MatLab Programming –
Eigenvalue Problems and Mechanical Vibration

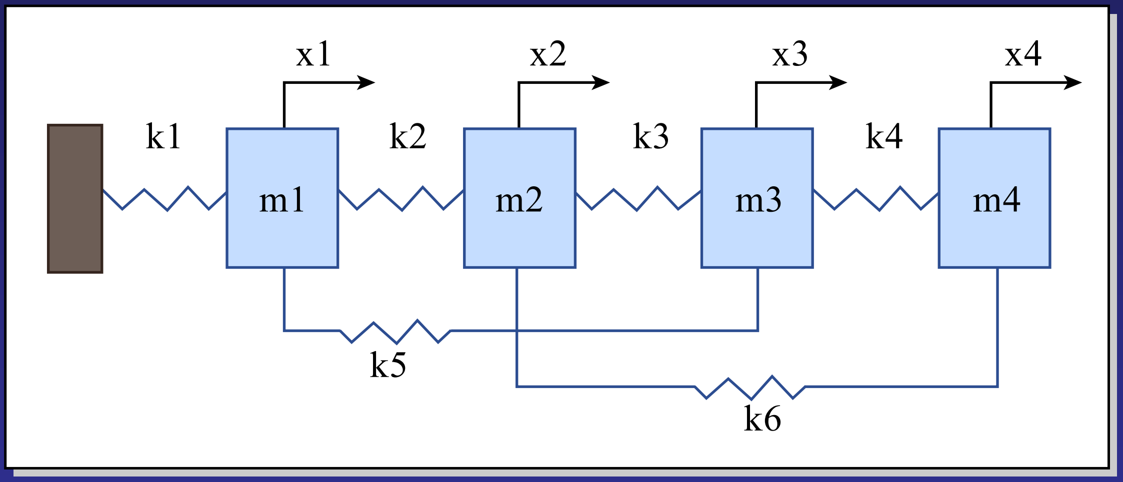

Figure by MIT OCW.

A Coupled Mass Vibration Problem

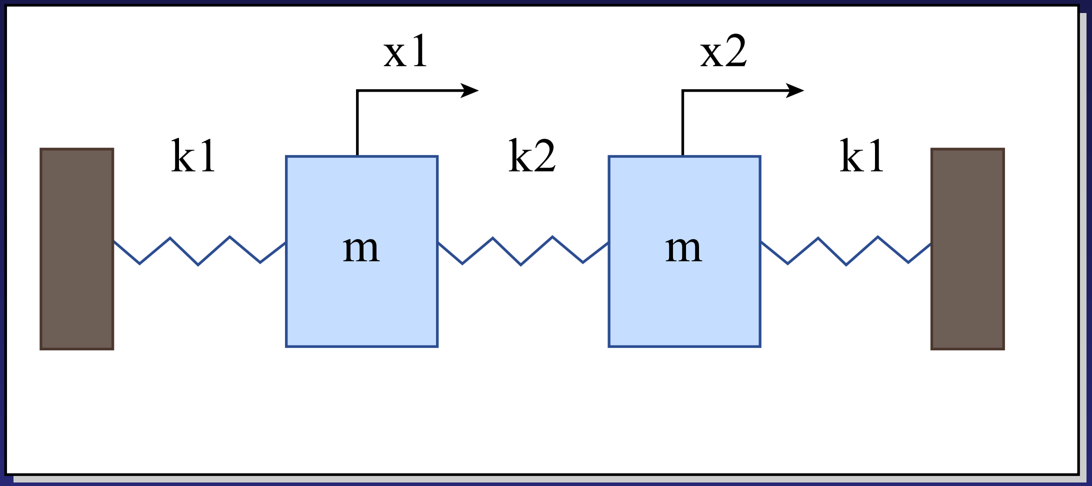

Figure by MIT OCW.

EOM:

$$m\ddot{x}_1 + k_1 x_1 - k_2(x_2 - x_1) = 0$$

$$m\ddot{x}_2 + k_1 x_2 + k_2(x_2 - x_1) = 0$$

Vibration Solutions – harmonic response

Trial solution:

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} \cos(\omega t + \varphi)$$

Matrix representation of EOM:

$$\begin{bmatrix} -\omega^2 + (k_1 + k_2)/m & -k_2/m \\ -k_2/m & -\omega^2 + (k_1 + k_2)/m \end{bmatrix} \cdot \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = 0$$

Vibrational Frequencies and Mode Shapes

Characteristic Equation (Determinant = 0):

$$k_1 + k_2 - m\omega^2 = \pm k_2$$

$$\omega_1^2 = \frac{k_1}{m} \qquad \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}_1 = A_1 \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\omega_2^2 = \frac{k_1 + 2k_2}{m} \qquad \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}_2 = A_2 \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

Vibrations as a general class of "Eigenvalue Problems"

Recast EOM:

$$\begin{bmatrix} -\omega^2 + (k_1 + k_2)/m & -k_2/m \\ -k_2/m & -\omega^2 + (k_1 + k_2)/m \end{bmatrix} \cdot \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = 0$$

As:

$$\begin{bmatrix} (k_1 + k_2)/m & -k_2/m \\ -k_2/m & (k_1 + k_2)/m \end{bmatrix} \cdot \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} \omega^2 & 0 \\ 0 & \omega^2 \end{bmatrix} \cdot \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$$

$$\begin{bmatrix} (k_1 + k_2)/m & -k_2/m \\ -k_2/m & (k_1 + k_2)/m \end{bmatrix} \cdot \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \omega^2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$$

$$A \cdot x = \lambda I \cdot x$$

$$(A - \lambda I) \cdot x = 0$$

Eigenvalue equation, Eigenvalues, Eigenvectors

Eigenvalue equation:

$$A \cdot x = \lambda x \quad (A - \lambda I) \cdot x = 0$$

Eigenvalues (angular frequencies of the vibration):

$$\lambda = \omega^2$$

Eigenvectors (mode shape of the vibration):

$$x = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$$

Solving Eigenvalue Problem in MatLab

Look at the problem numerically:

$$m = 1kg \quad k_1 = 1N/m \quad k_2 = 2N/m$$

Simple m-file:

```
m=1;
k1=1;
k2=2;
A=[(k1+k2)/m -k2/m; -k2/m (k1+k2)/m]
[X,L]=eig(A);
X
L
```

MatLab output of simple vibration problem

eigenvector 1 eigenvector 2

X =

-0.7071	-0.7071
-0.7071	0.7071

L =

eigenvalue 1

eigenvalue 2

1.0000	0
0	5.0000

Ok, we get the same results as solving the characteristics equation… so what is the big deal?