2.003SC Engineering Dynamics
Quiz 1

Directions: This exam is closed book. You are allowed one sheet of notes, front and back. No laptops or electronic communication devices are allowed in the exam. This includes cell phones.

It is strongly recommended that you show your work in symbolic terms first before you substitute in numbers. This makes it more likely that we can award partial credit, when numerical errors crop up.

There is a total of 100 points on this quiz. You have 1.5 hours from when you start the quiz in which to work.

1. *(10 points total)* For this problem, write the letter of the correct diagram for part 1(a) and 1(b) in your solution book.

   The disk of radius r rolls to the left without slipping and with constant angular velocity ω.

   (a) *(5 points)* Which figure shows the correct direction for the velocity, $\mathbf{v}_A$, of point A?
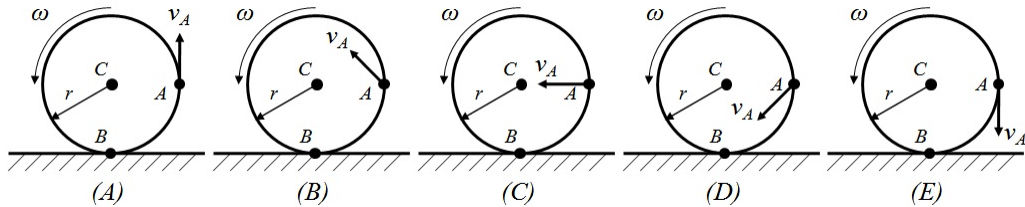

   (b) *(5 points)* Which figure shows the correct direction for the acceleration, $\mathbf{a}_A$, of point A?
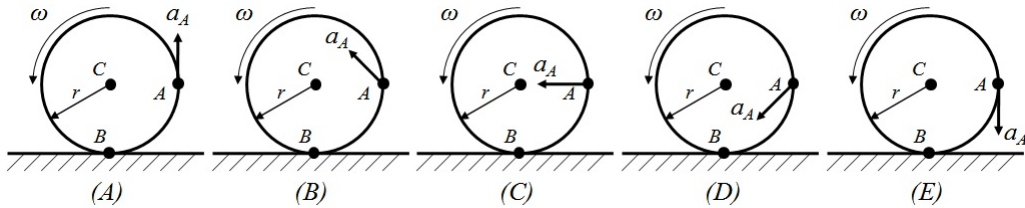
   Solution:

   (a) B

   (b) C

2. *(10 points)* The diagram shows a robotic arm which rotates with respect to the inertial frame O_{xyz} with angular velocity $\omega_1 \hat{k}$. A second arm is attached at A to the first arm and rotates with angular velocity $\omega_2 \hat{k}$ with respect to the first arm. For the particular instant in time shown in the figure below, the velocity of point A with respect to the O_{xyz} frame is given by $\mathbf{v}_{A/O} = R_A \omega_1 \hat{j}$. At that instant, the second arm is at right angles to the first arm as shown.
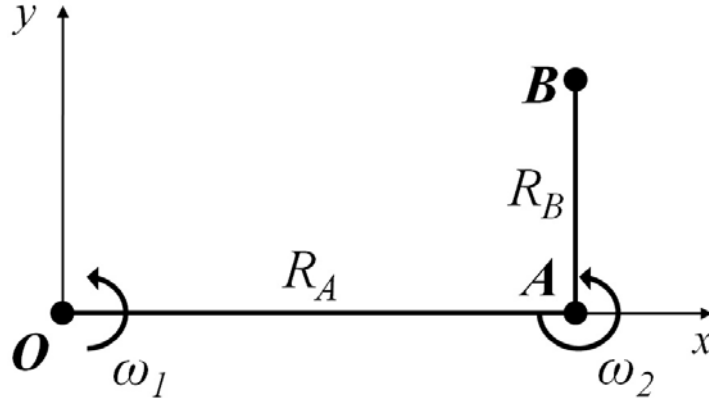

Find $\mathbf{v}_{B/O}$, the velocity of point B with respect to the inertial frame. Choose from the possibilities given below:

