**2.004 MODELING DYNAMICS AND CONTROL II** Spring 2003

**Problem Set No. 3**

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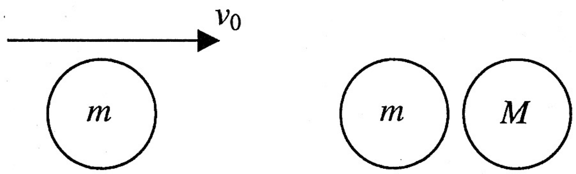
## Problem 1

The two masses on the right of the sketch below are slightly separated and at rest initially; the left mass is incident with speed $v_0$. Assuming head-on perfectly elastic collisions, determine the number of collisions that take place and find all final velocities.

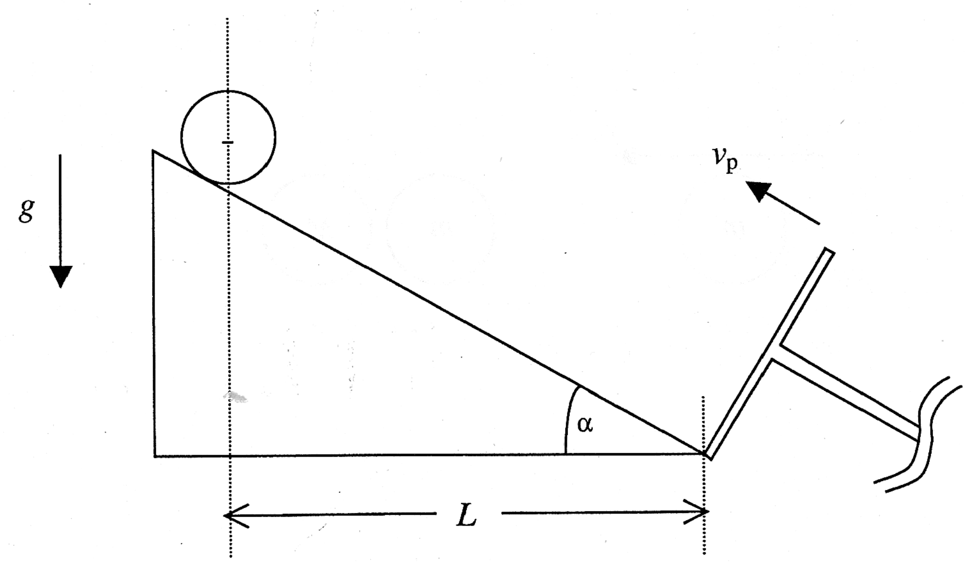



## Problem 2

A ball of negligible radius is released with zero initial velocity on a slope of angle $\alpha$, as shown below. At the same time, a piston starts moving up on the slope with constant velocity $v_p$. The initial horizontal distance between the ball and the piston is $L$, and the acceleration due to gravity is $g$. After its release, the ball slides down the slope without friction, and collides with the moving piston. The coefficient of restitution for this impact is $e$. Immeadiately after the impact, the piston is stopped.

(a) Find the time $t_1$ of the impact.

(b) Determine the maximal height $h_{max}$ that the ball will reach after the impact. ($h_{max}$ measured from the horizontal base of the slope).



**Problem 3**

Hammering a nail into a 2 × 4.

Consider the classical problem of hammering a nail into a piece of wood. A skilled carpenter swings the hammer in such a way that the hammer head (of mass $M$) strikes the nail (of mass $m$) centrally with a velocity $V_0$. As a result of the impact the nail, which had already penetrated to a depth $x_0$ begins to penetrate further into the wood with an initial velocity $v_1$. After an additional penetration $\delta$ the nail comes to rest with a final penetration of $x_0 + \delta$. Your assignment is to develop models for the relation between the additional penetration $\delta$ and the hammer velocity $V_0$. In particular, you are asked to develop models which use the data from a test in which

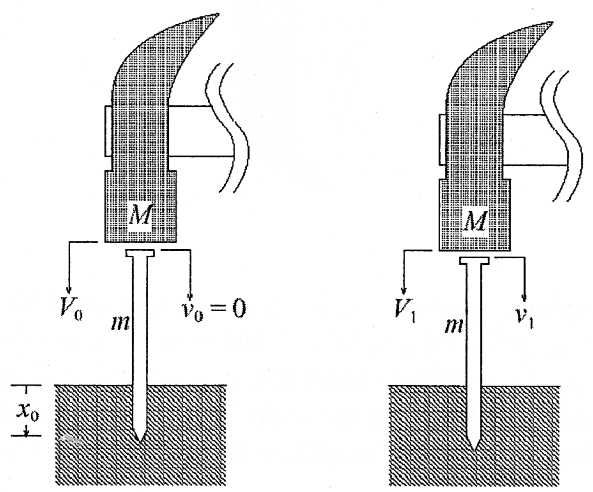
$M = 0.5$ kg $\qquad\qquad$ $m = 0.005$ kg

$x_0 = 10$ mm

$V_0 = 10$ m/s

$\delta = 2$ mm

to predict the magnitude of the additional penetration $\delta$ when, with the same hammer and the same nail and the same initial penetration $x_0$, the hammer velocity is reduced to $V_0 = 5$ m/s.

