2.004 Dynamics and Control II
Spring 2008

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

DEPARTMENT OF MECHANICAL ENGINEERING

2.004 *Dynamics and Control II*
Spring Term 2008

Problem Set 3
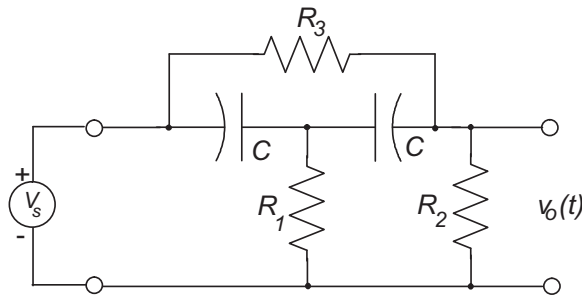
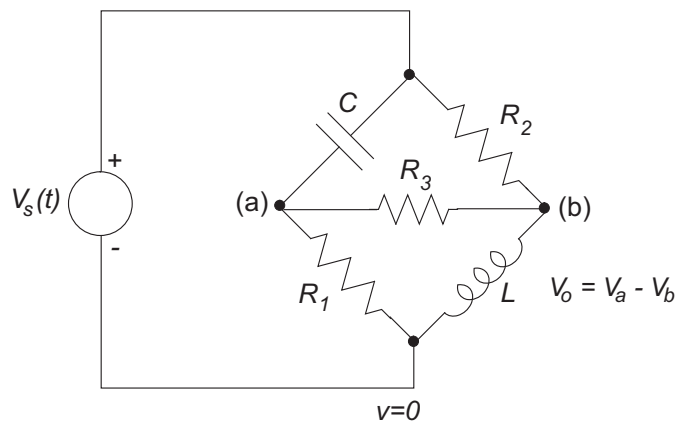Assigned: Feb. 22, 2008   Due: Feb. 29, 2008

Reading:

- Nise. Sec. 2.4.

Problem 1:   In class we looked briefly at a "bridged-T" electrical filter. Use mesh current methods to find the transfer function $H(s) = V_o(s)/V_s(s)$ of the filter shown below:.

Problem 2:   A "wheatstone bridge" is used in many sensors in control and measurement engineering. For example the elements in the bridge shown below might be sensitive to strain or chemicals in the environment.

A voltage source is connected across the bridge, and the output V_o is taken as the voltage difference between the nodes (a) and (b) as shown, that is
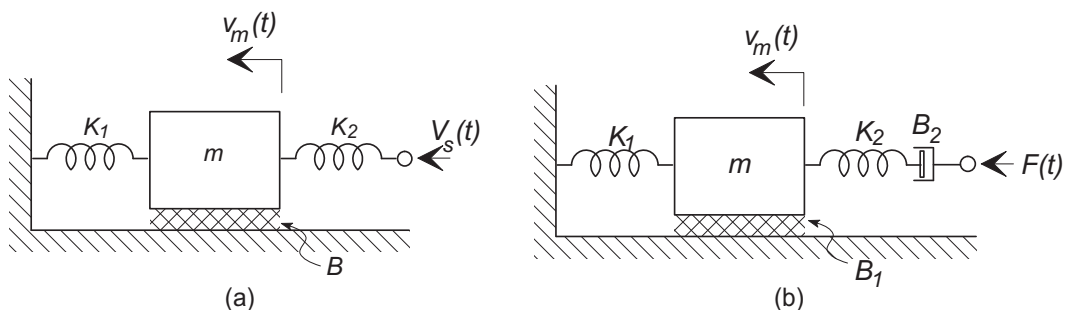
$$V_o = V_a - V_b.$$

Your task is to derive the transfer function relating $V_o(s)$ to the source voltage $V_s(s)$. However, you should use a method based on KCL and node equations that we have not covered in class (but is mentioned in Sec. 2.4 of Nise.) We will guide you through the method as follows.

(a) You will find it easier to deal with admittances in this problem. Draw an impedance graph representing the system and label the passive branches as admittances $(Y_1 \dots Y_5)$.

(b) With assumed currents in the branches, use KCL to write node equations for the nodes labelled (a) and (b).

(c) Realize that the voltage at the top and bottom nodes are known, and substitute for the branch currents in terms of the admittances and node voltages.

(d) Solve your equations for V_a and V_b, and form a transfer function relating $V_o(s)$ to $V_s(s)$ in terma of the admittances.

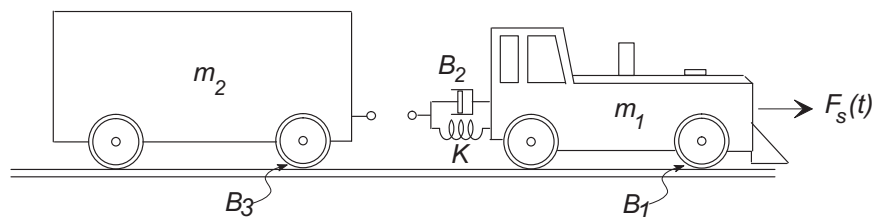**(e)** Then substitute in the component values, and tidy up your transfer function.

Note: A warning in advance - the algebraic substitutions can get really messy. You may use a symbolic software package, such as Maple, or the MATLAB Symbolic Toolbox, to help you with parts (c) through (e). Most of the credit will be given for parts (a) through (d).

Problem 3: Use impedance methods (of your choice) to find transfer functions relating the velocity of the mass m to the velocity source V_s in (a), and to the force source $F(t)$ in (b) below:

If any of the system elements do not show up in your transfer function, explain why.

Problem 4: A small railroad locomotive is sketched below uncoupled from a single car. The locomotive acts a force source $F_s(t)$ and has mass m_1 and viscous rolling resistance B_1. The coupling unit is modeled as a parallel spring K and damper B_2. The railroad car has mass m_2 and viscous rolling drag coefficient B_3.

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(a) Represent the locomotive (uncoupled) as a Thévenin equivalent model, and determine the equivalent velocity source and output (series) impedance.

(b) Use the Thévenin source model to draw an impedance graph of the system when the car is coupled to the locomotive.

Problem 5: Power transmission systems are designed for maximum efficiency, that is they must deliver maximum power to the load. Impedance matching of the load to the source is an important design criterion in both mechanical and electrical systems.

Suppose an electronic audio amplifier is modeled as a Thévenin source with a *resistive* output impedance $Z_{out} = R_o$. The amplifier is connected to a loudspeaker that exhibits a resistive characteristic R, and the acoustic power generated is proportional to the power dissipated in R.

(a) Write an expression for the acoustic power output from the speaker in terms of the Thévenin source parameters and the resistance R.

(b) Prove that the power transferred to the load is a maximum when $R = R_o$. (This is known as the *maximum power transfer* theorem.)