2.00AJ / 16.00AJ Exploring Sea, Space, & Earth: Fundamentals of Engineering Design
Spring 2009

# Gears & Linkages

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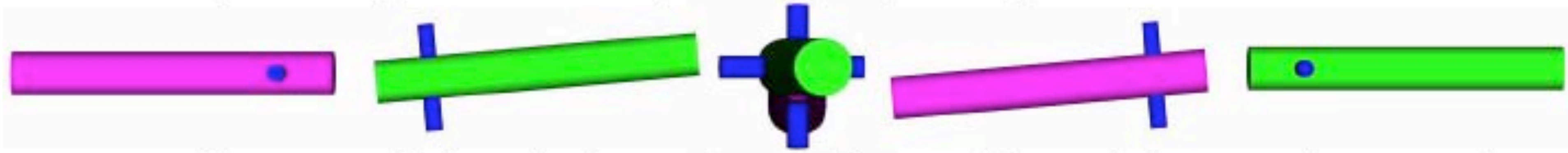
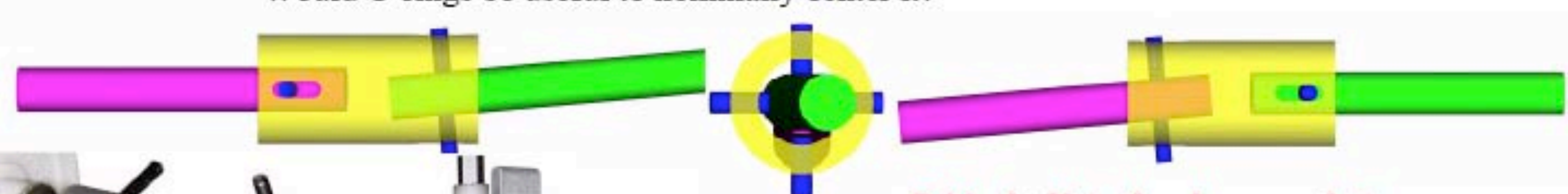
2.00A

# Couplings: *Cheap & Easy Example*

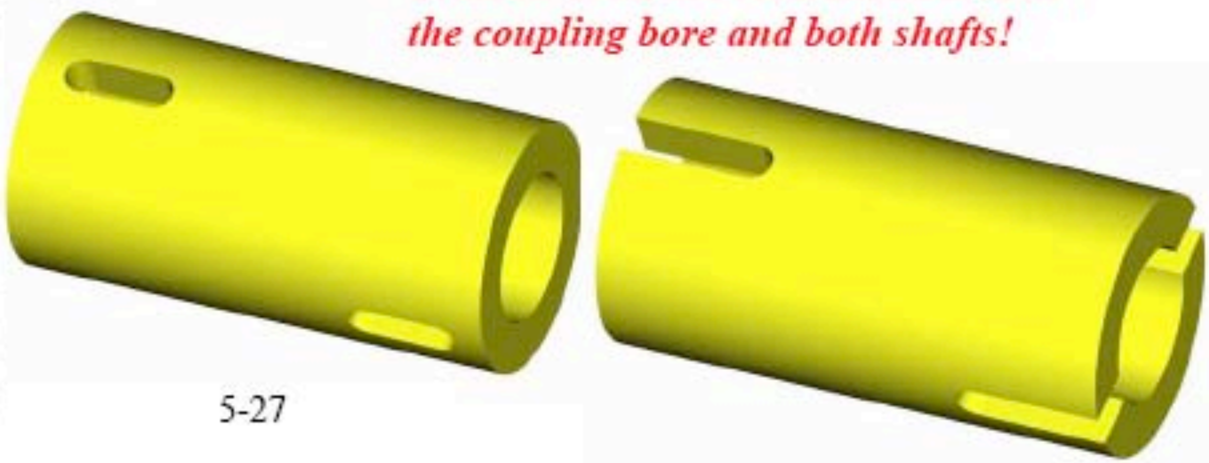
- What about in a robot design contest where two shafts may be linearly misaligned axially, vertically and horizontally, and angularly misaligned?



