2.007 Design and Manufacturing I
Spring 2009

Homework #2 Servomotors, Mechanisms & CAD

SOLUTION

Date Issued: Tuesday 24 FEB, 11AM
Date Due: Thursday 5 MAR, 11AM

1) (25 total) You're building a small, electric-powered vehicle to climb up a 20 degree ramp (similar slope as the ramp into the contest starting box). The vehicle will be operated by two servomotors that can continuously rotate the output shafts that attach directly to wheels on the left and right sides of the vehicle. The manufacturer's specifications indicate each servo is capable of a no load speed of 50 rpm and a stall torque of 0.3 N*m (similar to the specs of the HS-311 standard servos in the kit). The total vehicle weight along with its payload is 12 N (a little more than the weight of a liter of water). All the wheels have a radius of 6cm. The front and rear axles are 20cm apart. The center of gravity of the vehicle and payload is halfway between the front and rear wheels and is 3cm above the surface on which the vehicle runs.

    A) (5 points) Make a <u>free body diagram</u> of the vehicle driving up a 20 degree ramp after having reached its steady-state velocity. Label each force. Write three equations of equilibrium.
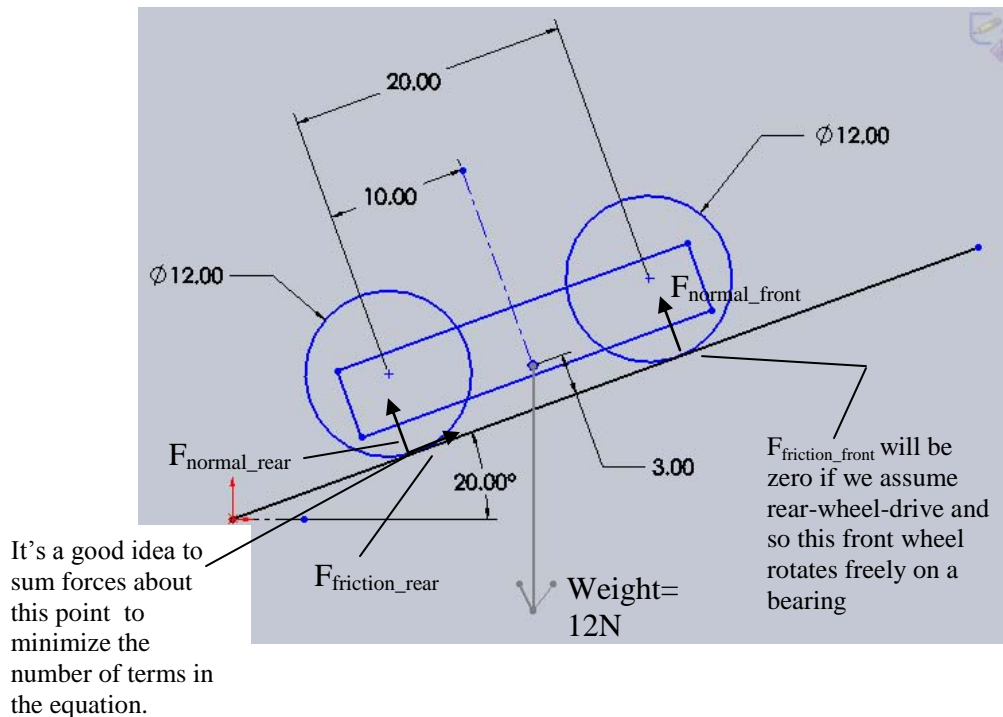
Three Equations of Equilibrium (assuming x is chosen along the direction parallel to the table)

Sum forces in the x direction $\quad F_{friction\_rear} - Weight \cdot \sin(20 \cdot degrees) = 0$

Sum forces in the y direction $\quad F_{normal\_rear} + F_{normal\_front} - Weight \cdot \cos(20 \cdot degrees) = 0$

Sum moments about rear contact point $\quad 20 \cdot cm \cdot F_{normal\_front} - (10 \cdot cm - 3 \cdot cm \sin(20 \cdot degrees)) \cdot Weight = 0$

B) (5 points) Estimate the <u>steady-state velocity</u> of the vehicle. Sketch the torque-speed curve and power-speed curve of the servos and indicate on these curves the shaft speed attained when climbing steadily up the 20 degree ramp.

