Unit I Quiz Question Sampler   2.086/2.090 Fall 2012

You may refer to your text and other class materials as well as your own notes and scripts. You may not use computers, tablets, calculators, or smartphones. If you wish to simulate real quiz conditions you should allocate roughly 15 minutes for each question.

NAME _____

All questions are multiple choice; *unless otherwise explicitly indicated, circle one and only one answer*. We provide a blank page at the end of these questions (and for each quiz) and we encourage you to work through each question carefully; but note that, under real quiz conditions, we *only grade your multiple choice selections*.

You may assume that all arithmetic operations are performed exactly (with *no* floating point truncation or round-off errors).

Answer Key: The answer key is provided at the end of this collection of questions. You should not expect a similar courtesy on the actual quiz.

Question 1. Consider the integral of a linear function,

$$I = \int_0^1 (\alpha + \beta x)\, dx \tag{1}$$

in which α and β are any *non-zero* constants (e.g., $\alpha = 2$, $\beta = 4.5$). We approximate this integral by first breaking up the full interval into $N - 1$ segments of equal length h and then applying an integration rule. Which of the following integration rules will yield the *exact* result for the integral of equation (1) for any value of h?

  (a) rectangle, left

  (b) rectangle, right

  (c) trapezoid

  (d) rectangle, midpoint

Please circle all answers that apply. You will get full credit for circling all the correct answers and no incorrect answers, proportional partial credit for circling some of the correct answer(s) and no incorrect answers, and zero credit if you circle any incorrect answers. (*Hint*: Draw a simple picture.)

Question 2. The function $f(x)$ is defined over the interval $0 \le x \le 1$ by

$$f(x) = \begin{cases} x & \text{for } 0 \le x \le 1/2 \\ 1-x & \text{for } 1/2 < x \le 1 \end{cases}.$$

We also introduce a set of points $x_i = \frac{i-1}{5}$, $1 \le i \le 6$, which define segments $S_i = [x_i, x_{i+1}]$, $1 \le i \le 5$ (for example, the first segment, S_1, is given by $0 \le x \le 0.20$). The function $f(x)$, points x_i, $1 \le i \le 6$, and segments S_i, $1 \le i \le 5$, are depicted in Figure 1.

In the below we consider two different interpolation procedures, "piecewise-constant, left endpoint" and "piecewise-linear," based on the segments S_i, $1 \le i \le 5$. In each case we evaluate the interpolant of $f(x)$ for any given value of x by first finding the segment which contains x and then within that segment applying the particular interpolation procedure indicated. Note your interpolants should not involve evaluations of the function $f(x)$ at points other than the x_i, $1 \le i \le 6$, defined above.

(i) Let $(\mathcal{I}^0 f)(x)$ be the "piecewise-constant, left endpoint" interpolant of f based on the segments S_i, $1 \le i \le 5$. The error in this "piecewise-constant, left, endpoint" interpolant at $x = 0.3$, defined as $|f(x = 0.3) - (\mathcal{I}^0 f)(x = 0.3)|$, is