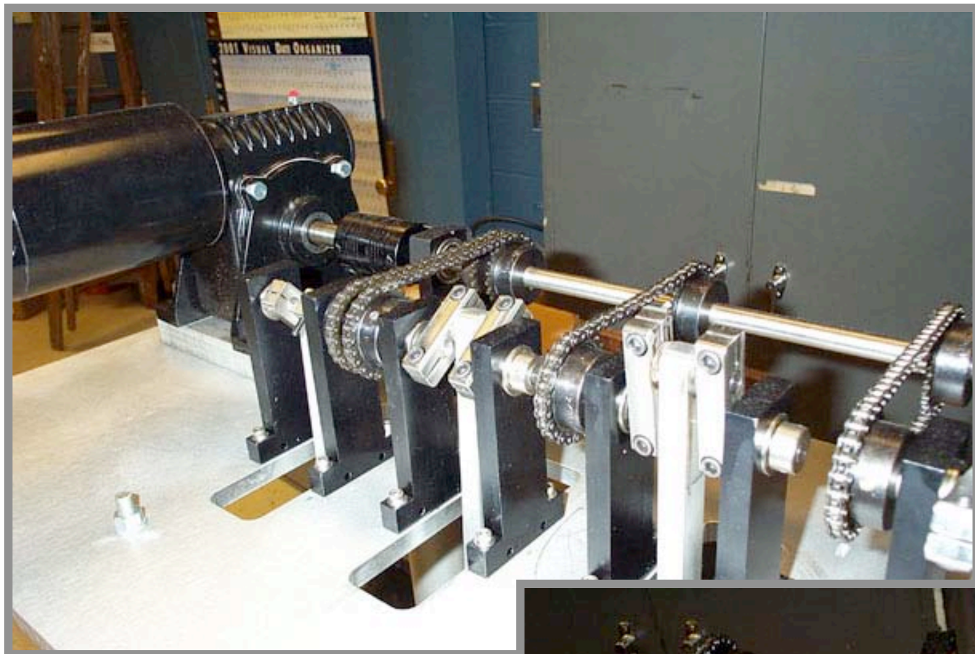
  - How can you design a simple one-piece coupling to enable one shaft to transmit torque to the other shaft?
    - Can the coupling be made from plastic tube to reduce shock loads?
    - Would O-rings be useful to nominally center it?

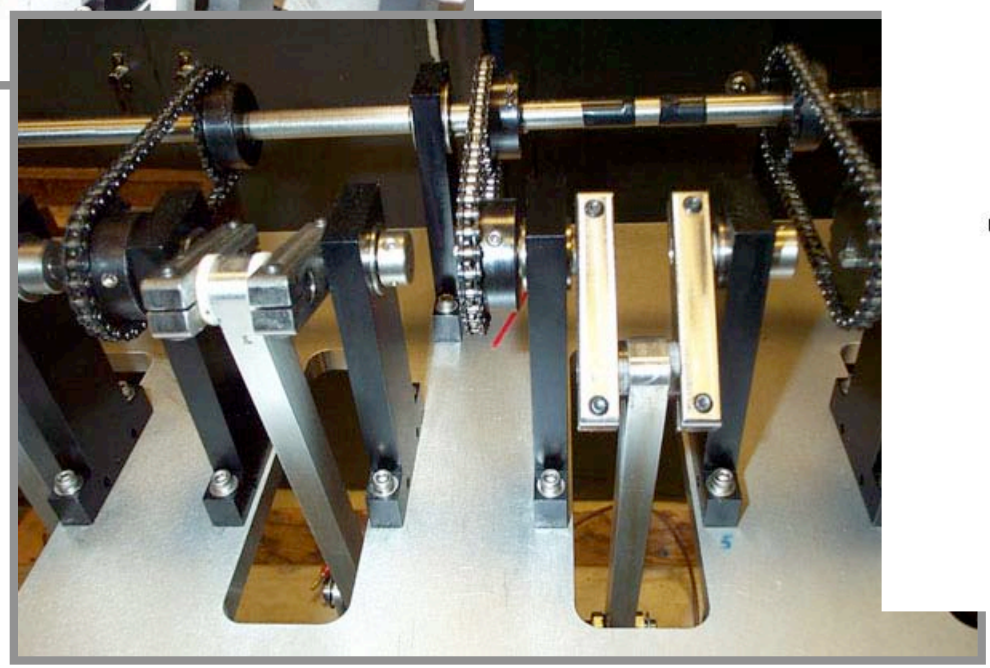


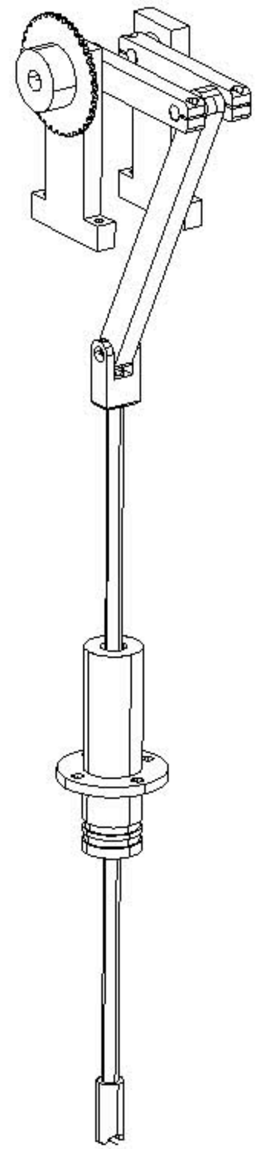
***Critical: Note the clearance between the coupling bore and both shafts!***