$\text{Ramp\_ang} := 20\,\text{deg}$ $\text{Stall\_torque} := 0.3\,\text{N·m}$

$\text{No\_load\_speed} := 50\,\dfrac{\text{rev}}{\text{min}}$ $\text{Weight} := 12\,\text{N}$ **These are some given values from the problem statement**

$R_{wheel} := 6\,\text{cm}$

$\tau(\omega) := \text{Stall\_torque} - \omega\cdot\dfrac{\text{Stall\_torque}}{\text{No\_load\_speed}}$ **This is just a linear function that fits the two points we know from the servo specifications.**

$P_{servo}(\omega) := \tau(\omega)\cdot\omega$ **This is straight from the defition of power.**

$\omega := 0, 1\cdot\dfrac{\text{rev}}{\text{min}} .. \text{No\_load\_speed}$ **This defines a range over which to plot the curves.**

This torque value required from the servos comes straight from summing forces along the direction parallel to the ramp.

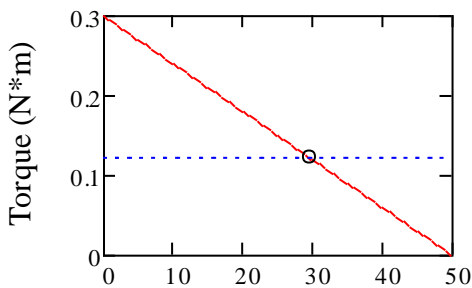**This speed of the car would result if there is no slip between the wheel and the ramp.**
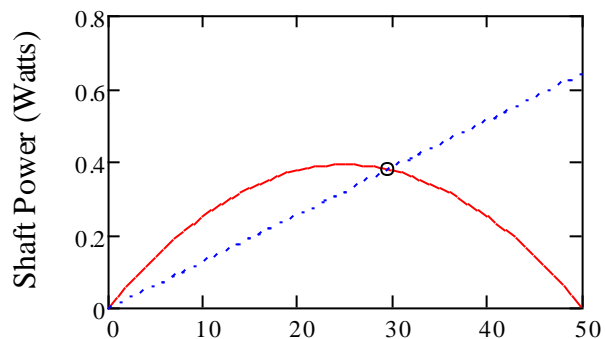
$$\text{speed}_{car}(\omega) := 6\cdot\text{cm}\cdot\omega$$

Power is speed of the car times the component of force parallel to the velocity vector.

$\tau_{req}(\text{Weight}) := \dfrac{\text{Weight}\cdot R_{wheel}\cdot\sin(\text{Ramp\_ang})}{2}$

$$P_{car}(\omega, \text{Weight}) := \text{speed}_{car}(\omega)\cdot\text{Weight}\cdot\sin(\text{Ramp\_ang})$$

If the weight is 12 N, then here is the result.

Torque (N*m) vs Shaft Speed (rpm)

— Servo performance
···· Requirement of car
⊖⊖⊖ Operating point

Shaft Power (Watts) vs Shaft Speed (rpm)

— Servo performance
···· Requirement of car
⊖⊖⊖ Operating point

$\omega_{guess} := 0\cdot\dfrac{\text{rad}}{\text{sec}}$

$\omega_{ss} := \text{root}\left[\left(\tau_{req}(12\,\text{N}) - \tau\left(\omega_{guess}\right)\right), \omega_{guess}\right] = 3.087\dfrac{1}{\text{s}}$

$\omega_{ss} = 29.479\dfrac{\text{rev}}{\text{min}}$

$V_{ss} := R_{wheel}\cdot\omega_{ss}$ $\boxed{V_{ss} = 0.185\dfrac{\text{m}}{\text{s}}}$