(A) $R_A \omega_1 \hat{j} - R_B \omega_2 \hat{i}$

(B) $R_A \omega_1 \hat{j} - R_B \left(\omega_1 + \omega_2 \right) \hat{i}$

(C) $-R_B \left(\omega_1 + \omega_2 \right) \hat{i}$

(D) $R_A \omega_1 \hat{j} + R_B \left(\omega_1 + \omega_2 \right) \hat{i}$

Solution:

$$v_{B/O} = v_{A/O} + v_{B/A},$$

$$v_{A/O} = R_A \omega_1 \hat{j},$$

$$v_{B/A} = \left(\frac{\partial \Omega_{B/A}}{\partial t} \right)\Bigg|_{\omega_{/O}=0}^{\;\;0} + \vec{\omega}_{/O} \times \vec{r}_{B/A}$$

$$= (\omega_1 + \omega_2) k \times R_b j$$

$$= -R_b (\omega_1 + \omega_2) \hat{i}$$

$$v_{B/O} = R_A \omega_1 \hat{j} - (\omega_1 + \omega_2) R_b \hat{i}$$

3. *(15 points total)* The figure shows a girl on a skateboard. For the purpose of this problem, the girl and skateboard may be treated as a point mass, M. Gravity acts. The shape of the curve traversed by the girl is $y = \frac{1}{4}x^2$. Assume no friction.

   At position A, $x_A = y_A = 0$ and the velocity of the girl is $\mathbf{v}_A$.

$$y = \tfrac{1}{4}\,x^2$$
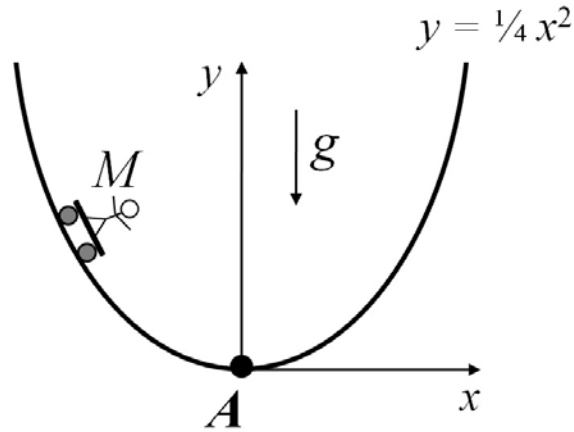
   (a) *(5 points)* Draw a free body diagram of the point mass at position A.

   (b) *(5 points)* Determine an expression for the magnitude and direction of $\mathbf{a}_n$, the acceleration of the mass normal to the curve at point A.

   (c) *(5 points)* Determine an expression for the normal force exerted on the skateboard, and calculate the normal force acting at point A when $M = 50\,\text{kg}$ and $v_A = 10\,\text{m/s}$.

Solution:

(a) Free body diagram of skateboard at $\mathbf{A}$:
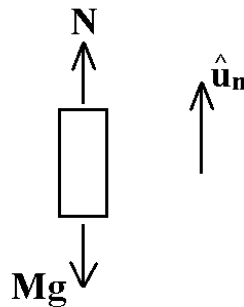
(b) Find normal acceleration at $\mathbf{A}$:

$$\sum F_{A,normal} = (N - Mg)\hat{u}_n = Ma_n\hat{u}_n$$

$$a_n = \frac{v_A^2}{\rho_A}, \qquad \rho_A = \frac{[1 + (\frac{dy}{dx})^2]^{\frac{3}{2}}}{\frac{d^2y}{dx^2}}\bigg|_{x_A=0}$$

3

$$a_n = \frac{v_A^2}{\rho_A} = \frac{10 \text{ m/s}^2}{2 \text{ m}},$$
$$a_n = 50 \ m/s^2 \hat{u}_n,$$

upwards at **A**.

$$y_A = \frac{1}{4} x_A^2 = 0,$$
$$\frac{dy}{dx} = \frac{x_A}{2} = 0,$$
$$\frac{d^2 y}{dx^2}\bigg|_{x_A} = \frac{1}{2},$$
$$\therefore \rho_A = \frac{1}{1/2} = 2 \text{ m}.$$

(c) from b).

$$N = Ma_n + Mg = 50 \text{ kg}(50\frac{m}{s^2} + 9.81\frac{m}{s^2}) = 50 \times 59.81 \text{ Newtons}.$$

$$\boxed{N \approx 2990 \text{ Newtons}}$$

4. *(25 points total)* Two carts are connected by a massless cable, which does not stretch. For the purposes of this problem the carts may be treated as particles. Gravity is present. Ignore friction.

