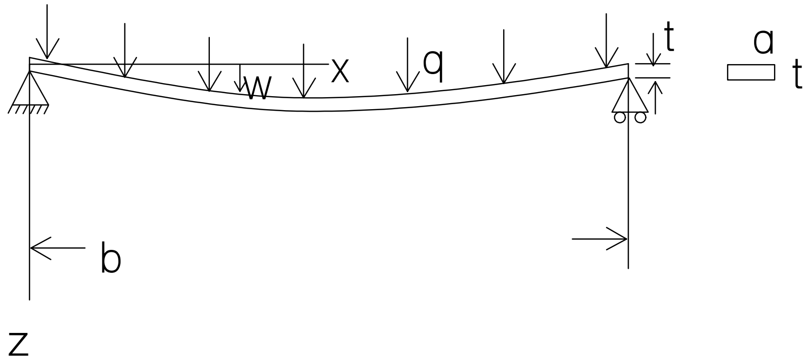
Plate Bending Introduction

review general beam, simply supported, clamped long plate
long plate, boundary conditions (end restrained) not so long plate

see: bending with z load sheet for derivations

simply supported beam:



$$Q(x) := \frac{q \cdot b}{2} - q \cdot x$$

$$\int_0^x Q(\xi)\, d\xi \rightarrow \frac{1}{2} \cdot q \cdot b \cdot x - \frac{1}{2} \cdot q \cdot x^2$$

$$M(x) := \frac{q \cdot b}{2} \cdot x - q \cdot x \cdot \frac{x}{2}$$

$$\frac{d}{dx} M(x) \rightarrow \frac{1}{2} \cdot q \cdot b - q \cdot x$$ = 0 @ x = b/2

$$M\left(\frac{b}{2}\right) \rightarrow \frac{1}{8} \cdot q \cdot b^2 \qquad M_{max} := \frac{1}{8} \cdot q \cdot b^2$$

$$\sigma_x := \frac{M(x)}{I} \cdot z$$

maximum when z = t/2, m(x) = M_{max}

$$\sigma_{x\_max} := \frac{M_{max}}{I} \cdot \frac{t}{2}$$

$$I := \frac{1}{12} \cdot t^3 \cdot a \qquad \sigma_{x\_max} := \frac{\frac{1}{8} \cdot q \cdot b^2}{I} \cdot \frac{t}{2} \qquad \sigma_{x\_max} \rightarrow \frac{3}{4} \cdot q \cdot \frac{b^2}{t^2 \cdot a}$$

+ tension other side of load

- compression on load side

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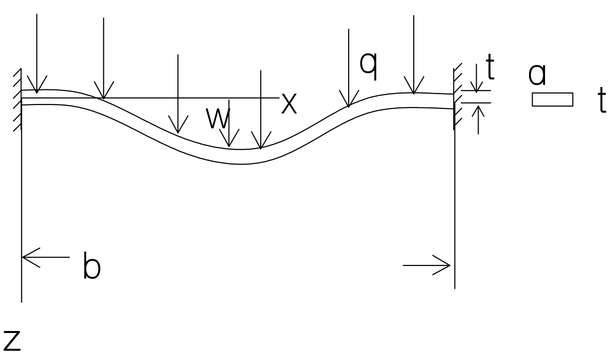
clamped beam:



need to use delection $\frac{d^4}{dx^4} w$ to solve

result: $$M(x) := -q \cdot \left(\frac{x^2}{2} - \frac{b \cdot x}{2} + \frac{b^2}{12}\right)$$

Given $\frac{d}{dx} M(x) = 0$ $\quad$ $\mathrm{Find}(x) \rightarrow \frac{1}{2} \cdot b$

$$M\left(\frac{b}{2}\right) \rightarrow \frac{1}{24} \cdot q \cdot b^2$$

$$M_{max} = M(x = 0, x = b)$$

$$M(0) \rightarrow \frac{-1}{12} \cdot q \cdot b^2 \qquad M(b) \rightarrow \frac{-1}{12} \cdot q \cdot b^2$$