If the weight is 6 N, then there is a different result with a slightly higher speed.
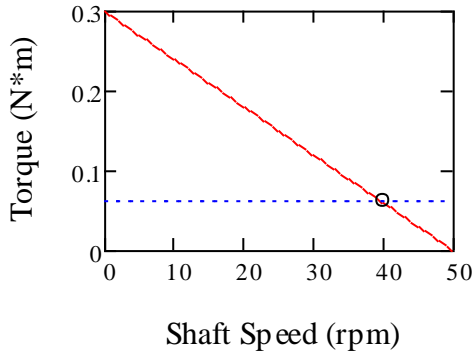
Torque (N*m) vs Shaft Speed (rpm)
— Servo performance
···· Requirement of car
⊖⊕⊖ Operating point

Shaft Power (Watts) vs Shaft Speed (rpm)
— Servo performance
···· Requirement of car
⊖⊕⊖ Operating point
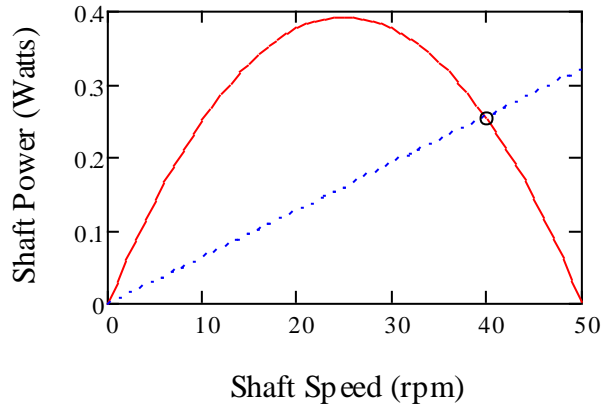
$$\omega_{guess} := 0 \cdot \frac{rad}{sec}$$

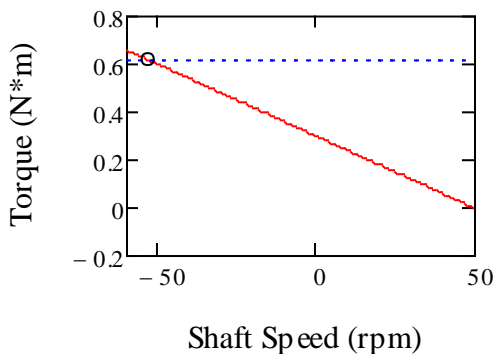$$\omega_{ss} := root\left[\left(\tau_{req}(6 \cdot N) - \tau\left(\omega_{guess}\right)\right), \omega_{guess}\right] = 4.161\frac{1}{s}$$

$$\omega_{ss} = 39.739\frac{rev}{min}$$

$$V_{ss} := R_{wheel} \cdot \omega_{ss}$$

$$V_{ss} = 0.25\frac{m}{s}$$

If the weight is 60 N, then the weight actually drives the vehicle down the ramp with the wheels spinning backwards and mechanical power of the servomotor being negative.

$$\omega := -60 \cdot \frac{rev}{min}, -59 \cdot \frac{rev}{min} .. No\_load\_speed$$

Torque (N*m) vs Shaft Speed (rpm)
— Servo performance
···· Requirement of car
⊖⊕⊖ Operating point

Shaft Power (Watts) vs Shaft Speed (rpm)
— Servo performance
···· Requirement of car
⊖⊕⊖ Operating point
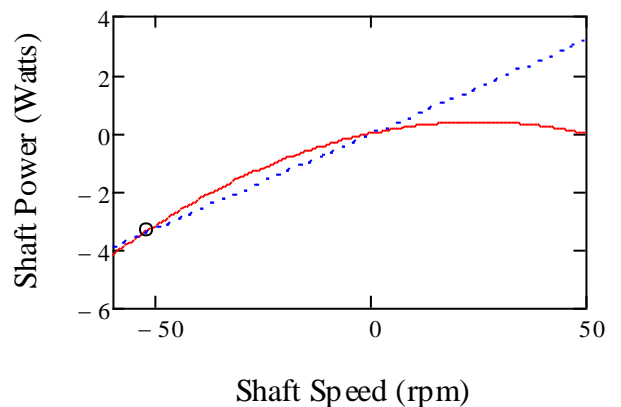
$$\omega_{guess} := 0 \cdot \frac{rad}{sec}$$