(a) 0

(b) 0.05

(c) 0.02

(d) 0.1

(e) 0.2

(ii) Let $(\mathcal{I}^1 f)(x)$ be the "piecewise-linear" interpolant of f based on the segments S_i, $1 \le i \le 5$. The error in this "piecewise-linear" interpolant at $x = 0.3$, defined as $|f(x = 0.3) - (\mathcal{I}^1 f)(x = 0.3)|$, is
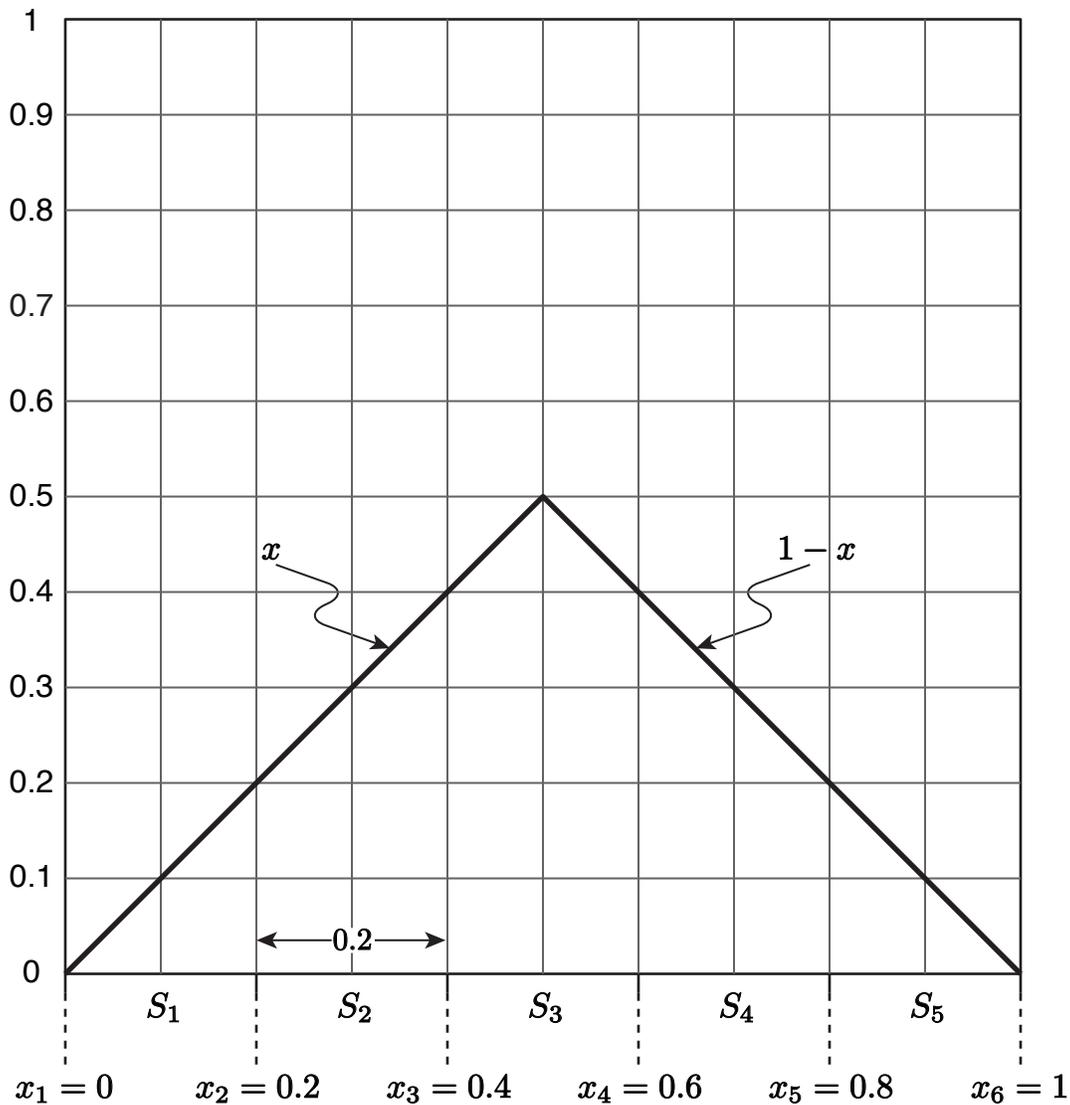
(a) 0

(b) 0.05

(c) 0.02

(d) 0.10

(e) 0.2

Figure 1: A plot of the function

$$f(x) = \begin{cases} x & \text{for } 0 \le x \le 1/2 \\ 1-x & \text{for } 1/2 < x \le 1 \end{cases},$$

as well as the points x_i, $1 \le i \le 6$, and associated segments S_i, $1 \le i \le 5$. Note the points x_i, $1 \le i \le 6$, are equidistant, and hence the segments S_i, $1 \le i \le 5$, are all the same length 0.2.

