18.701 Problem Set 8

This assignment is due Monday, November 8

1. Chapter 6, Exercise 11.1. *(operations of S_3 on set of 4)*

2. Chapter 7, Exercise 5.12. *(class equations of S_6 and A_6)*

3. Chapter 7, Exercise 5.11a,b. *(class equations of A_4 and A_5)*

4. Chapter 7, Exercise 2.15 *(expanded)*

(a) Let $F = \mathbb{F}_3$ and let $G = SL_2(F)$. Determine the centralizers and the orders of the conjugacy classes of the elements

$$\begin{pmatrix} 1 & 1 \\ & 1 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} & -1 \\ 1 & \end{pmatrix}.$$

(b) By considering the center of G, prove that G contains no conjugacy class of order 8 or 12.

(c) The vector space F^2 contains four subspaces of dimension 1, and G operates on the set of these subspaces. Determine the kernel and image of the corresponding permutation representation $\varphi : G \to S_4$.

(d) Verify the class equation (7.2.10) of G.

5. Chapter 6, Exercise M.4. *(hypercube)*

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