# Linkages



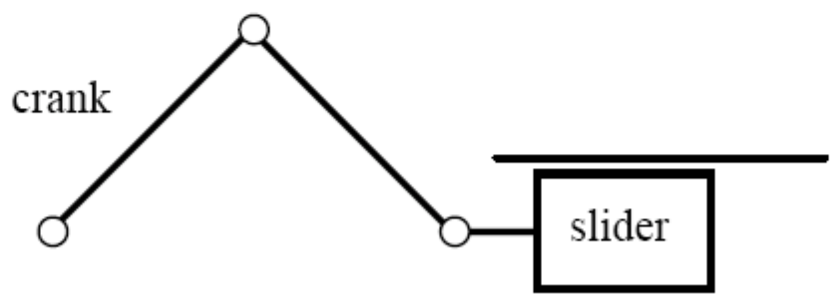




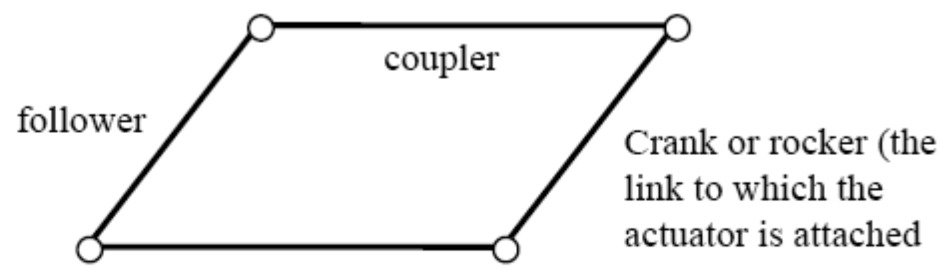
- *Linkage:* A system of *links* connected at *joints* with rotary or linear bearings
  - *Joint (kinematic pairs):* Connection between two or more links at their nodes, which allows motion to occur between the links
  - *Link:* A rigid body that possess at least 2 nodes, which are the attachment points to other links
- *Degrees of Freedom (DOF):*
  - The number of input motions that must be provided in order to provide the desired output, OR
  - The number of independent coordinates required to define the position & orientation of an object
  - For a *planar* mechanism, the degree of freedom (mobility) is given by *Gruebler's Equation*:

$$F = 3(n-1) - 2f_1$$

  - $n$ = Total number of links (including a fixed or single ground link)
  - $f_1$ = Total number of joints (some joints count as $f$ = ½, 1, 2, or 3)
    - Example: Slider-crank $n = 4, f_1 = 4, F = 1$
    - Example: 4-Bar linkage $n = 4, f_1 = 4, F = 1$
    - The simplest linkage with at least one degree of freedom (motion) is thus a 4-bar linkage!
    - A 3-bar linkage will be rigid, stable, not moving unless you bend it, break it, or throw it!




4 links (including ground), 4 joints




4 links, 4 joints

- Binary Link: Two nodes:

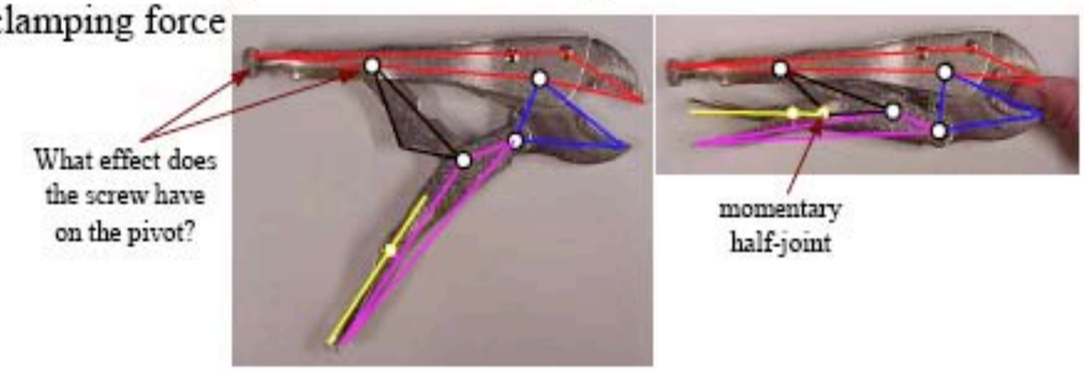
- Ternary Link: Three nodes:

- Quaternary Link: Four nodes:

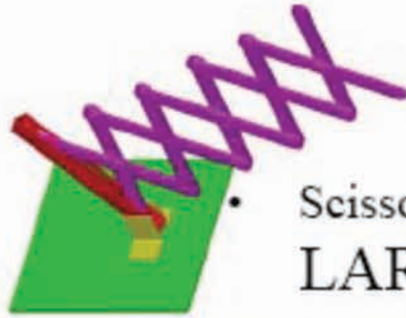
- Pentanary Link: Five nodes!
  *(Can you find it?!)*

# 5-Bar Linkages

- Compare a simple 4-bar linkage for pliers or small bolt cutters to a 5-bar linkage (5 bars, 5 joints, *2 DOF*) for bolt cutters
  - Where are the 2 degrees of freedom?
  - The FRs of the pliers are for wide range of motion and modest clamping force

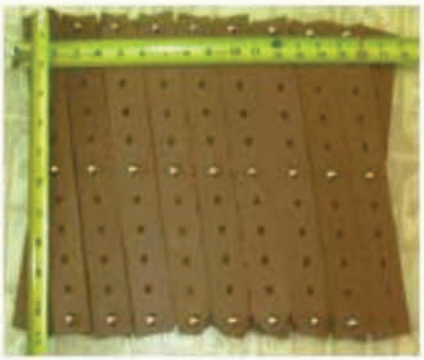



  - The FRs of the bolt cutters are for modest motion with extreme force
    - A 5-bar linkage can also act like a toggle mechanism