Question 3. The function $f(x)$ is defined over the interval $0 \le x \le 1$ by

$$f(x) = \begin{cases} x & \text{for } 0 \le x \le 1/2 \\ 1 - x & \text{for } 1/2 < x \le 1 \end{cases} .$$

We also introduce a set of points $x_i = \frac{i-1}{5}$, $1 \le i \le 6$, which define segments $S_i = [x_i, x_{i+1}]$, $1 \le i \le 5$ (for example, the first segment, S_1, is given by $0 \le x \le 0.20$). The function $f(x)$, points x_i, $1 \le i \le 6$, and segments S_i, $1 \le i \le 5$, are depicted in Figure 1.

We further define the definite integral

$$I = \int_0^1 f(x)\, dx .$$

In the below we shall consider two different numerical approximations to this integral , "rectangle rule, left" and "trapezoidal rule," based on the particular segments S_i, $1 \le i \le 5$. Note your numerical approximations should not involve evaluations of the function $f(x)$ at points other than the x_i, $1 \le i \le 6$, defined above.

(i) Let $I_h^{0,\,\text{left}}$ be the "rectangle rule, left" approximation to the integral I. The error in this "rectangle rule, left" approximation, defined as $|I - I_h^{0,\,\text{left}}|$, is

(a) 0

(b) 0.01

(c) 0.06

(d) 0.09

(e) 0.2

(ii) Let I_h^1 be the "trapezoidal rule" approximation to the integral I. The error in this "trapezoidal rule" approximation, defined as $|I - I_h^1|$, is

(a) 0

(b) 0.01

(c) 0.06

(d) 0.09

(e) 0.2

Question 4. We consider the differentiation of a very smooth function $f(x)$ by finite differences. (The function is very smooth, but not constant — for example, $f(x)$ might be $x^2 e^x$.) You are given the values of $f(x)$ at three points $\tilde{x}_1$, $\tilde{x}_2$, and $\tilde{x}_3$ which are equi-spaced in the sense that $\tilde{x}_2 - \tilde{x}_1 = h$ and $\tilde{x}_3 - \tilde{x}_2 = h$; you wish to approximate the first derivative of f at the particular point $x = \tilde{x}_2$, $f'(\tilde{x}_2)$.

The following finite difference formulas are proposed to approximate $f'(\tilde{x}_2)$:

(a) $\dfrac{f(\tilde{x}_2) - f(\tilde{x}_1)}{h}$

(b) $\dfrac{f(\tilde{x}_3) - f(\tilde{x}_1)}{2h}$

(c) $\dfrac{f(\tilde{x}_3) - f(\tilde{x}_2)}{2h}$

(d) $\dfrac{f(\tilde{x}_3) - f(\tilde{x}_2)}{h/2}$

(*i*) Circle all answers which converge to $f'(\tilde{x}_2)$ as h tends to zero.

You will get full credit for circling all the correct answers and no incorrect answers, proportional partial credit for circling some of the correct answer(s) and no incorrect answers, and zero credit if you circle any incorrect answers. (*Hint*: If the formula gives the wrong result for a linear function $f(x)$ then certainly the formula can not converge to the correct result in general.)

(*ii*) Mark "BEST" next to the answer above which will give the *most accurate* approximation to $f'(\tilde{x}_2)$ (for h sufficiently small).

Question 5. Consider the MATLAB script

```matlab
clear

y = [5,-3,2,7];

val = y(1);
for i = 2:length(y)
   % either && or & will work in the statement below
   if ( (y(i) <= val) && (y(i) >= 0) )
       val = y(i);
   end
end

val_save = val
```

(*i*) The *first time* through the loop, when `i` $= 2$, the expression `(y(i) <= val)` will evaluate to

    (a) (logical) `0`

    (b) (logical) `1`

    (c) `5`

    (d) `-3`

    where logical here refers to the logical data type or class.

