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2.004 Dynamics and Control II  
Spring 2008

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MASSACHUSETTS INSTITUTE OF TECHNOLOGY  
DEPARTMENT OF MECHANICAL ENGINEERING

2.004 *Dynamics and Control II*  
Spring 2008

Laboratory Project: Active Damping of Tall Building Vibrations<sup>1</sup>

Week 1: Modeling the System

Overview

The high-rise building is a modern miracle - miles of steel beams and welds, thousands of fasteners allowing graceful structures of one hundred or more stories in height. Like any high-aspect ratio structure, the skyscraper is flexible. You might not notice this until a strong wind-storm sets up large-scale vibrations in the first bending mode. Then the motions will make you ill, or at a minimum cause fatigue. The motions certainly cause damage to the building, notably in the loss of windows which can crack or fall, and in long-term fatigue life reduction. Among potential remedies for building sway, the most common today is the passive or active mass concept. In fact, our own Hancock Tower in Boston has two 300-ton masses near the top floor, that damp out vibrations caused by wind.

In these final three lab sessions you will study in detail a physical structure with a similar dynamic response, creating a linear model from first principles and using Simulink to characterize it, synthesizing an active control system design based on the model, and testing your controller on the actual device. The teaching staff will act as consultants.

The Experimental Plant

Figure 1 shows the experimental system that you will work with. The “building” consists of a block of metal atop a pair of steel side-plates with low lateral stiffness so that the mass can sway from side-to-side. The passive damping elements, along with the actuator and motion sensors are mounted on the steel plate. The moving mass is a steel shaft, mounted in “frictionless” air bearings. A wire spring couples the moving mass to the building.

At each end of the sliding shaft is a voice-coil actuator/sensor. Figure 2 shows a more detailed view of one side of the plant. One of the voice-coils is at the left side with one of the air-bearings. The wire-spring is at the right of the figure.

- The air-bearings are connected to a high pressure air supply. The shaft is suspended on a cushion of air, providing an almost frictionless suspension.
- The voice-coils are Lorentz force actuators, similar to loudspeakers, and produce a force proportional to the current flowing. At the same time they produce a back-emf voltage that is proportional to the velocity of the coil. They are energy conserving transducers (as we discussed in class) and so