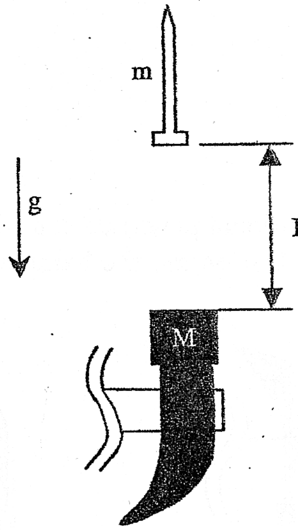



(a) $\qquad\qquad$ (b)

The total action involved in driving the nail can be divided into two stages: the *impact stage* in which there is a very large impulsive force between the hammer and the nail during the very short interval of contact between the hammer and the nail; and the *penetration stage* in which the initial velocity $v_1$ of the nail is arrested by the resisting forces in the wood. In the impact stage the effects of the ordinary forces such as gravity and the resisting forces in the wood are so small in comparison with the effects of the impulsive forces that they can be temporarily neglected. The situation just prior to the impact is shown in part (a) of the preceding figure. The hammer head has velocity $V_0$ and the nail is motionless ($v_0 = 0$). Immediately after the impact the nail has the velocity $v_1$ and the hammer head has the reduced velocity $V_1$ as shown in part (b) of the preceding figure. We assume, for simplicity, that the skilled carpenter controls the hammer so that there is no second strike during the penetration stage when the nail velocity is reduced.

(a) Establish an impact model with two isolated masses $M$ and $m$ and the initial conditions shown in part (a) of the preceding figure. Apply the conservation of linear momentum to obtain one equation relating the velocities $v_1$ and $V_1$ to the initial hammer head velocity $V_0$.

(b) To obtain a second equation we perform an experiment with the hammer and the nail in order to estimate the coefficient of restitution $e$.




We hold the hammer, upside down, in one hand and carefully drop the nail from a height $H$ and measure its rebound height $h$. This is fairly tricky as the nail must be dropped perfectly vertical. After several trials the average value of the ratio $h/H$ is found to be 0.1. Use this result to obtain an estimate of the coefficient of restitution $e$. Then derive a second equation relating the velocities $v_1$ and $V_1$ to the initial hammer head velocity $V_0$.

(c) Solve for the initial velocity $v_1$ of the nail.

In the penetration stage the nail of mass $m$ begins to further penetrate the wood with an initial velocity $v_1$. The forces acting on the nail are gravity and the reaction forces of the wood on the nail. Now before the impact, the forces on the nail are in equilibrium. The gravity force is balanced by an equal and opposite reaction from the wood. When the nail begins to move, an additional resistance to motion is exerted by the wood. To model this behavior, we assume that the wood continues to supply a force component to balance the gravity force and that an additional resistance $R$ is generated to oppose the velocity of the nail. You are asked to consider two different assumptions about the resistance force $R$.

(d) **Linear viscous drag.** The simplest model for resistance to velocity is the linear model used in Problem 1 of Problem Set No. 1

$$R = bv$$

This assumption is simple, but not very realistic for the nail penetration problem. It implies that a very small steady force on the nail would allow slow continued penetration. It also implies that the resistance is independent of the penetration depth $x$. Despite its shortcomings, use the linear assumption to derive an equation relating the additional penetration depth $\delta$ to the hammer-head velocity $V_0$.

(e) Use the formula just derived and the given data to evaluate the magnitude of the viscous drag coefficient $b$.

(f) **Penetration dependent Coulomb friction.** A more realistic model for the penetration resistance is a dry friction type behavior with the magnitude of the limiting static resistance force proportional to the depth of penetration $x$. In the standard model for Coulomb friction, a block is pressed against a stationary surface with a normal force $N$. Then a tangential force $F$ is applied to attempt to slide the block along the surface. In the Coulomb model, no motion occurs until $F = \mu_s N$, where $\mu_s$ is the coefficient of static friction which depends on the materials, the temperature, and the presence of any lubricants. Once the block begins to move, the kinetic friction force $F$ which acts to resist the sliding motion is usually equal or somewhat smaller than the limiting static friction force.

In the case of a nail penetrating a block of wood, the normal force is supplied by the radial pressure of the wood in contact with the penetrated segment of the nail. It is plausible to assume that the total effective normal force $N$ acting on the nail is proportional to the depth of penetration $x$; *i.e.*,

$$N = nx$$

where $n$ is a constant with the dimensions of force per unit length. Then the limiting static friction is

$$F = \mu_s nx$$

Once the nail begins to move you may assume that the kinetic resistance force has the form

$$R = \mu_k nx$$

where the kinetic friction coefficient $\mu_k$ is a constant only slightly smaller than the static coefficient of friction.

Formulate the equation of motion for the nail under these conditions. Integrate the equation between the limits of $x = x_0$ with $v = v_1$ and $x = x_0 + \delta$ with $v = 0$ to obtain a relation from which it is possible to determine the parameter $\mu_k n$ from the given data. Evaluate the parameter $\mu_k n$.

(g) Predict the magnitude of the additional penetration $\delta$ for the case where the hammer velocity $V_0 = 5$ m/s, but all other data remain unchanged, on the basis of the linear viscous drag model.

(h) Repeat (g) for the Coulomb friction model.