2.086 Numerical Computation for Mechanical Engineers

MINI-QUIZ 5 **Fall 2014**

You may refer to the textbook, lecture notes, Matlab® tutorials, and other class materials as well as your own notes and scripts.

You may use a calculator (for simple arithmetic operations and function evaluations). However, laptops, tablets, and smartphones are not permitted.

You have 30 minutes of recitation to complete the mini quiz. When you are finished, you can hand in your quiz and start working on your assignment.

NAME ____

There are a total of 100 points: four questions, each worth 25 points.

All questions are multiple choice; ***circle one and only one answer***. Make sure to fully erase or indicate "Retracted" on any other circles not associated with your single final answer.

We provide two blank pages at the end of the quiz which you may use for any derivations, but note that we will *only grade your multiple choice selections*.

This (same) quiz will be administered in all recitation sections. *You may not discuss this quiz with other people until the graded quizzes are returned to the class.*

Question 1 (25 points). Consider a least squares formulation for the model $y^{\text{model}}(x; \beta) = \beta_0 + \beta_1 x + \beta_2 x^2$, where $\beta = (\beta_0, \beta_1, \beta_2)^{\text{T}}$. We perform $m > 3$ measurements of y and arrange those in the $m \times 1$ (column) vector y^{meas}. The associated design matrix, which has independent columns, is denoted by X. We denote the least squares solution by $\hat{\beta} = (\hat{\beta}_0, \hat{\beta}_1, \hat{\beta}_2)^{\text{T}}$. Which of the following statements is correct?

(a) The least squares solution is given by $\hat{\beta} = X^{-1} y^{\text{meas}}$.

(b) The scalar $(y^{\text{meas}} - X\beta)^{\text{T}}(y^{\text{meas}} - X\beta)$ has a unique minimum when $\beta = \hat{\beta}$.

(c) For arbitrary vectors $\beta = (\beta_0, \beta_1, \beta_2)^{\text{T}}$, the scalar $(y^{\text{meas}} - X\beta)^{\text{T}}(y^{\text{meas}} - X\beta)$ may take on both positive and negative values.

(d) The design matrix X is of size $3 \times m$.

Question 2 (25 points). You are asked to perform a least squares fit using a function of the form $y^{\text{model}}(x;\beta) = \beta_0 + \beta_1 x$, where the vector $(\beta_0, \beta_1)^{\text{T}}$ is to be determined from the following data:

i	x_i	y_i^{meas}
1	0	0
2	2	2.5
3	2	1.5
4	4	4

The solution to the least squares problem is

(*a*) $\hat{\beta}_0 = 2,\ \hat{\beta}_1 = 0$

(*b*) $\hat{\beta}_0 = 1,\ \hat{\beta}_1 = 1$

(*c*) $\hat{\beta}_0 = -1,\ \hat{\beta}_1 = 1$

(*d*) $\hat{\beta}_0 = 0,\ \hat{\beta}_1 = 1$

Hint: You may answer this question without using computation. A sketch of the data points can be very helpful.

Question 3 (25 points). Consider the situation where the physical model describing the process of interest is given by the truth expression $y^{\text{true}}(x) = \alpha + \gamma x^2 + \delta x^4$. The following questions refer to performing a least squares fit for α, γ and δ using m measurement data and some proposed model. Which of the following statements is incorrect?

(a) Underfitting corresponds to the case of using $m < 3$ for performing the least squares.

(b) Overfitting needs to be avoided because it increases susceptibility to noise.

(c) Overfitting is particularly dangerous when the number of measurements, m, is small.

(d) Underfitting results in model error that cannot be rectified by increasing m.

(e) For the model $y^{\text{model}}(x; \beta_0) = \beta_0$ and $m = 4$ measurements, the design matrix, X, is $(1\ \ 1\ \ 1\ \ 1)^{\text{T}}$.

Question 4 (25 points). Which of the following models $\big(z = z^{\text{model}}(t; (\alpha, \gamma)^{\text{T}})\big)$ cannot be cast as a linear least squares problem (for the calculation of parameters α and γ) though appropriate transformations?

$$(a)\;\; z = \frac{1}{\alpha} + \gamma \sinh(t), \quad z, t > 0$$

$$(b)\;\; z = t^{\alpha t^{\gamma}}, \quad z, t > 0$$

$$(c)\;\; z = \alpha t + \exp(\gamma t), \quad z, t > 0$$

$$(d)\;\; z = \frac{1}{\alpha + \gamma t}, \quad z, t > 0$$

Recall that given a vector of measurements y^{meas}, a linear least squares problem of the form $y = y^{\text{model}}(x; (\beta_0, \beta_1)^{\text{T}})$ can be written as $y^{\text{meas}} = X\beta$, where $\beta = (\beta_0, \beta_1)^{\text{T}}$ and X is an appropriate design matrix.