$$\omega_{ss} := root\left[\left(\tau_{req}(60 \cdot N) - \tau\left(\omega_{guess}\right)\right), \omega_{guess}\right] = -5.509\frac{1}{s}$$

$$\omega_{ss} = -52.606\frac{rev}{min}$$

$$V_{ss} := R_{wheel} \cdot \omega_{ss}$$

$$V_{ss} = -0.331\frac{m}{s}$$

C) (5 points) Estimate the <u>smallest coefficient of friction</u> needed between the wheels and the ramp for which the vehicle will avoid slipping between the wheels and the ramp when climbing steadily up the 20 degree ramp.

Normal force on the front wheel (from sum of moments)

$$F_{normal\_front} := \frac{(10 \cdot cm - 3 \cdot cm \sin(20 \cdot deg)) \cdot Weight}{20 \cdot cm} = 2.692N$$

Normal force on the rear wheel (from sum of forces in the y direction)

$$F_{normal\_rear} := -F_{normal\_front} + Weight \cdot \cos(20 \cdot deg) = 2.946N$$

Required frictional force on the rear wheel (from sum of forces in the x direction)

$$F_{friction\_rear} := Weight \cdot \sin(20 \cdot deg) = 2.052N$$

$$\mu := \frac{F_{friction\_rear}}{F_{normal\_rear}} \qquad \mu = 0.697$$

Required frictional coefficient if we used front wheel drive

$$\mu := \frac{F_{friction\_rear}}{F_{normal\_front}} \qquad \mu = 0.762$$

Rear wheel drive is somewhat better for climbing hills without slipping.

NOTE: This answer is not affected by the weight of the vehicle that you assumed. In a Coulombic model of friction, the frictional force is proportional to the normal force, so the ratios are not a function of weight. Reality is a little different.

D) (5 points) Is the smallest coefficient of friction attained (from part C) in a front wheel drive configuration or a rear wheel drive configuration? Explain with a few sentences and/or sketches.

As seen in the calculations for part C. The required friction coefficient is lower if we assume rear wheel drive rather than front wheel drive. This is due to the shifting of weight from the frnt wheels to the rear wheels with increasing slope of the hill. This effect is mild in this vehicle because the center of gravity is so close to the surface. The effect would be more pronounced if the c.g. were higher. The effect will be extreme in any case when the vehicle is on the verge of tipping aftward and therefore all the weight is on the rear wheels.
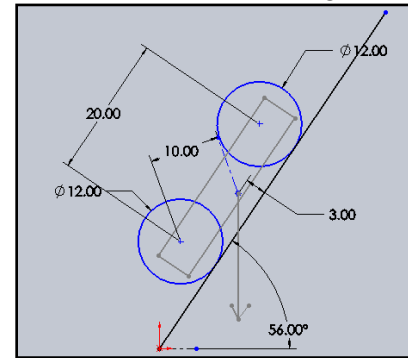
E) (5 points) Estimate the <u>largest angle</u> at which the vehicle will be able to proceed <u>up the ramp</u> very slowly, but without stalling. Check that the center of gravity is placed so that tipping does not occur before stall.

$\text{Stall\_torque} := 0.3 \, N \cdot m$

$$\text{max\_angle} := a sin\left(\frac{2 \cdot Stall\_torque}{12 \cdot N \cdot R_{wheel}}\right) = 56.443 deg$$

This answer is for an assumed weight of 12N.

If you assumed a weight of 60N, you'd find:

$$\text{max\_angle} := a sin\left(\frac{2 \cdot Stall\_torque}{60 \cdot N \cdot R_{wheel}}\right) = 9.594 deg$$

If you assumed a weight of 6N, you'd find that even at a 90 degree incline, the torque demanded of the servos is less than the stall torque. The vehicle could climb straight up as long as the wheels could somehow have adequate traction. This could be done by having the wheels ride up on a length of string that wraps around the wheels and is laid out along the path ahead.

2) (25 total) Consider the window glass mechanism shown in the two figures below.   The figure on the left shows shows the mechanism's pin locations and its disassembled links, and can be found on the course website in a file named "WindowMechanism". The figure on the right shows the assembled components (you'll have to make this yourself).