# Extending Linkages: *Scissor Linkages*

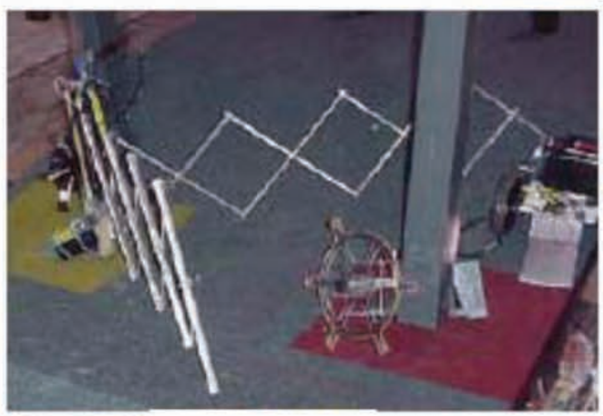
- Scissor Linkages (*Lazy Tongs*) are a great way to get a LARGE range of motion in a small package



- How does one develop a system as simple in principle, but as complex in detail as the Lazy Tongs?
  - The devil is in the details

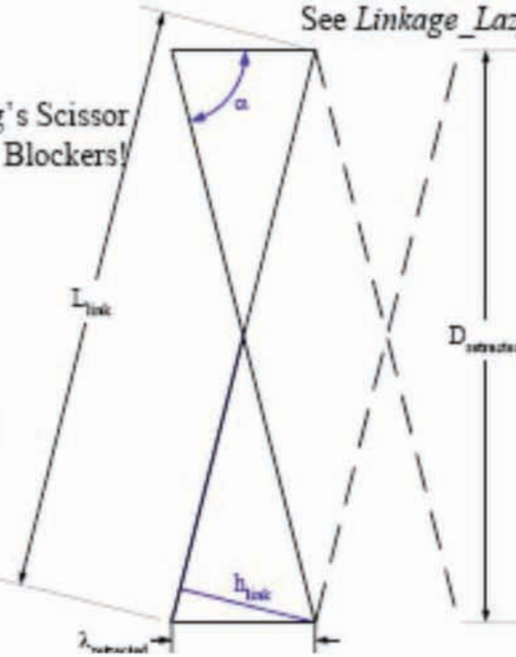
Bryan Ruddy's dovetail bearings to guide his scissors

Tolerances lead to scissor wobble...



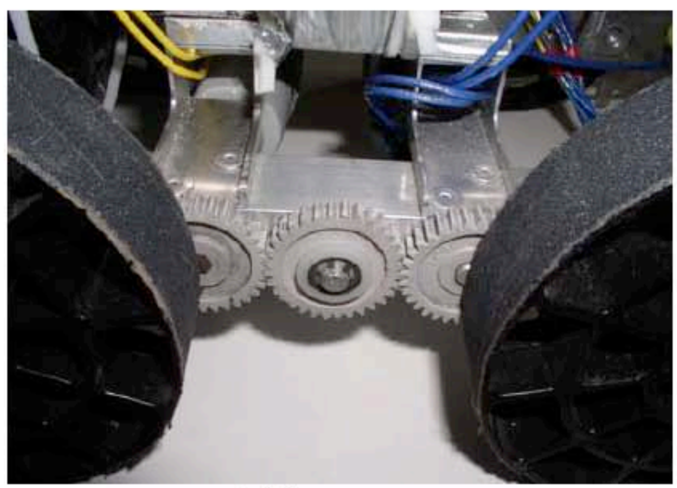
Eric Tung's Scissor Blockers!

See *Linkage_Lazy_Tongs.xls*

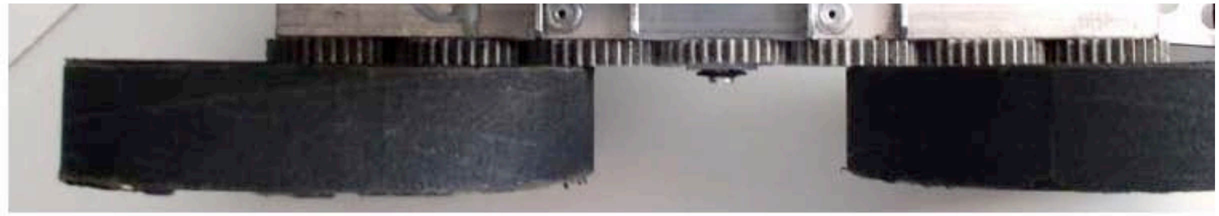


# Gears!

- Gears are most often used in transmissions to convert an electric motor's high speed and low torque to a shaft's requirements for low speed high torque:
  - *Speed* is easy to generate, because voltage is easy to generate
  - *Torque* is difficult to generate because it requires large amounts of current
- Gears essentially allow positive engagement between teeth so high forces can be transmitted while still undergoing essentially rolling contact
  - Gears do not depend on friction and do best when friction is minimized
- Basic Law of Gearing:
  - *A common normal (the line of action) to the tooth profiles at their point of contact must, in all positions the contacting teeth, pass through a fixed point on the line-of-centers called the pitch point*
  - Any two curves or profiles engaging each other and satisfying the law of gearing are *conjugate curves, the relative rotation speed of the gears will be constant*



