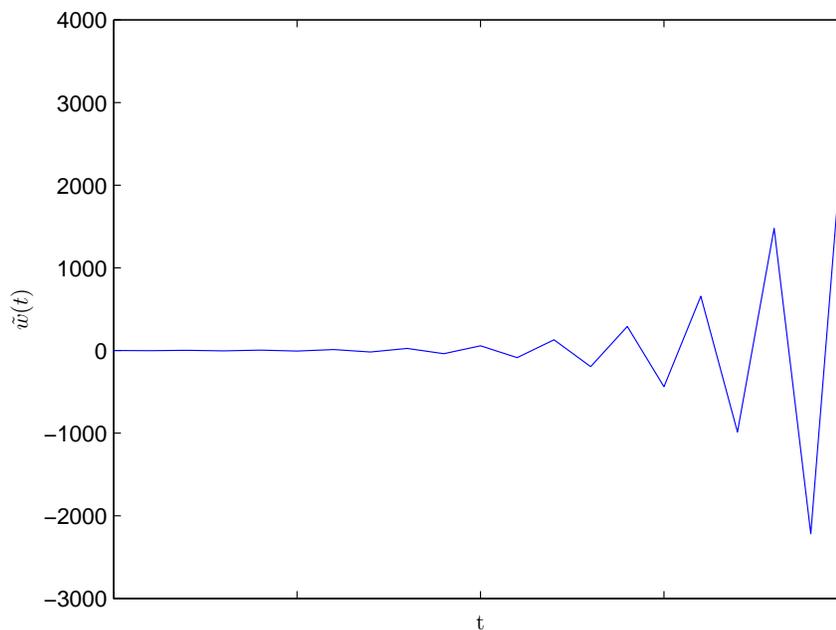
Question 2 (25 points). We would like to write a Crank-Nicolson solver for integrating the linear system of equations $\dot{z} = A z + f$ where

$$z = z(t) = \begin{pmatrix} z_1(t) \\ z_2(t) \end{pmatrix}, \ f = f(t) = \begin{pmatrix} f_1(t) \\ f_2(t) \end{pmatrix}$$

and A is a 2×2 square matrix of constant (time-independent) entries.

If I denotes the unity matrix of the same size as A, the update for $z(t + \Delta t)$ (for Crank-Nicolson) is

(a) $z(t + \Delta t) = (I + A \Delta t) z(t) + f(t) \Delta t$

(b) $z(t + \Delta t) = A z(t) \Delta t + f(t) \Delta t$

(c) $z(t + \Delta t) = (I + 0.5 \Delta t A) z(t) + 0.5 f(t)$

(d) $z(t + \Delta t) = (2I - \Delta t A)^{-1} \left[(2I + \Delta t A) z(t) + \Delta t \left[f(t) + f(t + \Delta t) \right] \right]$

(e) $z(t + \Delta t) = (2I - \Delta t A)^{-1} (2I + \Delta t A) z(t) + (2I - \Delta t A)^{-1} \left[f(t) + f(t + \Delta t) \right]$

(f) $z(t + \Delta t) = (2I - \Delta t A)^{-1} (2I + \Delta t A) z(t) + (2I - \Delta t A)^{-1} \left[f(t - \Delta t) + f(t) \right]$

(g) $z(t + \Delta t) = z(t - \Delta t) + 0.5 \Delta t \left[A z(t) + f(t) \right] + 0.5 \Delta t \left[A z(t - \Delta t) + f(t - \Delta t) \right]$

(h) $z(t + \Delta t) = (I - \Delta t A)^{-1} \left[z(t) + f(t + \Delta t) \right]$

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Question 3 The linear differential equation

$$2\frac{d^3y}{dt^3} + \frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 2y = \sin(t)$$

can be written in canonical form $\dot{z} = Az + f(t)$ where z is the vector $(y, dy/dt, d^2y/dt^2)^{\mathrm{T}}$, A is a 3×3 matrix of constant (time-independent) entries and $f(t)$ is a 3×1 vector. Which of the following represents the entries of A and $f(t)$ correctly?

(a) $A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 2 & 0.5 \end{pmatrix}$; $f(t) = \begin{pmatrix} 0 \\ 0 \\ 0.5\sin(t) \end{pmatrix}$

(b) $A = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 4 & 2 \end{pmatrix}$; $f(t) = \sin(t)\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

(c) $A = \begin{pmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 1 & 2 & 0 \end{pmatrix}$; $f(t) = \begin{pmatrix} 0.5\sin(t) \\ 0.5\sin(t) \\ 0.5\sin(t) \end{pmatrix}$

(d) $A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -0.5 \end{pmatrix}$; $f(t) = 0.5\ \sin(t)\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

(e) $A = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{pmatrix}$; $f(t) = \sin(t)\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$

(f) $A = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -0.5 \end{pmatrix}$; $f(t) = \begin{pmatrix} 0.5\sin(t) \\ 0 \\ 0 \end{pmatrix}$

(g) $A = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -0.5 \end{pmatrix}$; $f(t) = \begin{pmatrix} 0 \\ 0 \\ 0.5\sin(t) \end{pmatrix}$

Question 4 (25 points). This question concerns the numerical integration of the scalar ODE

$$\frac{dw}{dt} = -(1+t)w, \quad 0 < t < t_f$$
$$w(t=0) = w_0$$

using a timestep Δt. Which scheme produces

$$\tilde{w}^1 = \frac{w_0}{1 + \Delta t + \Delta t^2}$$

as the numerical solution after one timestep $[\tilde{w}^1 = \tilde{w}(t = j\Delta t), \ j = 1]$?

(a) Crank-Nicolson

(b) Euler Forward

(c) Euler Backward

(d) None of the above

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