A) (10 points) Make a 2D sketch in SolidWorks of the window glass mechanism, i.e. assemble the links. Once you have a sketch that moves properly, you will find it useful to apply a virtual work principle to attack part B.

The assembled mechanism is shown below with some notes:
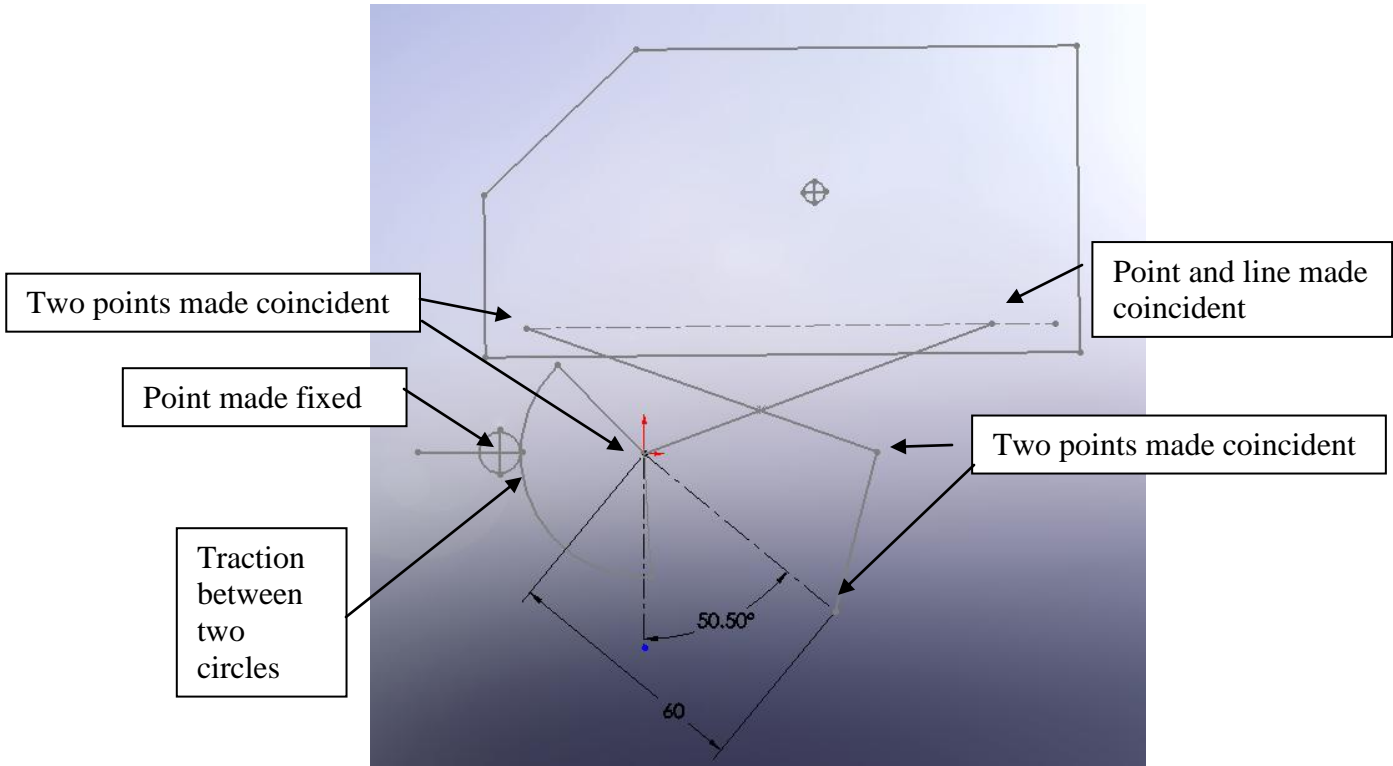
B) (15 points) Estimate the magnitide of a force applied perpenducular to the handle (labeled below) that would be just sufficient to keep the window glass from falling. You can assume: 1) the mechanism is in the position shown in the figure below; 2) there is negligable friction in the joints; and 3) the window glass has a mass of 1kg and that the mass of the links and gears is small by comparison.