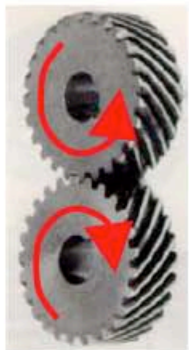
# Gears: *Gear Trains*



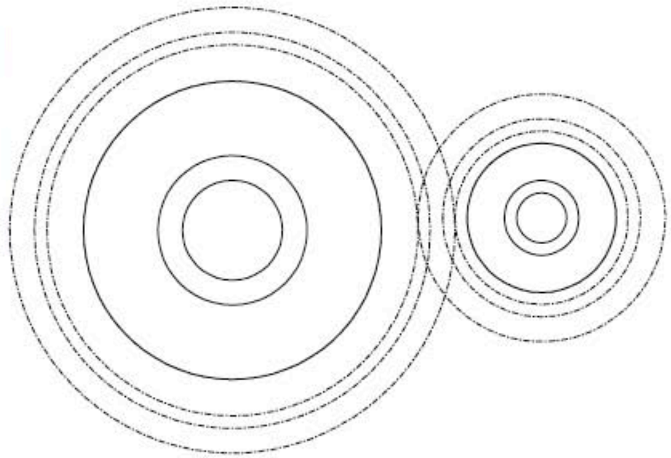
- A simple *gear train* to reduce motor speed and increase output torque:

The nice solid models are by Prof. Martin Culpepper, see http://psdam.mit.edu/2.000/start.html

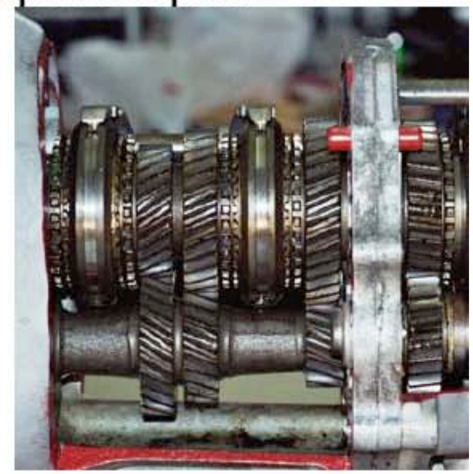
Worm Gears



Helical Gears



Spur Gears

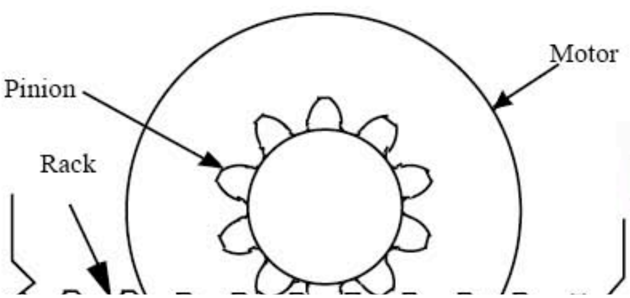


  - *Pinion:* smaller of two gears (typically on the motor) drives a gear on the output shaft
  - *Gear or Wheel:* Larger of the two gears
- Gears are highly efficient (90-95%) due to primarily rolling contact between the teeth; thus by conservation of energy:

$$\eta\Gamma_{motor\ torque}D_{input\ gear\ diameter} = \Gamma_{output\ torque}D_{output\ gear\ diameter}$$

$$\omega_{output} = \frac{\omega_{input}d_{input}}{d_{output}}$$

$$\Gamma_{output} = \frac{\eta\Gamma_{input}d_{output}}{d_{input}}$$




# Types of Gears

Image removed due to copyright restrictions. Please see http://www.weirdrichard.com/images/gearsbk.jpg

## Gears are generally used for one of four different reasons:

- •To reverse the direction of rotation
- •To increase or decrease the speed of rotation
- •To move rotational motion to a different axis
- •To keep the rotation of two axes synchronized

Image removed due to copyright restrictions.
Please see http://static.howstuffworks.com/gif/gearside1.gif

Most gears that you see in real life have teeth. The teeth have three advantages:

1. They prevent slippage between the gears. Therefore, axles connected by gears are always synchronized exactly with one another.
2. They make it possible to determine exact gear ratios. You just count the number of teeth in the two gears and divide. So if one gear has 60 teeth and another has 20, the gear ratio when these two gears are connected together is 3:1.
3. They make it so that slight imperfections in the actual diameter and circumference of two gears don't matter. The gear ratio is controlled by the number of teeth even if the diameters are a bit off.s are a bit off.

## Bevel Gears

Bevel gears have **teeth** cut on a cone instead of a cylinder blank. They are used in pairs to transmit **rotary motion** and **torque** where the bevel gear shafts are at **right angles** (90 degrees) to each other.