$$F = K_{vc} \times i$$

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<sup>1</sup>April 10, 2008

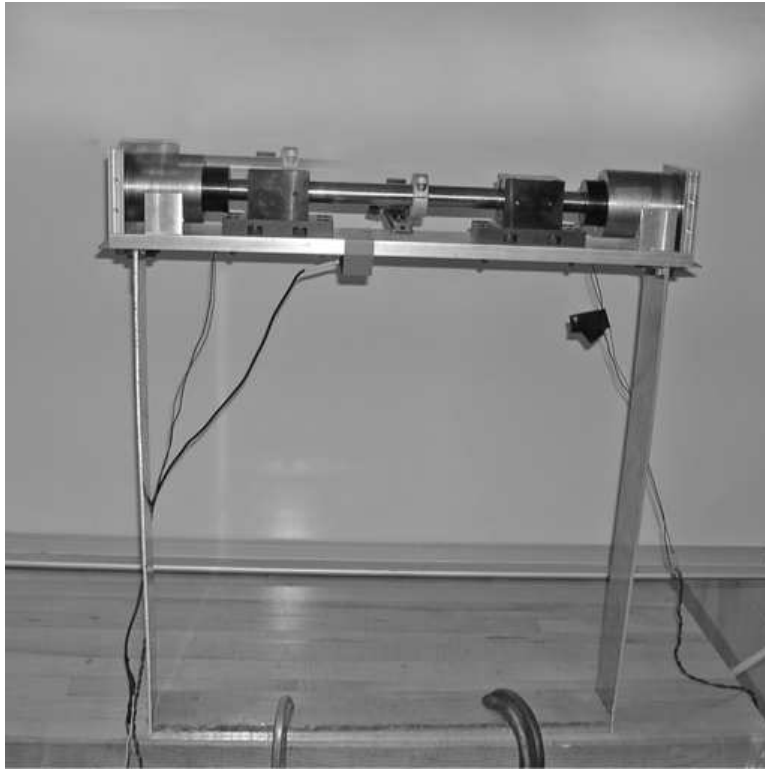


Figure 1: The experimental system

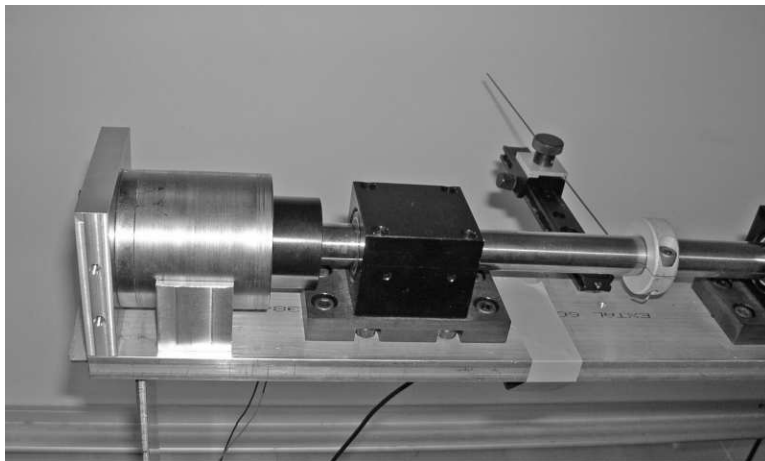


Figure 2: Detailed view of the left-side of the building.

$$v_b = K_c \times v$$

where  $F$  is the force produced,  $i$  is the current,  $v_b$  is the back-emf,  $v$  is the velocity, and  $K_{vc}$  is a constant. The value of  $K_{vc} = 7.1$  N/amp (or V-s/m). Thus the voice coil may be used as an actuator (by supplying current), or as a velocity sensor, by monitoring the voltage  $v_b$ . In this case we use one voice-coil as an actuator and a second one to monitor the relative velocity between the building and the sliding shaft.

The voice-coil consists of a copper coil, wound on an aluminum formed that slides in a strong magnetic field. As it moves, eddy-currents are set up in the former (similar to what happens in the lab rotating plant) leading to an inherent viscous drag as the coil moves.

- The wire spring provides a restoring force proportional to the displacement. Its length can be adjusted to provide a variable stiffness.
- In addition an accelerometer is attached to the building to sense its motion. The gain of the accelerometer is  $K_a = 0.453$  v-s<sup>2</sup>/m.

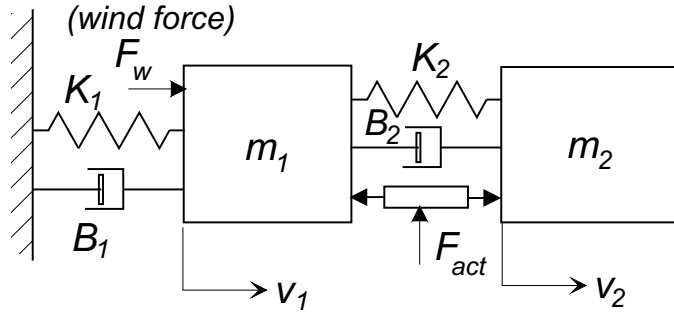


Figure 3: Lumped parameter model of the experimental plant.

Figure 3 shows a simple lumped parameter model of the system. In this model

$m_1$  is the lumped mass of the building.

$B_1$  represents the energy dissipation as the building moves.

$K_1$  is the lateral stiffness of the building structure.

$m_2$  is the mass of the sliding element (shaft and voice-coil formers).

$B_2$  is the viscous friction coefficient describing the eddy-current losses in the voice coils.

$K_2$  is the stiffness of the wire spring.

Two transfer functions for this system

$$\frac{V_1(s)}{F_w(s)} \quad \text{and} \quad \frac{V_1(s)}{F_{act}(s)}$$

are provided in the Appendix. You may need to generate additional transfer functions during the course of the project.

**Project Schedule** This is a three week project:

**Week 1:** Create a simplified model of the open-loop system, using the attached notes and data on the physical properties of the plant. You will write this model in state-space form, and use Matlab to (numerically) convert it to a transfer function.

**Week 2:** Employing MATLAB, design an active damping system built on the PID controller you have used for the flywheel plant. You will model your system in Simulink, and test out your controller in simulation.