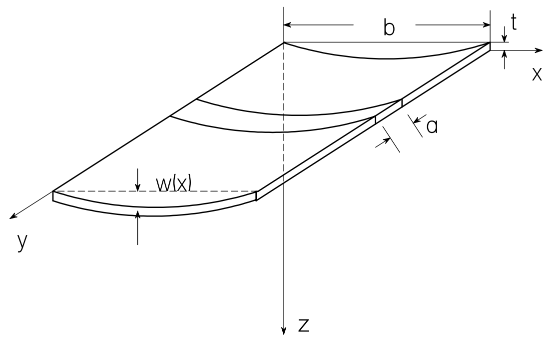
$$\sigma_{x\_max} := \frac{\frac{1}{12} \cdot q \cdot b^2}{I} \cdot \frac{t}{2} \qquad \sigma_{x\_max} \rightarrow \frac{1}{2} \cdot q \cdot \frac{b^2}{t^2 \cdot a}$$

- compression other side of load

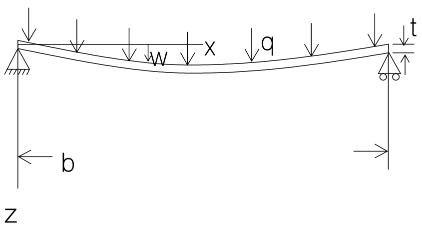
+ tension on load side

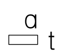
long plate:
treating unit length (away
from end effects)

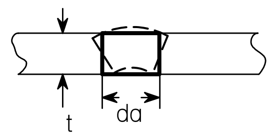
$a := 1$



section at a
simply supported
free to pull in







strain in y constrained by adjacent plate
anticlastic curvature Rν = 1/ν ∗ R

$\varepsilon_x := \frac{\sigma_x}{E} - \frac{\nu\cdot\sigma_y}{E}$ $\qquad \varepsilon_y := \frac{\sigma_y}{E} - \frac{\nu\cdot\sigma_x}{E} = 0 \qquad \frac{\sigma_y}{E} = \frac{\nu\cdot\sigma_x}{E}$

substituting => $\varepsilon_x := \frac{\left(1-\nu^2\right)\cdot\sigma_x}{E}$ or ... $\varepsilon_x := \frac{\sigma_x}{\frac{E}{\left(1-\nu^2\right)}}$ or ... $\varepsilon_x := \frac{\sigma_x}{E'}$ where E' = E/(1-ν^2)

rearranging => $\sigma_x := \frac{E\cdot\varepsilon_x}{1-\nu^2}$ $\qquad$ as in bending of beam: $\varepsilon_x := -\frac{z}{R}$ => $\sigma_x := -\frac{E}{1-\nu^2}\cdot\frac{z}{R}$

$$M := \int_{\frac{-t}{2}}^{\frac{t}{2}} \sigma_x\cdot z\,dz \qquad M := -\int_{\frac{-t}{2}}^{\frac{t}{2}} \frac{E\cdot z}{1-\nu^2}\cdot\frac{d^2}{dx^2}w\cdot z\,dz \qquad M := -\frac{E\cdot t^3}{12\cdot\left(1-\nu^2\right)}\cdot\frac{d^2}{dx^2}w \qquad M = M_y$$

define: $D := \frac{E\cdot t^3}{12\cdot\left(1-\nu^2\right)}$ $\qquad M := -D\cdot\frac{d^2}{dx^2}w$

moment relationships are the same:
simply supported:

$\sigma_{x\_max} := \frac{\frac{1}{8}\cdot q\cdot b^2}{I}\cdot\frac{t}{2} \qquad \sigma_{x\_max} := \frac{3}{4}\cdot q\cdot\frac{b^2}{t^2}$

clamped: (figure not shown)

$\sigma_{x\_max} := \frac{\frac{1}{12}\cdot q\cdot b^2}{I}\cdot\frac{t}{2} \qquad \sigma_{x\_max} := \frac{1}{2}\cdot q\cdot\frac{b^2}{t^2}$

Hughes 9.1.7 is of the form: $\sigma_{x\_max} := k\cdot q\cdot\frac{b^2}{t^2}$

k = 0.75 simply supported
k = 0.5 clamped
N.B. stress is + & - from bending