## Rack and Pinion

A **rack** and **pinion** mechanism is used to transform **rotary motion** into **linear motion** and vice versa. A round spur gear, the pinion, meshes with a spur gear which has teeth set in a straight line, the rack.

Image removed due to copyright restrictions.
Please see http://www.fi.edu/time/Journey/Time/Escapements/wormgear.gif

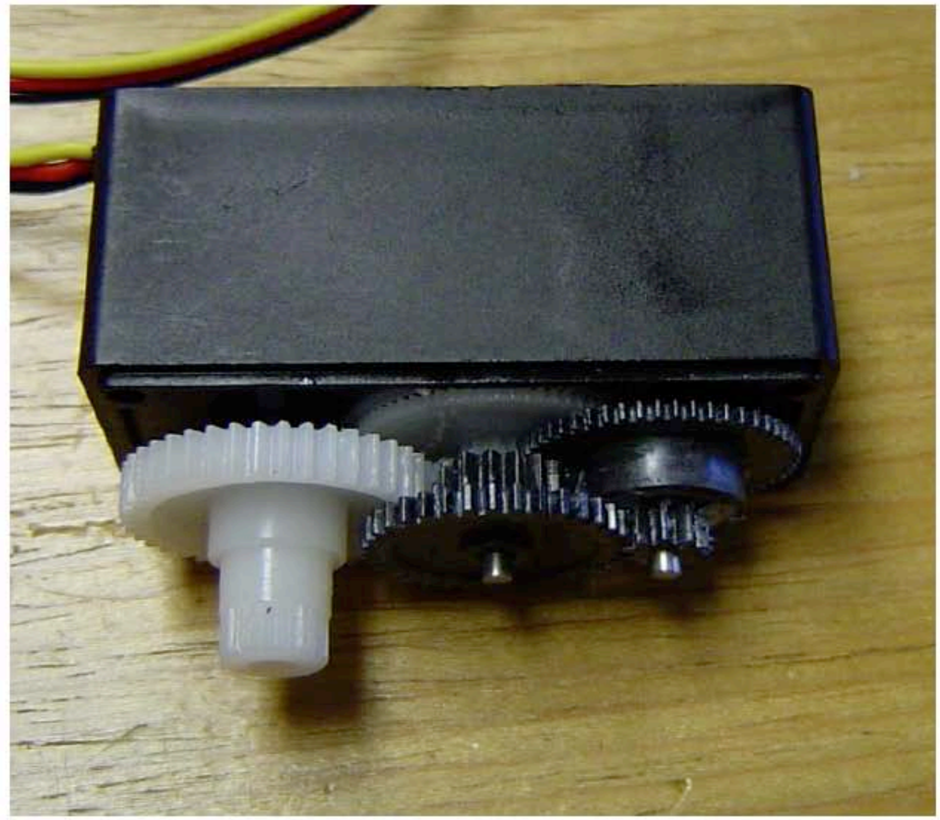
## Worm and Wormwheel

A gear which has one tooth is called a **worm**. The tooth is in the form of a **screw** thread. A wormwheel **meshes** with the worm. The wormwheel is a **helical gear** with teeth **inclined** so that they can engage with the thread-like worm. The wormwheel transmits **torque** and **rotary** motion through a **right angle**. The worm always drives the wormwheel and never the other way round. Worm mechanisms are very quiet running.


Image from Wikimedia Commons, http://commons.wikimedia.org

# Spur Gears

- Transmit motion between parallel shafts
- Teeth are parallel to the axis of rotation
- This is the simplest kind of gear we'll consider and most of today is dedicated to them



Courtesy of Dan Frey. Used with permission.

# Spur Gear

When two spur gears of different sizes mesh together, the larger gear is called a wheel, and the smaller gear is called a pinion. In a simple gear train of two spur gears, the input motion and force are applied to the driver gear. The output motion and force are transmitted by the driven gear. The driver gear rotates the driven gear without slipping

Gear Ratio: $$gr = \frac{\pi d}{\pi D} = \frac{d}{D} = \frac{r}{R}$$

$$v_d = v_D \longrightarrow \omega_d r = \omega_D R \longrightarrow \frac{r}{R} = \frac{\omega_D}{\omega_d}$$

