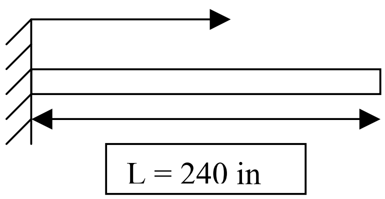
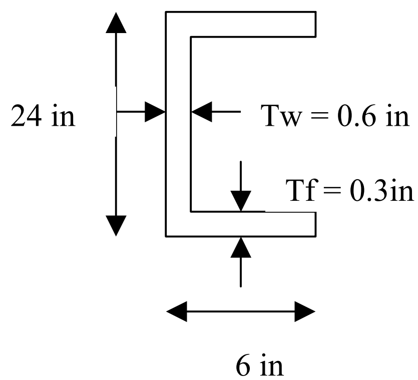
13.122 Ship Structural Design and Analysis

Problem Set 4 - 2003

1. Torsional Response of a Channel Cross-Section Cantilever Beam





For some structural members, the torsional response cannot be simply evaluated as pure torsion (Saint Venant torsion…torsion of the 13.10 variety) and pure warping torsion. The sum of these effects give the combined torsional resistance of a cross section,

$$M_z = M_z^{ST} + M_z^{w}$$

From Shames, we know that $M_z^{ST} = G \cdot K_T \cdot \varphi'$ and from our lecture notes we know that $M_Z^{ST} = -E \cdot I_\omega \cdot \varphi'''$. The concentrated torque M_z becomes

$$M_z = G \cdot K_T \cdot \varphi' - E \cdot I_\omega \cdot \varphi'''$$

For distributed torques along the length of the member,

$$m_z = \frac{-dM_z}{dz} = E \cdot I_\omega \cdot \varphi^{iv} - G \cdot K_T \cdot \varphi''$$

Given the above geometry and m_z = 400 ft-lbs/foot length:

a) show that $\varphi = A + B\cosh(\lambda z) + C\sinh(\lambda z) + \dfrac{m_z \cdot z^2}{2 \cdot G \cdot K_T}$ where $\lambda^2 = \dfrac{G \cdot K_T}{E \cdot I_\omega}$

b) Using the numerical techniques presented in the Lecture notes, calculate the cross-sectional area properties.

c) Identify the boundary conditions for the cantilever beam.

d) Plot ϕ, ϕ', and M_z versus length.

e) Calculate and plot the axial and shear stresses over the cross section at z = 0, .25L, .5L, .75L, and L. Note: you may want to set up b) with a few more s points over the cross section than what is needed to define the shape to better demonstrate the stress behavior over the cross section (i.e. linear versus parabolic).

f) What impact does a change in length have on d) and e)? Does Saint Venant torsion or warping torsion dominate for short, stubby beams or long, slender beams? Hint: Look at changes in λ with length.

2. A multi-cell cross section of nine equal square cells is subject to a torque T. The length of each cell wall is *a* and the wall thickness *t*. Calculate the shear flow in each cell. Assume since this is a closed cell, the problem is St. Venant's torsion.