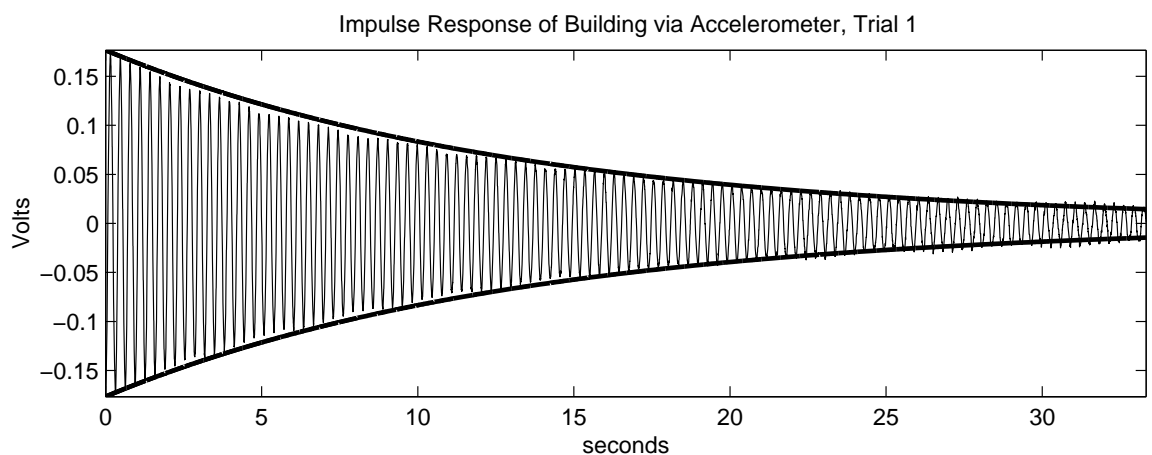
**Week 3:** Test your controller on the real plant; prepare and turn in a report detailing your model, the controller design, and your results. Be sure to document the process and rationale for your controller design!

**This Week's Goals:** In this first week you will:

- Be introduced to the experimental model building and the actuators and sensors that you will have available.
- Use experimental results to estimate the basic system parameters.
- Use MATLAB to determine some basic properties and responses of your model, and your model results against experimental measurements.

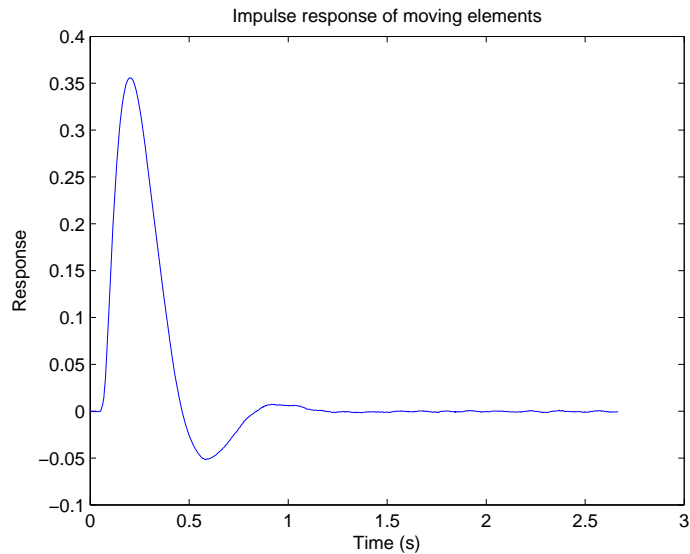
Steps:

1. Make a model of the “building”, consisting of  $m_1$ ,  $B_1$  and  $K_1$ .
2. The course staff removed the sliding components ( $m_2$ ,  $B_2$ ,  $K_2$ ) from the building and did some experiments and found:
  - (i) The building weighs 50.13 N.
  - (ii) The building was displaced a distance of 1 cm and released. The response was measured by an accelerometer and found to be highly oscillatory:



The response is contained in the MATLAB file `BuildingResponse.m` in the course locker. Download and use this data to estimate  $B_1$  and  $K_1$ .