$$\boxed{gr = \frac{\omega_D}{\omega_d}}$$

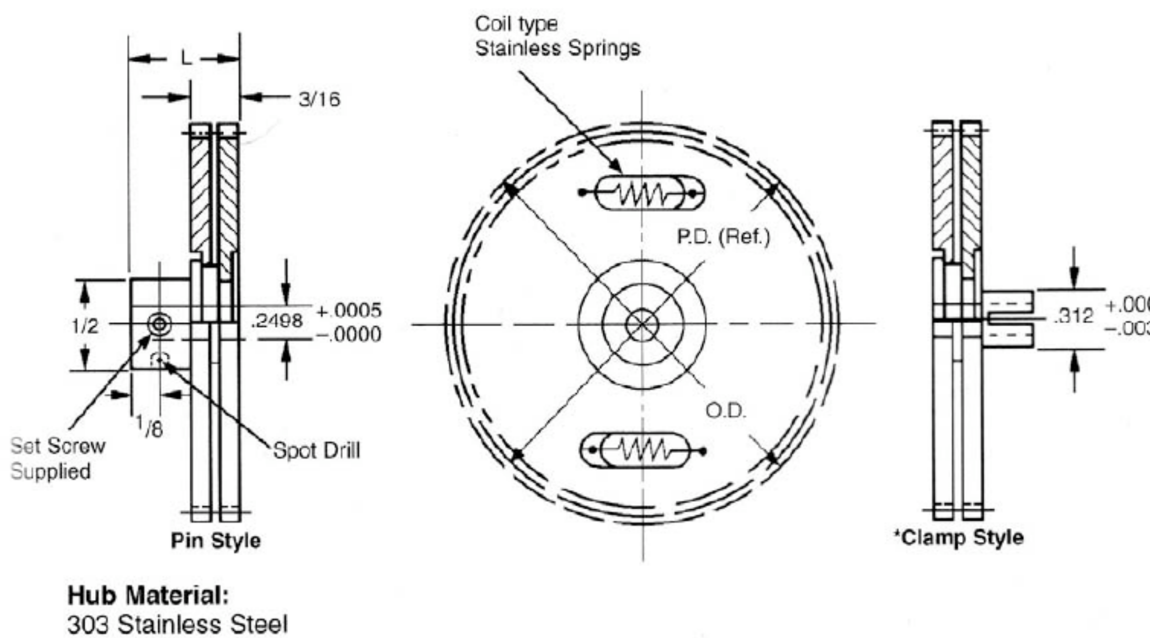
Images removed due to copyright restrictions.
Please see: http://www.fi.edu/time/Journey/Time/Escapements/spurgear.gif
http://commons.wikimedia.org/wiki/File:Gears_large.jpg

*The gear ratio is proportional to ratio of the gear diameters and inversely proportional to the ratio of gear speeds.*

http://www.osha.gov/SLTC/etools/machineguarding/animations/gears.html

# Backlash

- If you want any tolerance for
  - Center distance errors
  - Thermal growth
- There will be backlash when gears reverse




An “anti-backlash” gear

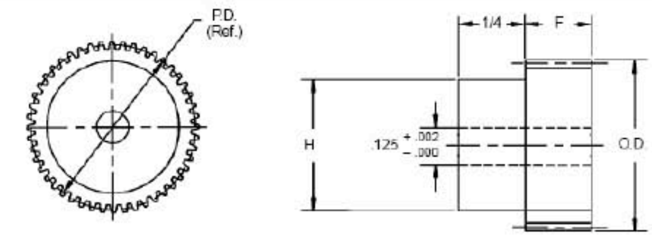
Courtesy of W. M. Berg, Inc. Used with permission.

# Gear Selection

- Pitch
- Face width
- Material
- Pressure angle
- # of teeth
- Hub style, bore, etc.

