18.600: Lecture 31

Strong law of large numbers and Jensen's inequality

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Outline

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A story about Pedro

Strong law of large numbers

Jensen's inequality

Pedro's hopes and dreams

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- Compute $E[R_1] = .53 \times 1.15 + .47 \times .85 = 1.009$.
- Then $E[T_{120}] = 1.009^{120} \approx 2.93$. And $E[T_{1200}] = 1.009^{1200} \approx 46808.9$

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- Let's do some simulations.

Logarithmic point of view

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- This means that, when n is large, S_n is *usually* a very negative value, which means T_n is *usually* very close to zero (even though its expectation is very large).
- Bad news for Pedro's grandchildren. After 100 years, the portfolio is probably in bad shape. But what if Pedro takes an even longer view? Will T_n converge to zero with probability one as n gets large? Or will T_n perhaps always *eventually* rebound?

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- Recall: **weak law of large numbers** states that for all $\epsilon > 0$ we have $\lim_{n\to\infty} P\{|A_n - \mu| > \epsilon\} = 0$.
- The **strong law of large numbers** states that with probability one $\lim_{n\to\infty} A_n = \mu$.
- It is called "strong" because it implies the weak law of large numbers. But it takes a bit of thought to see why this is the case.

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- So $\lim_{n\to\infty} P\{|A_n - \mu| > \epsilon\} \leq \lim_{n\to\infty} P\{Y_\epsilon \geq n\} = 0$.
- If the right limit is zero for each ϵ (strong law) then the left limit is zero for each ϵ (weak law).

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- Expand $(X_1 + \ldots + X_n)^4$. Five kinds of terms: $X_iX_jX_kX_l$ and $X_iX_jX_k^2$ and $X_iX_j^3$ and $X_i^2X_j^2$ and X_i^4.

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- The first three terms all have expectation zero. There are $\binom{n}{2}$ of the fourth type and n of the last type, each equal to at most K. So $E[A_n^4] \leq n^{-4}\left(6\binom{n}{2} + n\right)K$.

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- Thus $E[\sum_{n=1}^{\infty} A_n^4] = \sum_{n=1}^{\infty} E[A_n^4] < \infty$. So $\sum_{n=1}^{\infty} A_n^4 < \infty$ (and hence $A_n \to 0$) with probability 1.

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- **Proof:** Let $L(x) = ax + b$ be tangent to graph of g at point $(E[X], g(E[X]))$. Then L lies below g. Observe

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- **Note:** if g is **concave** (which means $-g$ is convex), then $E[g(X)] \leq g(E[X])$.
- If your utility function is concave, then you always prefer a safe investment over a risky investment with the same expected return.

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- He repeats this yearly until fund collapses.
- With high probability Pedro is rich by then.

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- This is a special case of the "principal-agent" problem of economics. How do you ensure that the people you hire genuinely share your interests?

18.600 Probability and Random Variables
Fall 2019