(*ii*) Upon running the script above (to completion) `val_save` will have the value

    (a) `5`

    (b) `-3`

    (c) `2`

    (d) `7`

Question 6. Consider the MATLAB script

```
clear

x = [1,0,0,2];
y = [0,2,4,5];

z = (x.^2) + y;

ind_vec = find( z > 3 );

r = z(ind_vec);
```

(i) This script will calculate z to be

(a) [1,2,4,7]

(b) [1,2,4,9]

(c) [2,2,4,9]

(d) [1,0,0]

(ii) This script will calculate ind_vec to be

(a) [0,0,1,1]

(b) [0,1,1,0]

(c) [2,3]

(d) [3,4]

(iii) This script will calculate r to be

(a) [4,7]

(b) [4,9]

(c) [3,4]

(d) [25]

Question 7. We wish to approximate the integral

$$I = \int_0^1 xe^x \, dx$$

by two different approaches (A and B) in the script

```
% begin script

clear

numpts = 21;              % number of points in x

h = 1/(numpts-1);         % distance between points in x
xpts = h*[0:numpts-1];    % points in x

fval_at_xpts = xpts.*exp(xpts);

weight_vecA = [0,h*ones(1,numpts-1)];
weight_vecB = [0.5*h,h*ones(1,numpts-2),0.5*h];

IA = sum(weight_vecA.*fval_at_xpts);
IB = sum(weight_vecB.*fval_at_xpts);

% end script
```

The first approach (A) yields `IA`; the second approach (B) yields `IB`. We can readily derive (by integration by parts) that the exact result is $I = 1$.

Hint: Run the script "by hand" for `numpts = 3` and draw a picture representing the areas associated with `IA` and `IB`. The graph in Figure 2 may prove helpful.
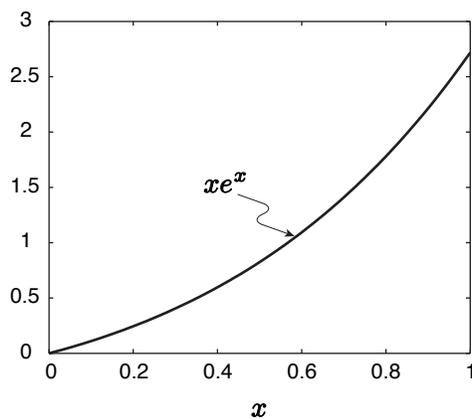

Figure 2: A graph of the integrand xe^x. Note that the second derivative of xe^x is positive for all x, $0 \le x \le 1$.

We now run the script (for `numpts = 21`).

(i) We will obtain

    (a) `IA` greater than 1

    (b) `IA` less than 1

    (c) `IA` exactly equal to 1

    where we recall that the exact result is $I = 1$.

(ii) We will obtain

    (a) `IB` greater than 1

    (b) `IB` less than 1

    (c) `IB` exactly equal to 1

(iii) We will find

    (a) `abs(IA - 1)` less than `abs(IB - 1)`

    (b) `abs(IA - 1)` greater than `abs(IB - 1)`

    (c) `abs(IA - 1)` equal to `abs(IB - 1)`

Note that `abs` is the MATLAB built-in absolute value function: `abs(z)` returns the absolute value of `z`. Hence in (iii) we are comparing the absolute value of error in the two approaches.

Answer Key

Q1 (c), (d)

Q2 *(i)* (d)
   (ii) (a)

Q3 *(i)* (b)
   (ii) (b)

Q4 *(i)* (a), (b)
   (ii) (b) is BEST

Q5 *(i)* (b)
   (ii) (c)

Q6 *(i)* (b)
   (ii) (d)
  (iii) (b)

Q7 *(i)* (a)
   (ii) (a)
  (iii) (b)

2.086 Numerical Computation for Mechanical Engineers

Fall 2012