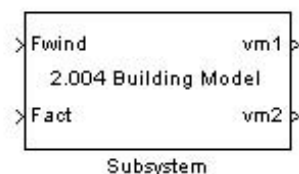
3. The sliding shaft with the two voice coil formers and the collet for attaching the wire spring were weighed and found to have a weight of 8.53 N.
4. Make a model of the sliding components  $m_2$ ,  $B_2$ ,  $K_2$ , assuming that the base on which they sit is fixed, ie  $v_{m_1} = 0$ .
5. The response of the sliding components was measured, and was found to be:



The raw data from these measurements is contained in the MATLAB file `DamperResponse.m`. Use the data in this file to determine  $B_2$  and  $K_2$ .

6. Once you have all of the parameters enter them into the model of the system and plot the impulse response of  $v_{m_1}$  to the wind force  $F_w(t)$ . Also make a pole/zero plot of this system. Your lab instructor may suggest some other methods to verify your model.

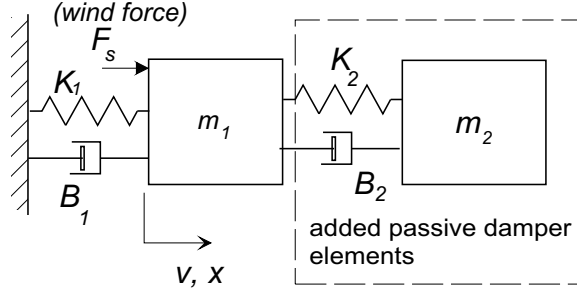
**Tools:** You have available to you the two data files, Matlab, and Simulink (as well as all other software on the lab computers). For Simulink we have created a special block for this project.



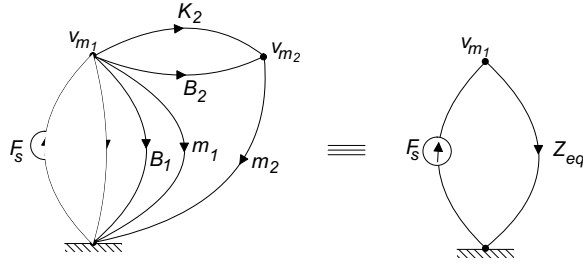
This block has two inputs ( $F_w$  and  $F_{act}$ ) and two outputs ( $v_{m1_1}$  and  $v_{m1_1}$ ), and effectively simulates the four transfer functions relating these inputs and outputs. It takes the six parameter values (which must be named `m1`, `m2`, `B1`, `B2`, `K1`, `K2`) from the MATLAB workspace. In order to use this block you must therefore define all of these values. This block is located in the course locker in the file `BasicModel.mdl`.

## Appendix: Transfer Function Generation

(a) For the system representing the effect of wind load on the building



The impedance graph below may be reduced to a single impedance as shown:



The required transfer function is

$$G(s) = \frac{V(s)}{F(s)} = Z_{eq}$$

It is more convenient to work with admittances (since there are a lot of parallel elements):

$$\begin{aligned} Y_{eq} &= \frac{1}{Z_{eq}} = Y_{m_1} + Y_{B_1} + Y_{K_1} + \frac{(Y_{K_2} + Y_{B_2})Y_{m_2}}{Y_{K_2} + Y_{B_2} + Y_{m_2}} \\ &= \frac{(Y_{m_1} + Y_{B_1} + Y_{K_1})(Y_{K_2} + Y_{B_2} + Y_{m_2}) + (Y_{K_2} + Y_{B_2})Y_{m_2}}{Y_{K_2} + Y_{B_2} + Y_{m_2}} \end{aligned}$$

Then

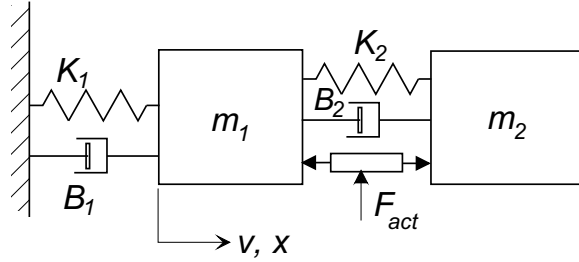
$$\begin{aligned} G(s) &= \frac{1}{Y_{eq}} \\ &= \frac{Y_{K_2} + Y_{B_2} + Y_{m_2}}{(Y_{m_1} + Y_{B_1} + Y_{K_1})(Y_{K_2} + Y_{B_2} + Y_{m_2}) + (Y_{K_2} + Y_{B_2})Y_{m_2}} \\ &= \frac{m_2 s + B_2 + K_2/s}{(m_1 s + B_1 + K_1/s)(K_2/s + B_2 + m_2 s) + (K_2/s + B_2)m_2 s} \\ &= \frac{m_2 s^3 + B_2 s^2 + K_2 s}{a_4 s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0} \end{aligned}$$