Assume that the questions in this problem apply only to the period of time prior to the mass, M_1, impacting the pulley wheel.
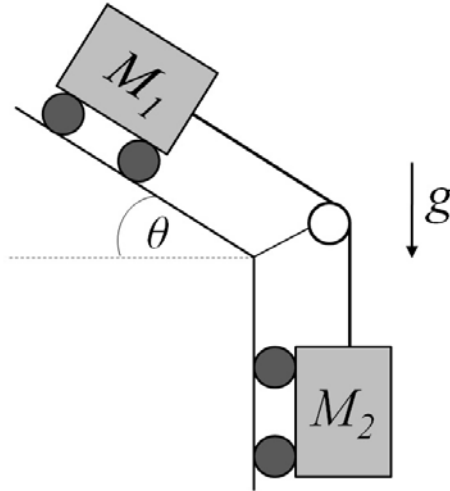
(a) *(5 points)* How many independent coordinates are necessary to completely describe the motion of this system?

(b) *(10 points)* Draw a free diagram for each of the carts and designate the necessary coordinates.

(c) *(10 points)* Find an expression for the acceleration of mass M_1.

Solution:

(a) How many degrees of freedom (DOFs)?
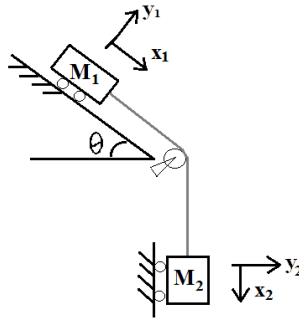
$$\# \text{ DOFs} = 3 \cdot 2 - C$$
$$C = 5 \text{ constraints}$$
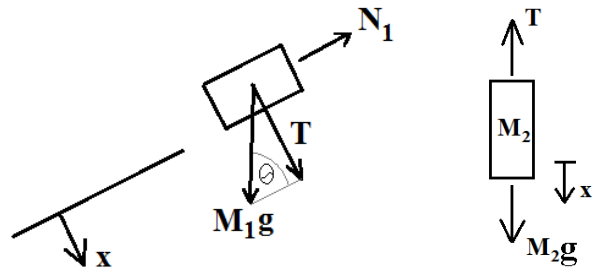$$\ddot{y}_1 = \ddot{y}_2 = \ddot{z}_1 = \ddot{z}_2 = 0$$
and
$$x_1 = x_2 = x$$
$$\#\text{DOF} = 6 - 5 = 1$$

One independent coordinate, x, is required.

(b) Free body diagrams:

(c)

$$\left.\begin{array}{l} F_{M_1,x} = T + M_1 g sin\theta = M_1\ddot{x} \\[2mm] F_{M_2,x} = -T + M_2 g = M_2\ddot{x} \end{array}\right\}$$ Eliminate T, solve for $\ddot{x}$.

$$\boxed{\ddot{x} = \frac{M_1 g sin\theta + M_2 g}{M_1 + M_2}}$$

5. *(40 points total)* The figures below show a top view and side view of a rotating arm attached at a right angle to the axle at D. The axle is supported by two bearings at points A and C. A pulley is fixed to the axle at point B.

A cord is wrapped around the pulley and is used to apply a constant horizontal force, Q, to the pulley. The radius of the pulley is R. The length of the rotating arm is L. The pulley, axle, and arm are considered to be massless. There is a concentrated mass, M, attached to the end of the arm. Gravity is present. The distance between points C and D is b.


Top view

Side view
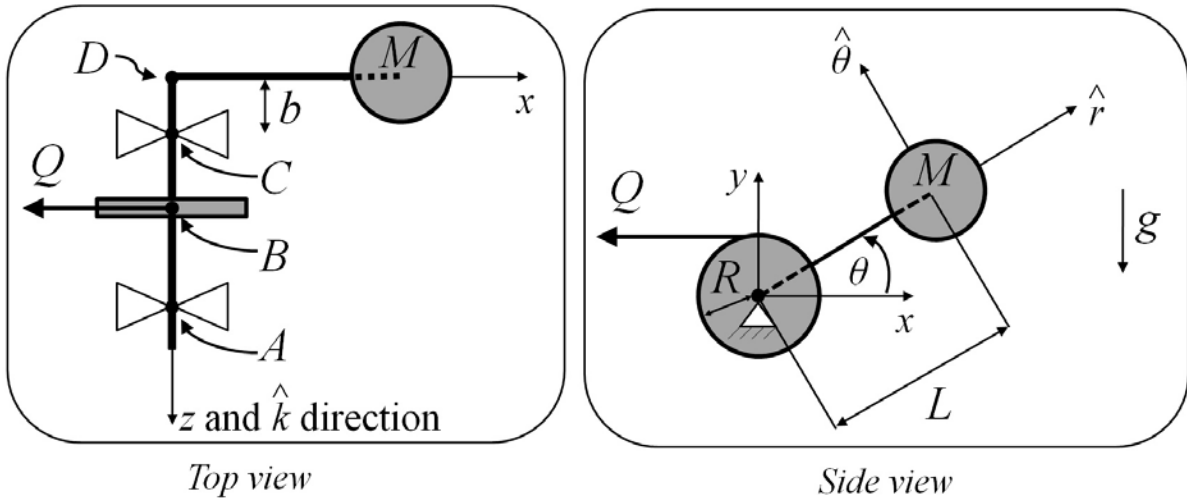
(a) *(10 points)* Find the angular momentum, $\mathbf{h}_{/C}$, of the mass M with respect to the point C in terms of the angular velocity $\dot{\theta}$.