## Spur Gears

**24, 32, 48, and 64 Pitch 1/8" Bore AGMA Quality 4**
**Cold Rolled Steel and Brass 20° Pressure Angle**

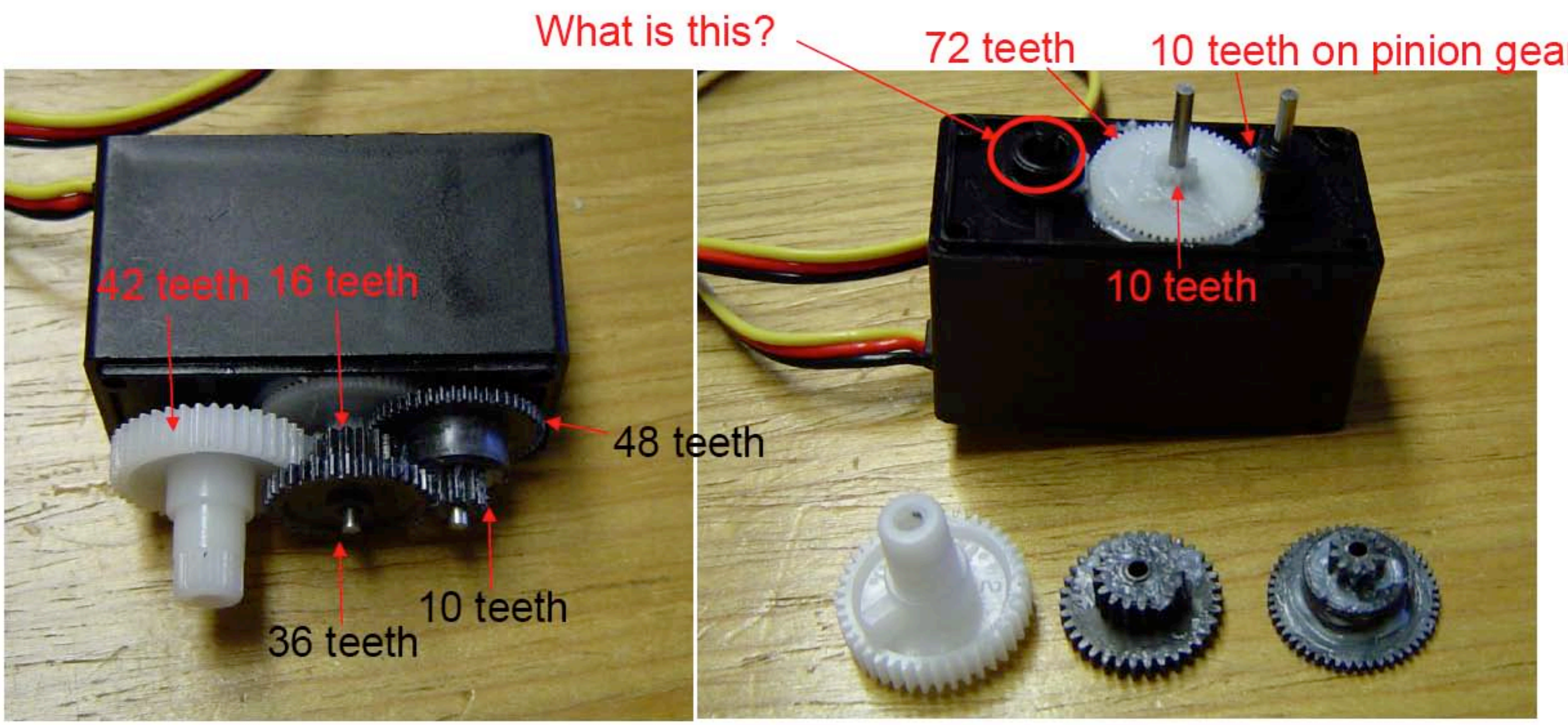



B
9

| COLD ROLLED STEEL C12L14 OR C12L15 WITH SELENIUM | BRASS ALLOY 360 | | | | | |
|---|---|---|---|---|---|---|
| STOCK NUMBER | STOCK NUMBER | NO OF. TEETH | PITCH DIA. | OUTSIDE DIA. | H | F |
| 24 PITCH (.1309) | | | | | | |
| PX24S-8 | PX24B-8 | 8 | .333 | .416 | .208 | |
| PX24S-9 | PX24B-9 | 9 | .375 | .458 | .250 | |
| PX24S-10 | PX24B-10 | 10 | .417 | .500 | .291 | |
| PX24S-12 | PX24B-12 | 12 | .500 | .583 | .375 | 1/4 |
| PX24S-16 | PX24B-16 | 16 | .666 | .750 | .542 | |
| PX24S-18 | PX24B-18 | 18 | .750 | .833 | .625 | |
| — | PX24B-22 | 22 | .916 | 1.000 | .792 | |
| 32 PITCH (.0981) | | | | | | |
| PX32S-10 | PX32B-10 | 10 | .312 | .375 | .218 | |
| PX32S-11 | PX32B-11 | 11 | .344 | .406 | .250 | |
| PX32S-12 | PX32B-12 | 12 | .375 | .437 | .281 | |
| PX32S-14 | PX32B-14 | 14 | .438 | .500 | .343 | |
| PX32S-15 | PX32B-15 | 15 | .469 | .531 | .375 | 1/4 |
| PX32S-16 | PX32B-16 | 16 | .500 | .562 | .406 | |
| PX32S-18 | PX32B-18 | 18 | .563 | .625 | .468 | |
| PX32S-20 | PX32B-20 | 20 | .625 | .688 | .532 | |
| | PX32B-24 | 24 | .750 | .813 | .656 | |
| 48 PITCH (.0654) | | | | | | |
| PX48S-14 | PX48B-14 | 14 | .292 | .333 | .229 | |
| PX48S-15 | PX48B-15 | 15 | .312 | .353 | .250 | |
| PX48S-16 | PX48B-16 | 16 | .333 | .375 | .271 | |
| PX48S-18 | PX48B-18 | 18 | .375 | .417 | .312 | 1/8 |
| PX48S-24 | PX48B-24 | 24 | .500 | .542 | .437 | |
| PX48S-32 | PX48B-32 | 32 | .666 | .708 | .604 | |
| | PX48B-36 | 36 | .750 | .792 | .687 | |
| | PX48B-40 | 40 | .833 | .875 | .770 | |
| 64 PITCH (.0490) | | | | | | |
| PX64S-15 | PX64B-15 | 15 | .234 | .265 | .187 | |
| PX64S-16 | PX64B-16 | 16 | .250 | .281 | .203 | |
| PX64S-18 | PX64B-18 | 18 | .281 | .312 | .234 | 1/8 |
| — | PX64B-24 | 24 | .375 | .406 | .328 | |
| — | PX64B-40 | 40 | .625 | .656 | .578 | |
| — | PX64B-48 | 48 | .750 | .781 | .703 | |

Berg Manufacturing "The Mark of Quality" **1-800-232-BERG**

- Given the top output shaft speed, what is the motor shaft speed (in rpm)?



http://auto.howstuffworks.com/wiper1.htm