18.965 Fall 2004
Homework 4

Due Monday 10/19/04

Exercise 1. Proof that the evaluation map

$$ev : C^k(M, \mathbb{R}^n) \to \mathbb{R}^n$$

is C^1 and that it is a submersion.

Exercise 2. Prove that there is an immersion of $T^2 \setminus \{pt\}$ into $\mathbb{R}^2$. Prove that there is an immersion of $T^n \setminus \{pt\}$ into $\mathbb{R}^n$. (This is Exercise 2 pg 27 of Hirsch's "Differential Topology.")

Exercise 3. An s-fold point of a map $f : M \to \mathbb{R}^n$ is a point $x \in M$ so that there are s distinct points $x = x_1, \ldots x_s$ so that

$$f(x_1) = \ldots = f(x_s).$$

Let M and N be manifolds whose dimensions satisfy

$$s + 1/s < n/\dim M < s/s - 1$$

Show that there is a residual set in $C^k(M, \mathbb{R}^n)$ so the set of $s+1$-fold points is empty and the set of s-fold points is a smooth submanifold of dimension $ms-(s-1)n$. (This is derived from Exercise 7 pg 27 of Hirsch's "Differential Topology.")