(b) *(10 points)* Take the time derivative of $\mathbf{h}_{/C}$. The result will be in the form:

$$\frac{d}{dt}\mathbf{h}_{/C} = [\text{fill in}]\hat{r} + [\text{fill in}]\hat{\theta} + [\text{fill in}]\hat{k}$$

Fill in the blanks for all three terms.

(c) *(15 points)* Using the results of part (b), find an equation of motion that relates the angular acceleration, $\ddot{\theta}$, to the applied external torques. In other words, find the equation of motion for the system.

(d) *(5 points)* One of the terms of $\frac{d}{dt}\mathbf{h}_{/C}$ in (b) is in the $\hat{\theta}$ direction. Describe the force that creates the torque associated with this term. Said another way, explain the purpose of this force in the mechanical system.
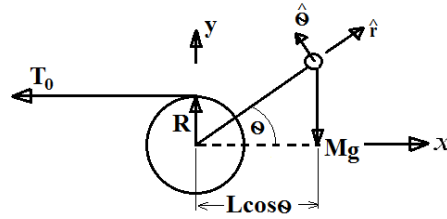
Solution:
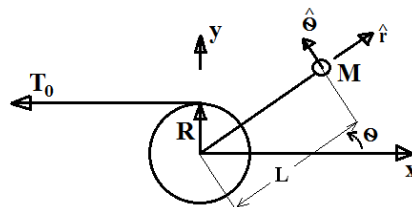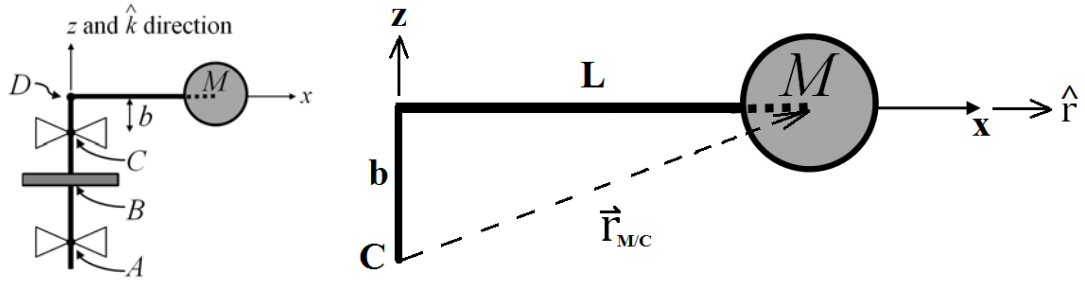
z and $\hat{k}$ direction

(a)

Find $\vec{h}_{/C} = \vec{r}_{M/C} \times \vec{P}_{M/O}$

$$= (L\hat{r} + b\hat{k}) \times ML\dot{\theta}\hat{\theta}$$

$$\boxed{\vec{h}_{/c} = ML^2\dot{\theta}\hat{k} - MLb\dot{\theta}\hat{r}}$$

(b)

$$\boxed{\frac{d}{dt}(\vec{h}_{/c}) = -MLb\ddot{\theta}\hat{r} - MLb\dot{\theta}^2\hat{\theta} + ML^2\ddot{\theta}\hat{k}}$$

(c) Free body diagram:

$$\tau_{/c} = \frac{d\vec{h}_{/c}}{dt} + \vec{v}_{c/O} \times \vec{P} = \frac{dh_{/c}}{dt}, \quad \vec{v}_{c/O} = 0$$

In the $\hat{k}$ direction, $\boxed{\tau = +T_0 R - MgL\cos\theta = ML^2\ddot{\theta}}$

(d) The force $-ML\dot{\Theta}^2\hat{r}$ is required to make a mass travel in a circle. It is the mass times the centripetal acceleration. This force is associated with a torque at C in the $-\hat{\theta}$ direction.

$$\tau_\theta = \vec{r} \times \vec{F} = (L\hat{r} + b\hat{k}) \times (-ML\dot{\theta}^2\hat{r})$$

$$= \boxed{-MLb\dot{\theta}^2\hat{\theta}}$$

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