Initial Position with Reference Measurements:
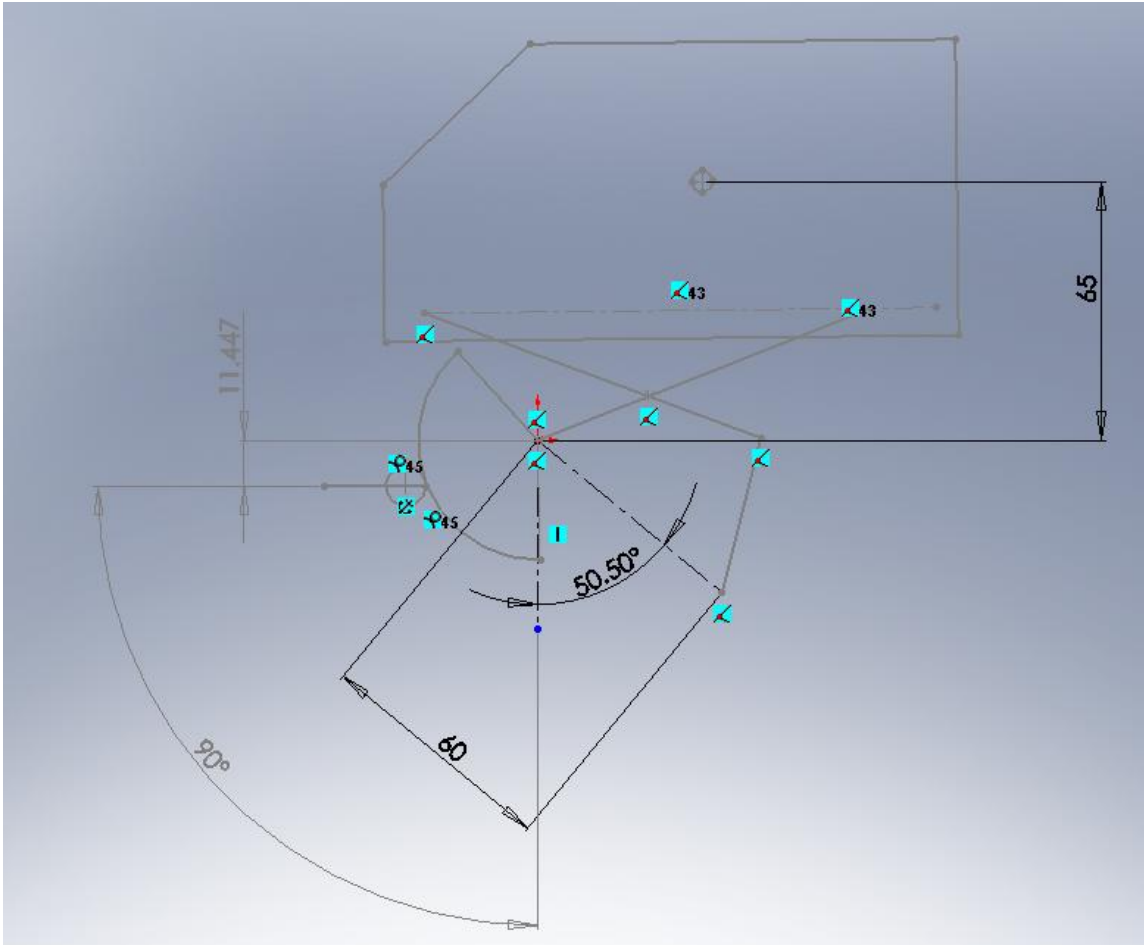
Note: Without loss of generality, the handle has been chosen to be in the horizontal position at the starting configuration to facilitate small displacement measurements.