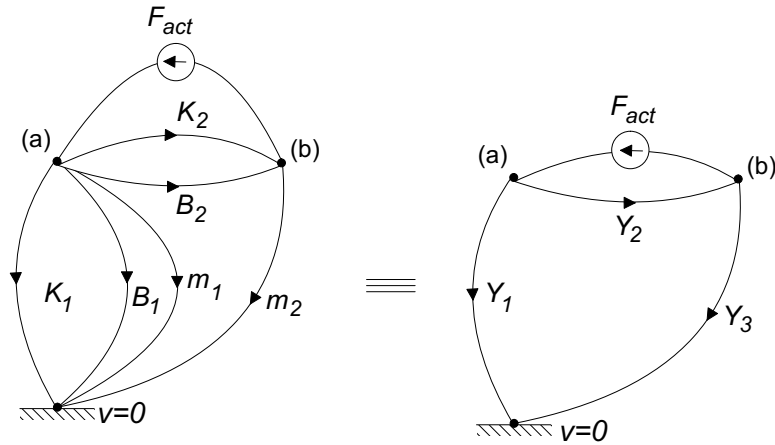
where

$$\begin{aligned}
 a_4 &= m_1 m_2 \\
 a_3 &= (m_1 + m_2) B_2 + m_2 B_1 \\
 a_2 &= (m_1 + m_2) K_2 + m_2 K_1 + B_1 B_2 \\
 a_1 &= K_1 B_2 + K_2 B_1 \\
 a_0 &= K_1 K_2
 \end{aligned}$$

(b) For the transfer function relating the building motion to the actuator force:



Reduce the system graph to a reduced impedance graph as shown below:



where

$$\begin{aligned}
 Y_1 &= \frac{1}{Z_1} = m_1 s + B_1 + \frac{K_1}{s} \\
 Y_2 &= \frac{1}{Z_2} = B_2 + \frac{K_2}{s} \\
 Y_3 &= \frac{1}{Z_3} = m_2 s
 \end{aligned}$$

Use node equations:

$$\begin{aligned}
 \text{At node (a)} \quad F_{Z_1} + F_{Z_2} &= F_{act} \\
 \text{At node (b)} \quad F_{Z_2} - F_{Z_3} &= F_{act}
 \end{aligned}$$

Substitute admittances

$$\begin{aligned} v_a Y_1 + (v_a - v_b) Y_2 &= F_{act} \\ (v_a - v_b) Y_2 - v_b Y_3 &= F_{act} \end{aligned}$$

and express in matrix form

$$\begin{bmatrix} Y_1 + Y_2 & -Y_2 \\ Y_2 & -(Y_2 + Y_3) \end{bmatrix} \begin{bmatrix} v_a \\ v_b \end{bmatrix} = \begin{bmatrix} F_{act} \\ F_{act} \end{bmatrix}$$

Use Cramer's Rule to solve for  $v_a$ :

$$\begin{aligned} v_a &= \frac{\begin{vmatrix} F_{act} & -Y_2 \\ F_{act} & -(Y_2 + Y_3) \end{vmatrix}}{\begin{vmatrix} Y_1 + Y_2 & -Y_2 \\ Y_2 & -(Y_2 + Y_3) \end{vmatrix}} \\ &= \frac{Y_3 F_{act}}{Y_1 Y_2 + Y_1 Y_3 + Y_2 Y_3} \end{aligned}$$

Substitution for the admittances gives

$$G(s) = \frac{v_a(s)}{F_{act}(s)} = \frac{m_2 s^3}{a_4 s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0}$$

$$\begin{aligned} \text{where } a_4 &= m_1 m_2 \\ a_3 &= (m_1 + m_2) B_2 + m_2 B_1 \\ a_2 &= (m_1 + m_2) K_2 + m_2 K_1 + B_1 B_2 \\ a_1 &= K_1 B_2 + K_2 B_1 \\ a_0 &= K_1 K_2 \end{aligned}$$

and we note that the denominator is the same as in (a) above.