Final Position:
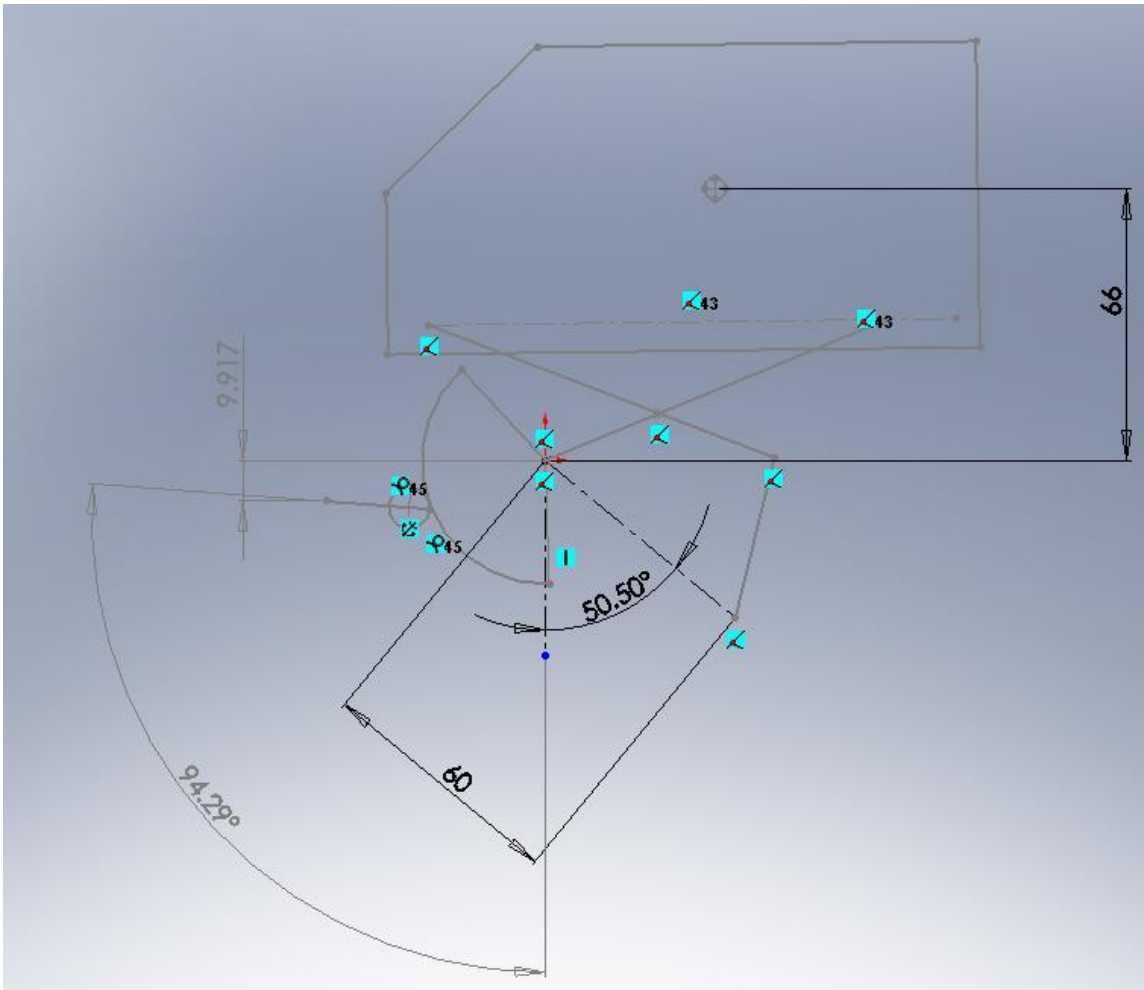
Using Small Angular Displacement:
(Note: measured handle length from center is 20.4mm)

$$F_y \Delta y = \tau_{handle} \Delta\theta_{handle} \Rightarrow (9.8N)(0.001m) = \tau_{handle}\left(\pi \frac{4.29}{180}\right) \Rightarrow \tau_{handle} = 0.131Nm$$

$$F_{handle} = \frac{\tau_{handle}}{0.0204m} = 6.42N$$

Using Small Linear (Tangential) Displacement:

$$F_y \Delta y = F_{handle} \Delta x_{handle} \Rightarrow (9.8N)(0.001m) = F_{handle}(0.011447m - 0.0009917m)$$
$$F_{handle} = 6.41N$$